Probabilistic Fuzzy Clustering Algorithm for Fuzzy Rules Decomposition

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Abstract

The Fuzzy C-Means (FCM) clustering algorithm is the best known and the most used method for fuzzy clustering and is generally applied to well defined sets of data. In this work a generalized Probabilistic Fuzzy C-Means (PFCM) algorithm is proposed and applied to fuzzy sets clustering. The methodology presented leads to a fuzzy partition of the fuzzy rules, one for each cluster, which corresponds to a new set of fuzzy sub-systems. When applied to the clustering of a flat fuzzy system the result is a set of decomposed sub-systems that will be conveniently linked into a Parallel Collaborative Structure.

1. Introduction

One of the most popular object data clustering algorithms is the FCM algorithm, proposed by Dunn [1] and extended by Bezdek [2], which can be applied if the objects of interest are represented as points in a multi-dimensional space. FCM relates the concept of object similarity to spatial closeness and finds cluster centers as prototypes. Several examples of application of FCM to real clustering problems have proved the good characteristics of this algorithm with respect to stability and partition quality.

From this method a large variety of clustering techniques was derived with more complex prototypes, which are mainly interesting in data analysis applications. However, the generalization of these techniques to clustering imprecisely or uncertainly data or objects is not yet explored. Recently, fuzzy set theory is more and more frequently used in intelligent systems, because of its simplicity and similarity to human reasoning. The theory has been successfully applied to many fields such as manufacturing, engineering, diagnosis, economics, and others [7]. In this context, a generalization of the previous methods in order to be used in clustering of fuzzy data (or fuzzy numbers) would be a meritorious research.

In this work, a new fuzzy relational clustering algorithm, based on the fuzzy c-means algorithm is proposed to cluster fuzzy data, which is used in the antecedent and the consequents parts of the fuzzy rules. This clustering process divides the fuzzy rules of a Fuzzy System into a set of classes or clusters of fuzzy rules based on similarity. From this new strategy, a flat fuzzy system $f(x)$ can be organized into a hierarchical structure of fuzzy systems [3][6].

We propose a new technique, the Probabilistic Fuzzy Clustering of Fuzzy Rules (FCFR), based on cluster methodology, to decompose a flat fuzzy system $f(x)$ into a set of n fuzzy sub-systems $f_1(\text{x})$, $f_2(x)$, ..., $f_n(x)$, organized in a collaborative structure. Each of these clusters may contain information related with particular aspects of the system $f(x)$. The proposed algorithm allows grouping a set of rules into c subgroups (clusters) of similar rules. It is a generalization of the Probabilistic Clustering Algorithm (FCM), here applied to rules instead of points. With this algorithm, the system obtained from the data is transformed into a new system, organized into several subsystems, in PCS structures [4 - 5].

2. The Probabilistic Clustering Algorithm for Fuzzy Rules

The objective of fuzzy clustering partition is to separate a set of fuzzy rules $\Im=\{R_1, R_2,..., R_M\}$ in c clusters in the antecedent space and e clusters in the consequent space, according to a “similarity” criterion. This process allows finding the optimal clusters centres, V and Z, respectively in the input and output space, the partition matrix, U, of combined input-output partition and the matrix W of scalars values. Each value u_{ijl} represents the membership degree of the l^{th} rule, R_l, belonging to the i^{th} cluster of the input space and j^{th} cluster of the output space. w_{jl} is a value that express the translation of the consequent of the l^{th} rule fuzzy sets in direction of the center of j^{th} the output center of cluster. So, the center of each rule l in the cluster j is θ_i^l, with $\theta_i^l = w_{il}\theta^l$ and is expectable that:

$$\sum_{j=1}^{e} w_{jl} = 1, \quad l = 1,\cdots,M \tag{1}$$

with $w_{jl} \in \mathbb{R}$.

