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The use of ore microscopy data for flotation process control by means of a liberation model — A case study

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ABSTRACT

Any realistic description of ore processing dealing with a particulate system requires an adequate characterization of its properties, mainly particle size and grade, because these are the most important variables that influence the performance of a mineral processing separation, such as froth flotation. Experience has shown that an appropriate model for liberation of solid phases must always integrate all available mineralogical data by comminution.

The present work aims to describe a case study for calibration of a liberation model with data obtained from semi-quantitative mineralogical analysis (ore microscopy). For the specific case of random comminution, a limit probability argument is used to justify the introduction of Euler's Beta Law as a well-suited tool for describing the liberation state of a comminuted ore, because it is able to predict the grade distribution of a given iso-size fraction of such a particulate system.

The liberation algorithm is linked with a phenomenological froth flotation model, based on an explicit dependence of the kinetic constants of the rate process (seen as non-wettability parameters) on the ore grade and joint size distribution.

Finally, both models are fitted to time-recovery and time-concentrate grade curves of batch kinetic flotation tests carried out on a copper sulphide ore from Neves-Corvo mine with different feed size distributions, in order to perform a global assessment of the proposed approach. The possibility of adjusting metal recoveries and concentrate grades seems to be an important improvement on flotation kinetic studies. This goal was achieved by the integration of the ore microscopy data with the proposed liberation model.

Describing a Mineral Processing System

Any realistic description of a mineral processing system dealing with a particulate system requires an adequate characterization of its properties, mainly particle size, grade and ore texture, because these are the most important variables that affect the performance of a mineral processing separation such as froth flotation. Analytical approaches have to be complemented by an appropriate model of liberation of solid phases by comminution (Leite, 1991).

Available data, such as from bulk chemical analysis, ore microscopy (paragenetic and textural data), image analysis or separation tests (such as heavy liquid separation or magnetic separation with the Franz separator), needed for characterizing the complexity of a given metallurgical system must always be integrated with a liberation model.

Liberation Model

Liberation is considered the most important operation of mineral processing because it determines the separability of a given ore. Liberation is a dynamic state evolving along comminution time. It is a powerful concept in mineral processing because it makes the particle properties, mainly size, *s*, and grade, *g*, dependent on the method in which the particulate system is affected by the comminution process.

Some authors (Andrews and Mika, 1975; Madureira, 1978; Madureira and Machado Leite, 1982; Madureira et al., 1988; Herbst et al., 1985; Barbery, 1991) felt the need to describe comminution in mineral processing technology, not only as a simple size-reduction process, but as a more complex phenomenon designed to produce liberated particles, and

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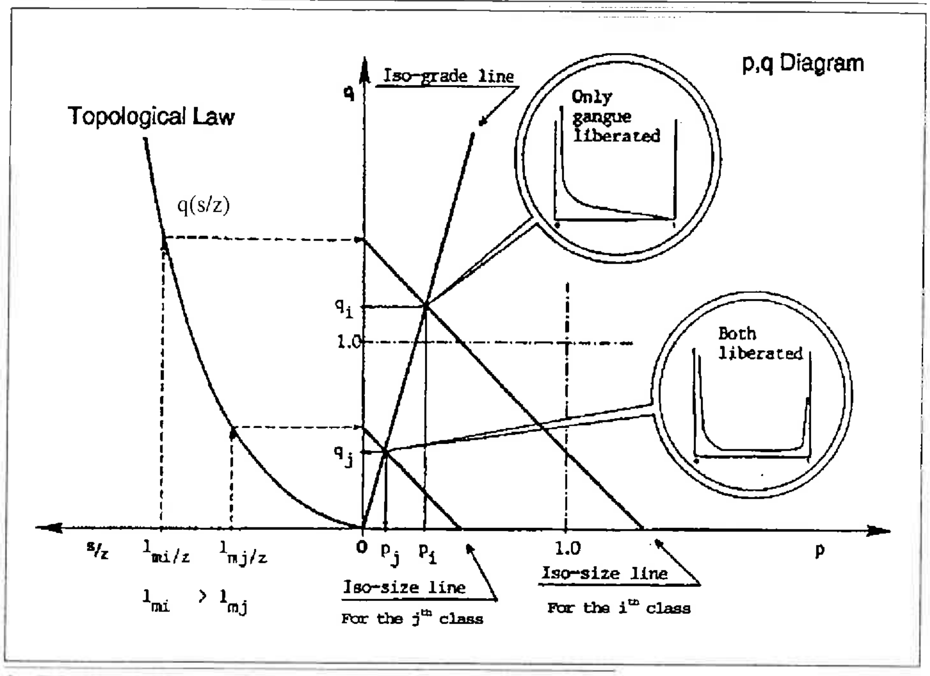
Fig. 3. Topological Law.



