Modeling an Aggregate Production Planning

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Abstract. This paper aims to present and discuss the benefits that small- and medium-sized companies can get when using mathematical programming to help in the decision-making process regarding aggregate production planning. The aggregate plan is concerned with determining the quantity of a good or a family of goods to be produced and also scheduling its production for the medium-range period, in which the main objective is to meet forecast demand while minimizing costs. To exemplify an aggregate production planning problem, a simplified model of a real company, considering four months, was proposed. Future developments should improve the model to consider other data such as inventory management and costs; raw material storage and logistics; employee costs, contracting and subcontracting and overtime.

Keywords: Aggregate Production Planning · Integer Programming · Mathematical Model.

1 Introduction

Production planning is an essential and complex activity inside a company that requires simultaneous cooperation between everyone responsible for the decision-making process. Usually, production planning can be separated into three different types: short-range plans; medium-range plans, and long-range plans [2].

Short-range plans are highly influenced by job assignments, ordering, job scheduling, and dispatching. Medium-range plans are elaborated to always be aligned with the long-range plans and strategy adopted by the company. Sales and operations planning (S&OP), budgeting, and inventory are some of the pillars that must be considered in every intermediate-range plan. Finally, a long-range plan is most of the time related to new infrastructure, research and development, new products, and facility location and/or capacity.

Lachtermarcher [5] stated that when it comes to solving problems involving short-, medium-, and long-term production planning, it is possible to develop a linear programming (LP) mathematical model and use it to find the optimal solution. Furthermore, it is possible to simulate different production scenarios and amplify the analysis horizon. The objective of this work is to present and develop an integer programming mathematical model for aggregate production planning, which is included in the medium-range plans.

Thus, the paper is organized as follows: the next section describes what an aggregate production plan is, and how integer programming can be used to develop it; the following section presents a simplified case study of a real problem of a given company, where an integer programming model is proposed; section 4 discusses the outputs of the proposed model; Finally, section 5 presents the conclusions and further work.

2 Literature Review

2.1 Aggregate Production Planning

The act of elaborating a strategic plan of operations among all possible horizons, to find the balance between resources and forecasted demand, is known as Sales and Operations Planning (S&OP) [2]. This planning has to consider every aspect possible of the production system, from the supply chain itself to the final customer [2].

One of the most important output information of S&OP is the capability to determine the feasible plans among all possibilities, considering any constraints, concerning the firm itself or the supply chain. The output of S&OP, which is included in the medium-range plan, is called an aggregate plan. As stated by Heizer et. al [2], the aggregate plan is concerned with determining the quantity of a good or a family of goods to be produced and also scheduling its production for the medium-range period. Most of the time, its main objective is to meet forecast demand while minimizing costs.

For an S&OP to be effective and generate a good aggregate plan, it has to have the following four features: a logical way to measure sales and output; a forecast of demand for the period considered; a defined method to determine the costs involved; a model that combines demand, costs, and production capacity so it is possible to develop a schedule [3].

Every aggregate plan must be capable of absorbing demand fluctuations, both caused by internal and external phenomena. In the first case, these phenomena are related to machinery defects, strikes, and product nonconformities, among others. On the other hand, everything that is out of the company's control is classified as an external phenomenon, such as supplier delays, unexpected worldwide events that affect the supply chain, etc. [3].

When it comes to aggregate planning strategies, considering a qualitative perspective, the decision-maker must know how the company wants to absorb any kind of demand variability. Therefore, several questions must be answered, such as: Is the company expanding its inventory capacity? Can the workforce size be variable? Are part-timers, outsourcing, and overtime an option? Should prices or market campaigns be used to influence the demand? [2].

In this context, the act of developing an aggregate plan involves both manipulating and determining a large set of controllable variables such as inventory levels, workforce size, production rates, subcontracting, part-time workers, outsourcing, etc. [3]. This can be solved by graphical techniques or mathematical approaches [2].

The first is a trial-and-error analysis that is easy to use but does not guarantee to find an optimal solution because it is mostly done by plotting one or more graphs, relating forecast demand, production capacity for regular time, costs, and any policy that may apply, for then using basic math to come to conclusions. The second method is more precise since it is purely mathematical based, being able to achieve an optimal solution.

2.2 Mathematical Based Decision Support Systems for Aggregate Production Planning

The currently available computational resources make it possible to develop various decision support systems based on the mathematical approach. These systems, usually based on quantitative models, are flexible and can be used both in academia and companies [6].

