Hierarchization Process by Possibilistic Fuzzy Clustering of Fuzzy Rules

Paulo Salgado[1], Manuela Cunha[2] , João Pavão[3] and Getúlio Igrejas[4]

[1] *CITAB – Universidade de Trás-os-Montes e Alto Douro, Vila Real, Portugal*

[2] *Escola Secundária S. Pedro, Vila Real, Portugal*

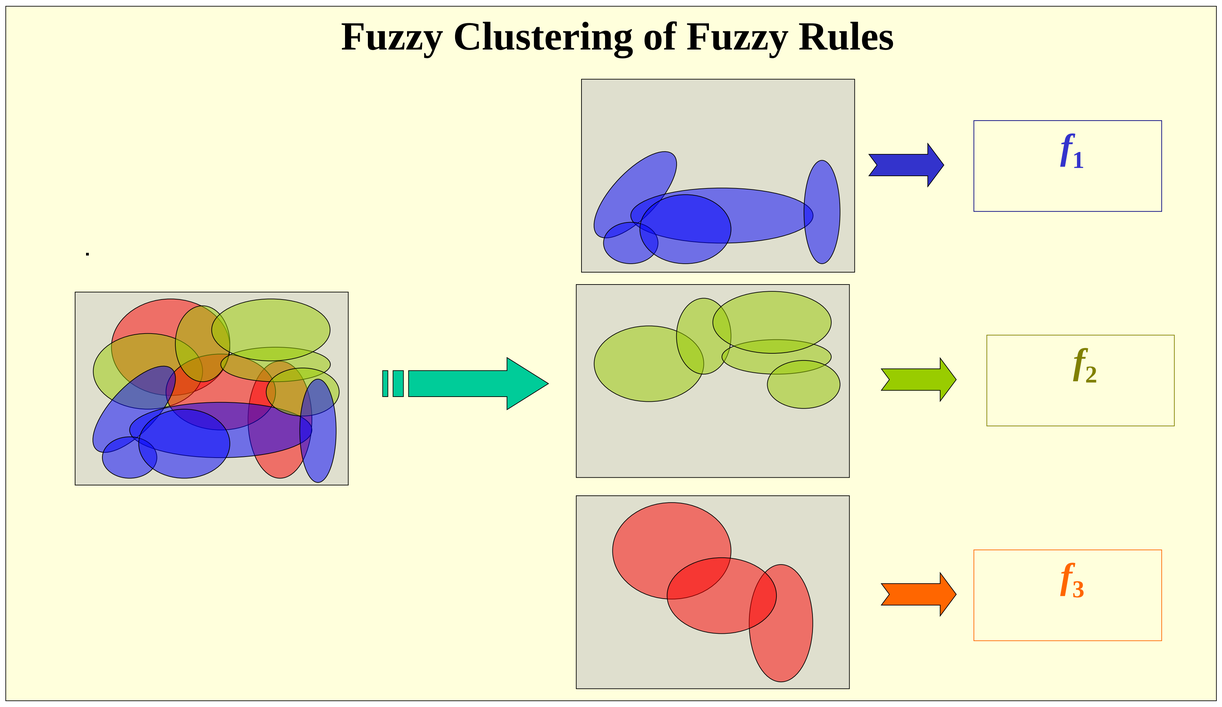
[3] *Depto Eng./ECT – Universidade de Trás-os-Montes e Alto Douro, Vila Real, Portugal*

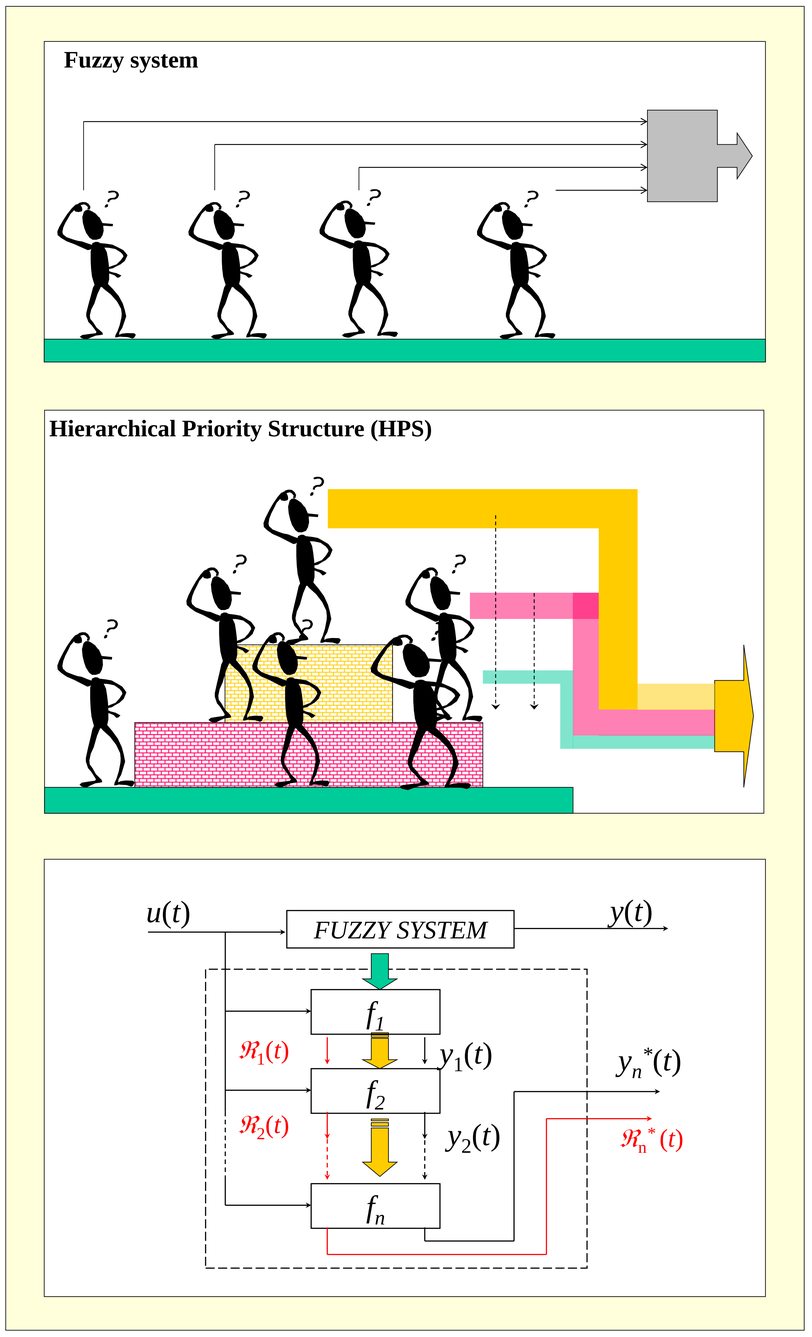
[4]*ESTiG-Instituto Politécnico de Bragança, Portugal*