Cavalheiro (1984, 1990) proposed for the Topological Law:

$$\Phi\,(s/z) = K\cdot\frac{s}{z}\cdot exp(K\cdot\frac{s}{z})$$

where *(p,q)* area calculated as: $p(s/z) = g_0\cdot\Phi(s/z)$

$q(s/z) = (1-g_0)\cdot\Phi(s/z)$

K is termed the topological constant, because it depends on texture type alone.

In the graphic, l_{m_i}/z represents the mean size of the i^{th} size class, measured in units of the *in situ* grain size of the ore texture, z.

size decreases, to a point where further particle size reduction does not significantly increase liberation.

Liberation Model for Random Comminution

Comminution is said to be random if there is no relationship between the mechanism of fracture and the properties of the parent ore texture and composition. This concept is idealized, however, it is a good starting point for a conceptual discussion of liberation.

Some authors (Madureira and Machado Leite, 1982) have demonstrated that, for the specific case of random comminution, any given ore, characterized by its average grade, is represented in the *(p,q) Diagram* by a straight line passing through the origin. The slope of each one of these *iso-grade lines* is a direct function of the ore average grade, $\bar{g}$:

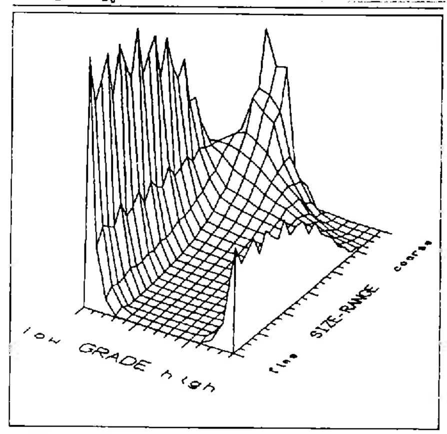
$$slope = \frac{(1-\bar{g})}{\bar{g}} \quad \ldots\ldots\ldots\ldots (6)$$

The authors also showed that each iso-size fraction has a very simple representation in the same *(p,q) Diagram* for the case of random comminution: straight lines with a slope of -1, called *iso-size lines* (Madureira et al., 1988) (Fig. 2).

Cavalheiro (1984, 1990), for the specific case of random comminution, developed a methodology to fit the Incomplete-beta-function ratio to data from ore microscopy and chemical analysis of the products of a Cyclosizer. The methodology developed by this author to obtain the so-called *Topological Law* allowed the calibration of the iso-size lines of the *(p,q) Diagram* to a size scale (for example, a traditional standard sieve series), with mineralogical data (grade histograms obtained by point counting) collected from iso-size fractions of a given ore. The value, z, of the *in situ* grain size of the ore texture before grinding, is back-calculated with the help of the Marquardt non-linear algorithm to minimize the differences between the experimental grade distributions by size and those predicted by the Topological Law.

Figure 3 shows a graphical summary of the procedure proposed by Cavalheiro to improve the Beta Liberation Model. First, for each size class the mean size is computed and placed on the left-hand side of the x-axis of the graphic where the adjusted Topological Law is plotted. This function gives the origin intercept of the iso-size line of the given size class. The values of parameters *p* and *q* are the co-ordinates of the intersection point between the iso-size line and the iso-grade line which represents the ore average grade. With these parameters we are able to calculate the Incomplete-beta-function ratio to obtain the desired grade histogram of the respective size classes.

Fig. 4. Example of a joint size *s* and grade *g* distribution for a given g_0.