In the commercial environment, there are already a lot of computational systems used to plan and determine the production sequence. These programs are referred to as Advanced Planning and Scheduling (APS), and they use a vast variety of techniques to determine production planning, material acquisition, and delivery logistics. Nonetheless, these programs are not worth it for medium and small companies. The reason behind it relies on two main factors: (i) big companies are well organized and most of the time already have the required data in a management information system (MIS) that can be integrated with an APS system; (ii) the cost involved to customize, implement and train people to use an APS system is relatively high [1].

One feasible solution for small and medium companies is to develop their particular mathematical model to support the decision-making process regarding aggregate production planning. Among the large number of techniques that can be applied to solve such a problem, one very common is integer or linear programming. These models can be solved in almost any mathematical software, and therefore it is very accessible for the vast majority of companies [6].

2.3 Integer Programming: a Brief Introduction

Integer programming is a method that uses a set of mathematical linear functions to describe a certain problem and then solve it to obtain an optimal result expressed in integer numbers. It is important to state that the word programming, in this context, does not mean strictly computer programming, although computers are commonly used to solve this type of mathematical model [4].

When defining an integer or linear programming mathematical model, there are always four groups of elements: (i) objective function: a mathematical function that represents the goal; (ii) decision variables: the main entities that influence the goal, that is, variables on which the objective function depends; (iii) constraint functions: a set of equations that limit the solutions within the domain by representing real-world limitations to the decision variables; (iv) nonnegativity constraints: a set of constraints that prevent the decision variables to be negative. Fig. 1 summarizes the concepts presented so far.

3 Modeling an Aggregate Production Planning Using Integer Programming

3.1 Definition of the hypothetical scenario

To exemplify and apply the content discussed so far, it is proposed to model a simplified aggregate production planning of a real company, considering four months, which is approximately 65 working days. During this period, the firm is going to manufacture five different products using three different machines two shifts a day, as summarized in Table 1.

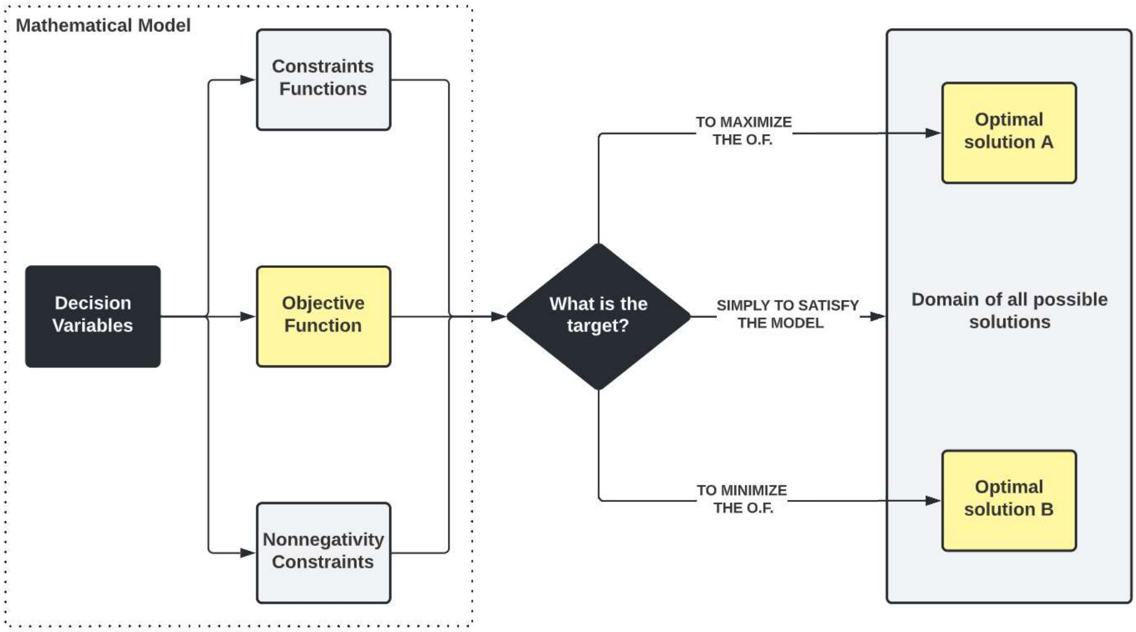

Fig. 1: Flowchart representing an integer or linear programming process.