psal@utad.pt, manuelasalg@gmail.com, jpavao@utad.pt, igrejas@ipb.pt

Abstract

This paper presents a possibilistic fuzzy clustering algorithm that is applied to a multidimensional fuzzy set or fuzzy rules. This method can be used to decompose the fuzzy system into an hierarchical structure. The methodology presented leads to a fuzzy partition of the fuzzy rules, one for each cluster, which corresponds to a new set of fuzzy sub-systems. This technique is tested to organize the fuzzy model into a new and more comprehensive structure.





Fuzzy Clustering algorithms of fuzzy rules – FCAFR

Step 1– Sets: X=$\{x_1,\ldots, x_n\}$ of points; $\Im=\{R_1, \ldots, R_M\}$ of rules with relevance $\Re_l(\boldsymbol{x_k})$. c is the number of clusters. Initialize $U(0)$.

Step 2– On the r^{th} iteration, with $r = 0, 1, 2, \ldots$, compute:

$$v_i^{(r)} = \frac{\sum_{l=1}^{M} u_{il}^m \cdot \sum_{k=1}^{n} \left(\Re_l(x_k)\cdot x_k\right)}{\sum_{l=1}^{M} u_{il}^m \cdot \sum_{k=1}^{n} \Re_l^m(x_k)}$$

Step 3– Compute the new partition matrix $U(r+1)$:

$$u_{il}^{(r+1)} = \frac{1}{1+\left(\sum_{k=1}^{n} \Re_l^m(\boldsymbol{x}_k)\cdot D_{ilk} \Big/ \eta_i\right)^{1/(m-1)}}$$

where $D_{ilk} = \|\boldsymbol{x}_k - \boldsymbol{v}_i\|^2 + \alpha_i \left\|\theta_l - h_i - \sum_{j=1}^{e} w_{ij}\boldsymbol{z}_j(\boldsymbol{x}_k)\right\|^2$.

Step 4 – Compute the clusters offsets, $h^{(r+1)}$:

$$h_i^{(r+1)} = \frac{\sum_{l=1}^{M} u_{il}^m \cdot \left(\bar{\Re}_l\cdot\theta_l - \sum_{j=1}^{e} w_{ij}^{(r)} \sum_{k=1}^{n} \Re_l(x_k)\cdot\boldsymbol{z}_j(\boldsymbol{x}_k)\right)}{\sum_{l=1}^{M} u_{il}^m \cdot \bar{\Re}_l}, \; i=1,\cdots,c \quad (6)$$

Step 5 – Compute the new partition matrix $W(r+1)$:
For each cluster i minimizes:

$$W_i^{(r+1)}: \underset{W_i}{Min} J = \underset{W_i}{Min} \sum_{k=1}^{n}\sum_{l=1}^{M} \left\|\theta_l - h_i^{(r+1)} - \sum_{j=1}^{e} w_{ij}^{(r)}\boldsymbol{z}_j(\boldsymbol{x}_k)\right\|_{Q^{(i)}}^2 \quad (7)$$

where $Q^{(i)}$ is the weigh matrix of the i^{th} cluster problem with non-null elements $Q_{lk}^{(i)} = u_{il}{}^m \Re_l(\boldsymbol{x}_k)$.

Step 6– If $\|U(r+1)-U(r)\| < \varepsilon$ then the process ends. Otherwise let $r = r + 1$ and go to step 2.

Histrogram + PFCFR Segmentation