As we can see in Figure 3, the liberation of the coarsest size class, l_{mi}/z, plotted on the left hand side of the *s/z*-axis, is represented by an Incomplete Beta Function with p<1 and q>1, indicating that only gangue is liberated. On the other hand, in the finest size class, l_{mi}/z, represented by a Beta Function with p<1 and q<1, both valuable and gangue minerals are liberated.

Fitting the Topological Law to real data, we can estimate, by back calculation, the *in situ* grain size, z. Using this algorithm with a given size distribution, the joint size and grade distribution (Fig. 4) that represents the liberation level of the ore for that size distribution can be computed.

A Case Study — Batch Flotation of a Copper Sulphide Ore from Neves-Corvo Orebody (Portugal)

Step 1 — Experimental Set-up

In order to test the model with real data, an experimental procedure was designed:

- selection of a sample of 250 kg of a biphase complex sulphide, consisting of chalcopyrite in a pyritic matrix;
- optical microscopic analysis of the ore texture in polished section for paragenetic identification and visual estimation of the *in situ* grain size;
- batch grinding tests, performed with two different residence times to obtain one product with a K_{80} = 40 µm (which is the liberation size in the mill) and another product with a K_{80} = 50 µm;
- size classification of these products in a cyclosizer and semi-quantitative mineralogical analysis, using an optical microscope and a point counter to determine the grade histograms of each size class;
- batch flotation tests to determine the time-recovery and the time-concentrate grade curves.

thus be mathematically treated as a bivariate system, represented by the size and grade joint distribution, $f(s,g)$, in which particle *size* and *grade* are time dependent variables.

In his pioneering work, Madureira (1978) pointed out that, "since particle formation by comminution is a complex random process, arising from a multitude of macroscopically different events and unpredictable factors, which suggest that an error propagating mechanism is at work and gives rise to macroscopically different products, a limit probability argument would lead to a Beta form Function."

The further assumption that no single factor, or small group of factors, fundamentally controls the process is consistent with the non-discriminating argument. The multiplicity and equipotency of all these factors as a whole allow us to invoke the limit distribution theorem justifying the use of Euler's Incomplete Beta Function for describing the *Grade Formation Function*, $C(g|s,g_0)$ of a given comminution process, which is a function conditional on daughter particle size s, and original parent particle volume grade g_0:

$$C(g|s,g_0) = C_{p,q} \cdot g^{p-1} \cdot (1-g)^{q-1} \quad \ldots\ldots\ldots (1)$$

with

$$C_{p,q} = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \text{ and } g_0 = \frac{p}{p+q} \quad \ldots\ldots (2)$$

Γ being the factorial function, $\Gamma(x+1)=x.\Gamma(x)$, and p and q parameters that control the shape of Beta function.

According to Korn and Korn (1968), the Incomplete Beta Function is defined as:

$$B_g(p,q) = \int_0^g x^{p-1} \cdot (1-x)^{q-1} dx \quad \ldots\ldots\ldots (3)$$

and the Incomplete-beta-function-ratio is defined as:

$$I_g(p,q) = B_g(p,q)/B(p,q) \quad \ldots\ldots\ldots (4)$$

with

$$B(p,q) = \int_0^1 x^{p-1} \cdot (1-x)^{q-1} dx = \frac{1}{C_{p,q}} \quad \ldots\ldots (5)$$

Thus, the Grade Formation Function is described by the Incomplete-beta-function-ratio integrand and is used as the probability distribution of the volume grade g of iso-size particles. The parameters p and q control the shape of this distribution. Figure 1 shows the *(p,q) Diagram* in which can be seen that the Beta Law is well suited to describe different degrees of liberation, ranging from completely locked to completely liberated. Thus, these two parameters *(p,q)* must be functions of the daughter particle size, s, in order to describe the desired dependence on particle size, because liberation will increase as the particle

Fig. 1 . (p,q) Diagram of Euler's Incomplete Beta Function.