Table 1: Company's simplified scenario

Working days	**Shifts/day**	**Nº of Products**	**Nº of Machines**
65	2	5	3

Table 2: Production distribution between products and machines

Product	**Machine 01**	**Machine 02**	**Machine 03**
Model 01	10 units/shift	Not produced	18 units/shift
Model 02	15 units/shift	Not produced	21 units/shift
Model 03	20 units/shift	Not produced	18 units/shift
Model 04	Not produced	28 units/shift	33 units/shift
Model 05	Not produced	19 units/shift	21 units/shift

Each one of the three machines is capable of processing a different set of products. Also, each product is manufactured at a different rate, depending on the machine, as shown in Table 2.

The demand for each product for the period is defined and known. There will be necessary to produce 1500 units of product 1; 1300 units of product 2; 700 units of product 3; 950 units of product 4; and 1500 units of product 5, totaling 5950 units. The company disposes of enough raw material to produce a maximum of 7500 products. Along with that, the company knows the profit margin for each product, which is summarized in Table 3.

Table 3: Demand and profit margin per product

Product	**Demand**	**Profit per unit**
Model 01	1500 units	€ 23,00
Model 02	1300 units	€ 22,00
Model 03	700 units	€ 21,00
Model 04	950 units	€ 19,00
Model 05	1500 units	€ 20,00

Since this is an oversimplified model, it does not consider any costs involving inventory management; raw material storage and logistics; employee, contracting and subcontracting; overtime; among others.

3.2 Integer Programming Model

Considering that one product can be manufactured in more than one machine, it is important to define X_{ij} as the quantity of product i processed in machine j. These quantities represent the decision variables of the linear programming model, thus, they have to be an integer number. Furthermore, it is important to notice that there are no X_{12}, X_{22}, X_{32}, X_{41}, and X_{51}, because some machines are not capable of manufacturing certain products (see Table 2).

As in this example, most of the time, when integer programming is used to develop aggregate production planning, the major objective is to maximize profit [2]. Thus, in such a case, the objective function is going to represent the profit itself.

The profit equation must consider every possible aspect: the more accurate the objective function is, the better the model. In a simplified way, the profit is a function of the quantity of goods that are produced times its margin, i.e., the difference between the selling price and the production cost. For the company under consideration, the profit equation can be written as follows in (1).

$$\begin{aligned} P = C_1(X_{11} + X_{13}) + C_2(X_{21} + X_{23}) + C_3(X_{31} + X_{33}) \\ + C_4(X_{42} + X_{43}) + C_5(X_{52} + X_{53}) \end{aligned} \quad (1)$$

Where P is the total profit, given in euros and the C_i is the profit for the product

i, expressed in euros/unit, accordingly to Table 3. In this context, the profit will be maximum by determining the optimal quantities X_{ij} to be produced.

The constraint functions are what limit the solution within the domain. In the model that is being presented here, there are three constraints: the available raw material; the usage of all machines during the four months; the minimum demand for every product.

As it is known, the company has enough raw material only to produce 7500 units of all products combined. In terms of the mathematical model, this can be written as follows:

$$X_{11} + X_{13} + X_{21} + X_{23} + X_{31} + X_{33} + X_{42} + X_{43} + X_{52} + X_{53} \leq M_{max} \tag{2}$$

Where M_{max} is the maximum quantity of goods that can be produced with the available raw material.

As previously mentioned, the second constraint is related to the limited machinery usage during the four months. As stated in Table 2, each machine produces a certain product at a certain rate. For example, both machines 1 and 3 can manufacture product 2, but their production rate is 15 units/shift and 21 units/shift, respectively. Because of this, it is needed to establish one constraint for each of the three machines, as stated below.

$$\frac{X_{11}}{T_{11}} + \frac{X_{21}}{T_{21}} + \frac{X_{31}}{T_{31}} \leq T_1 \tag{3}$$

$$\frac{X_{42}}{T_{42}} + \frac{X_{52}}{T_{52}} \leq T_2 \tag{4}$$

$$\frac{X_{13}}{T_{13}} + \frac{X_{23}}{T_{23}} + \frac{X_{33}}{T_{33}} + \frac{X_{43}}{T_{43}} + \frac{X_{53}}{T_{53}} \leq T_3 \tag{5}$$

Where T_j is the amount of work shifts that machine j can operate at its full capacity, during the 65 working days; T_{ij} is the average quantity of products i manufactured per shift when processed in the machine j, given in units/shift. Each constraint (3), (4) and (5) is measured in number of shifts, since each one is the sum of X_{ij}/T_{ij}, for every product i and machine j.