Let $x_k \in S$ be a point covered by one or more fuzzy rules. Naturally, the membership degree of point x_k belonging to $(ij)^{th}$ cluster is:

$$\sum_{i=1}^{c}\sum_{j=1}^{e} u_{ijl} = 1, \quad \forall x_k \in S \tag{2}$$

and the relevance of the rules l in x_k point:

$$\sum_{l=1}^{M} \Re_l\left(x_k\right) = 1, \quad \forall x_k \in S \tag{3}$$

The rule decomposition into $c \times e$ sub-relations will lead to an output fuzzy set decomposition as well. For the Fuzzy Clustering of Fuzzy Rules Algorithm (FCFRA) the objective is to find $U=[u_{ijl}]$, $\boldsymbol{V} = [v_1,\cdots,v_c] \in R^{n\times c}$ and $\boldsymbol{Z} = [z_1,\cdots,z_e] \in R^e$ where:

$$J = \sum_{k=1}^{n}\sum_{l=1}^{M}\sum_{i=1}^{c}\sum_{j=1}^{e} u_{ijl}^m \Re_l^m\left(\boldsymbol{x}_k\right)\left[\left(\boldsymbol{x}_k - \boldsymbol{v}_i\right)^2 + \left(\theta_l w_{jl} - \boldsymbol{z}_j\right)^2\right] \tag{4}$$

is minimized, with a weighting constant $m > 1$, with equation (1), (2) and (3) as constraints.

It can be shown that the following algorithm may lead the triplet (U*,V*,W*) to a minimum. The results can be expressed by the following algorithm:

Probabilistic Fuzzy Clustering algorithms of fuzzy rules – FCFRA

Step 1– For a set of points X={x_1,..., x_n}, with $x_i \in S$, and a set of rules $\Im$={R_1, R_2,..., R_M}, with relevance $\Re_l\left(\boldsymbol{x_k}\right)$, keep c, $2 \le c < np$, and initialize $U(0) \in M_{fcm}$.

Step 2– On the r^{th} iteration, with r = 0, 1, 2, ... , compute the c mean vectors.

$$v_i^{(r)} = \sum_{k=1}^{n}\left(\sum_{l=1}^{M} U_{il}^m \cdot \Re_l^m\left(x_k\right)\cdot x_k\right) \Big/ \sum_{k=1}^{n}\left(\sum_{l=1}^{M} U_{il}^m \cdot \Re_l^m\left(x_k\right)\right) \tag{5}$$

where $U_{il}^m = \sum_{j=1}^{e} u_{ijl}^m$, i =1, 2, ... , e.

Step 3– Compute the new partition matrix $U(r+1)$ using the expression:

$$u_{ijl}^{(r+1)} = 1 \Bigg/ \sum_{r=1}^{c}\sum_{s=1}^{e}\left(\frac{\sum_{k=1}^{n}\Re_l^m\left(\boldsymbol{x}_k\right)\cdot D_{ijlk}}{\sum_{k=1}^{n}\Re_l^m\left(\boldsymbol{x}_k\right)\cdot D_{rslk}}\right)^{\frac{1}{m-1}} \tag{6}$$

where $D_{ijlk} = \left(\boldsymbol{x}_k - \boldsymbol{v}_i\right)^2 + \left(\theta_l w_{jl} - \boldsymbol{z}_j\right)^2$, with $1 \le i \le c, 1 \le l \le M$.

Step 4 – Compute the new partition matrix $W(r+1)$ with the expression:

$$w_{jl}^{(r+1)} = \left(1 - \hat{\theta}_l^T \sum_{r=1}^{e} V_r\right) \Bigg/ \sum_{r=1}^{e}\left(\frac{\bar{U}_{jl}}{\bar{U}_{rl}}\right)^m + \hat{\theta}_l V_j \tag{7}$$

with $\hat{\theta}_l = \theta_l \big/ \left(\theta_l^T \theta_l\right)$ and $\bar{U}_{jl}^m = \sum_{i=1}^{c} u_{ijl}^m$.

Step 5 – Compute z_j with:

$$z_j^{(r+1)} = \sum_{l=1}^{M}\left[U_{jl}^m \cdot \bar{\Re}_l^m w_{jl}\theta_l\right] \Bigg/ \sum_{l=1}^{M}\left[U_{jl}^m \cdot \bar{\Re}_l^m\right] \tag{8}$$

where $\bar{\Re}_l^m = \sum_{k=1}^{n} \Re_l^m\left(x_k\right)$.

Step 6– If $\|U(r+1)-U(r)\| < \varepsilon$ then the process ends. Otherwise let $r = r + 1$ and go to step 2.

More details about this method can be found in [6].

3. Conclusions

In this work, the mathematical fundaments for Possibilistic fuzzy clustering of fuzzy rules were presented. In the FCFRA the relevance concept has a significant importance. Based on this concept, it is possible to make a possibilistic fuzzy clustering algorithm of fuzzy rules, which is naturally a generalization of possibilistic clustering algorithms.

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