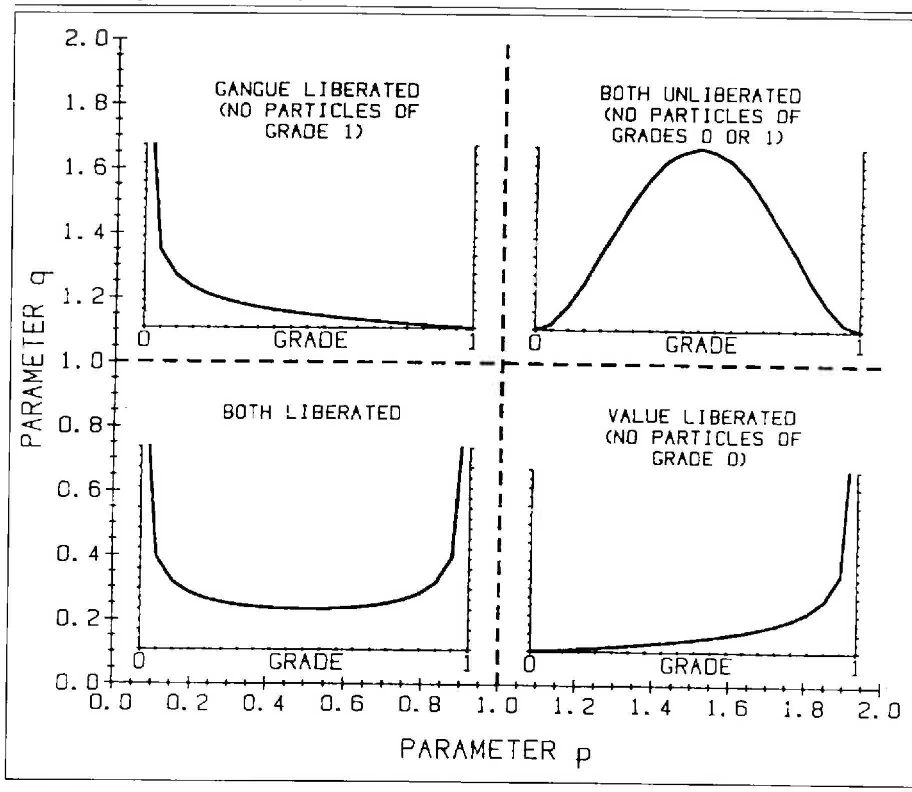
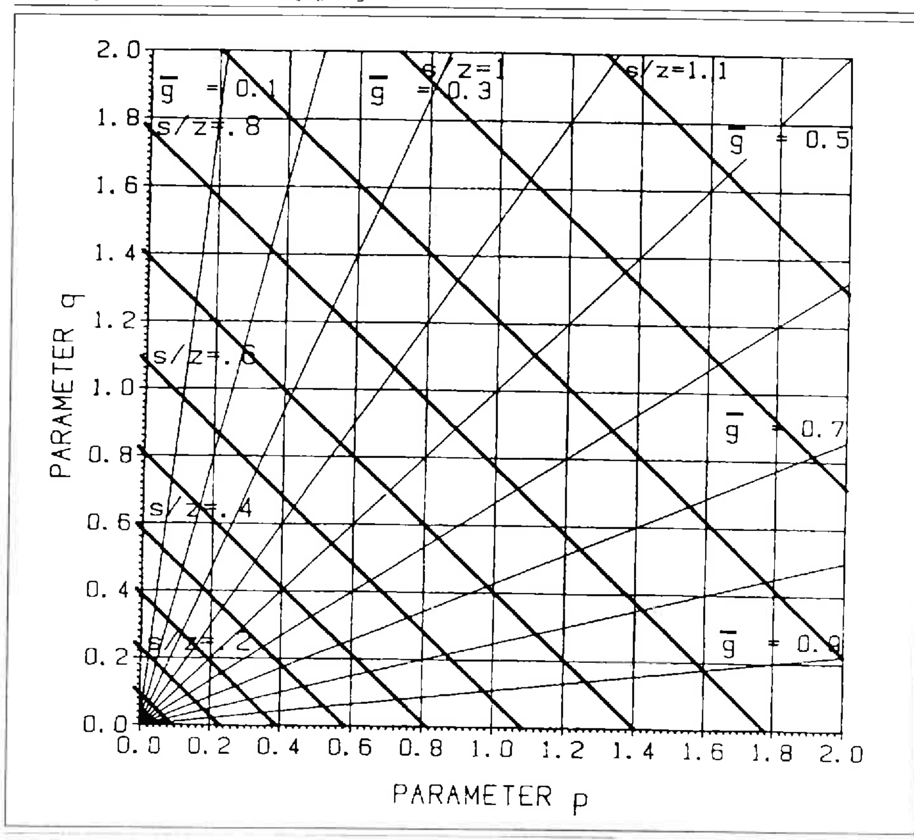


Fig. 2. Iso-grade and Iso-size lines in *(p,q)* diagram.