The third set of constraints is the minimum quantity of each product that has to be produced, i.e., the individual demand, which can be filled by the quantities produced in every machine. Therefore, it can be defined as follows.

$$X_{11} + X_{13} \geq D_1 \tag{6}$$

$$X_{21} + X_{23} \geq D_2 \tag{7}$$

$$X_{31} + X_{33} \geq D_3 \tag{8}$$

$$X_{42} + X_{43} \geq D_4 \tag{9}$$

$$X_{52} + X_{53} \geq D_5 \tag{10}$$

Where D_i is the product's i demand in the considered period, given in units, accordingly to Table 3.

The last mathematical definition that is required to complete the model is the non-negativity constraint - a premise of linear programming - which defines that it is not possible to produce a negative quantity of any product. This can be determined as follows in (11).

$$X_{ij} \in Z | X_{ij} \geq 0 \tag{11}$$

This assumption means that the quantity of product i manufactured in machine j, X_{ij} is greater or equal to zero and, as stated before, must be an integer because products cannot be produced partially, since the company is characterized by a discrete process.

The summary of the aggregate planning model proposed is presented in 3.2. It gathers all the information about the integer programming model discussed.

$$\text{Maximize } P = \sum_{i=1}^{5} \sum_{j=1}^{3} C_i X_{ij}$$

$$\text{s.t.} \quad \sum_{i=1}^{5} \sum_{j=1}^{3} X_{ij} \leq M_{max}$$

$$\frac{X_{11}}{T_{11}} + \frac{X_{21}}{T_{21}} + \frac{X_{31}}{T_{31}} \leq T_1$$

$$\frac{X_{42}}{T_{42}} + \frac{X_{52}}{T_{52}} \leq T_2$$

$$\frac{X_{13}}{T_{13}} + \frac{X_{23}}{T_{23}} + \frac{X_{33}}{T_{33}} + \frac{X_{43}}{T_{43}} + \frac{X_{53}}{T_{53}} \leq T_3$$

$$X_{11} + X_{13} \geq D_1$$

$$X_{21} + X_{23} \geq D_2$$

$$X_{31} + X_{33} \geq D_3$$

$$X_{42} + X_{43} \geq D_4$$

$$X_{52} + X_{53} \geq D_5$$

$$X_{12} = X_{22} = X_{32} = X_{41} = X_{51} = 0$$

$$X_{ij} \in Z | X_{ij} \geq 0$$

4 Results and Discussion

First of all, it is important to state that this model is oversimplified. In reality, much more variables must be taken into consideration, such as inventory management; raw material storage and logistics; employee costs; among others.

When involving up to 8000 variables, an integer programming problem can be solved through Excel using the Solver add-on. Therefore, since the model described in section 3 is relatively simple, it was imported into Excel and the add-on Solver was used. Thus, the results are shown in the Figure 2.

Aggregate Planning					
	Product 1	Product 2	Product 3	Product 4	Product 5
Machine 01:	0	843	1476	-	-
Machine 02:	-	-	-	950	1825
Machine 03:	1948	457	0	0	0
Sum:	1948	1300	1476	950	1825
Demand:	1500	1300	700	950	1500
Surplus:	448	0	776	0	325

Fig. 2: Results of the integer programming model

As can be seen, the maximum profit possible is € 158.950,00, which corresponds to € 37.735.50 a month. All machines are considered to operate almost at their maximum, once their usage is very close to 100%. The only goods that have been manufactured in surplus are products 1, 3, and 5, with 448, 776, and 325 surplus units, respectively. This was made to maximize the usage of every machine because the demand can be satisfied in less than 65 working days; therefore, those surplus items are going to be stored. The fact that only products 1, 3, and 5 are being produced in surplus is because the model identified that in such a way, the profit function is maximized.

5 Conclusion

It can be concluded that the model satisfies the optimization premise, since, after supplying the demand, the model seeks solutions in which the surplus products are more profitable, and then maximizes their production to ensure that no machine is idle. Furthermore, as each machine produces the same product at a different rate, it can be seen from the results that the model allocated most of the production of a given product to the most efficient machine, as it is expected.

Face all that has been presented in this paper, one can assure that integer programming consists of a powerful tool in developing aggregate production planning. No matter the size of the company, the possibility of predicting a vast number of scenarios is fundamental for any firm that wants to prosper and grow [2].

Along with that, it is clear that any small- or medium-sized company is capable of creating and operating its aggregate production planning using integer programming. Although the model shown in this paper is very simple, the importance of good management has been evidenced one more time.

Future developments should improve the objective function and the constraints functions to consider inventory management and costs; raw material storage and logistics; employee costs, contracting and subcontracting; overtime; among others. This will make the model more realistic, and, therefore, the results more reliable.

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