According to Cavalheiro (1984, 1990), $\bar{g}$ represents the average ore grade, s is the size of broken particles, and z is the *in situ* grain size of the ore texture before grinding. Thus, s/z is the size of broken particles as function of the texture *in situ* grain size.

Step 2 — Fitting the Liberation Model to Mineralogical Data

In Figure 5, grade histograms of the cyclosizer products obtained with mineralogical analysis (Gaspar and Pinto, 1991) are shown. The Beta Liberation Model was fitted to these data with the help of the Marquardt algorithm and a bulk *in situ* mean grain size of 100 μm was estimated.

Figure 6 shows the *Grade Histograms by Size* predicted by the model. The fit is sufficient for simulation purposes, as will be shown in the next section. However, some discrepancies, such as lower liberation values predicted by the Beta Law Model, indicate that the non-discriminating argument does not completely justify the comminution behaviour of the ore in terms of liberation, provided the methodology of mineralogical analysis is correct. From a statistical standpoint, the chi-square goodness of fit test with a rejection region of 5%, shows that the Beta Law is acceptable for locked particles Cpy (10% to 35%), Cpy (60% to 90%) as well as for free gangue — Cpy (< 10%).

Step 3 — Fitting the Model to Experimental Data from Batch Flotation Tests

Because flotation is usually considered a rate process, predicting the joint size and grade distributions of the ore at a given comminution level allows the development of the *distributed rate constant*. This concept has been proposed by authors such as Huber-Panu et al. (1976), Kapur and Mehrotra (1989), and Kelly and Carlson (1991). This method describes the kinetic constants as explicit functions of particle size and grade.

First order kinetics for flotation is represented by:

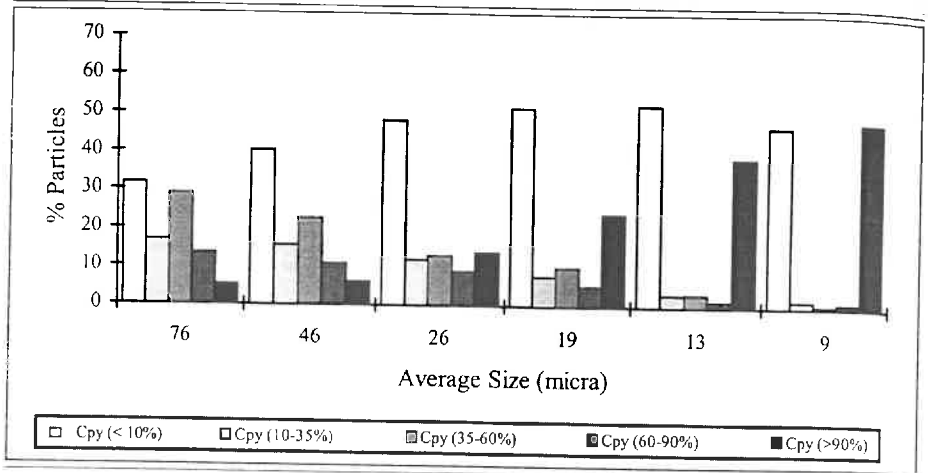
$$\frac{dm_{i,j}(t)}{dt} = -K_{i,j} . m_{i,j}(t) \quad (7)$$

where $m_{i,j}(t)$ represents the mass of particles in the i^{th} size class and j^{th} grade class that float with the flotation rate constant $K_{i,j}$.

This allows the calculation of the flotation rate constant $K_{i,j}$ as a function of size s_i and grade g_j by using a model that includes the effect of particle collision with the bubbles and the adhesion of particles to bubbles after collision (Leite, 1991). We now assume that the rate constant $K_{i,j}$ can be broken up into the product of:

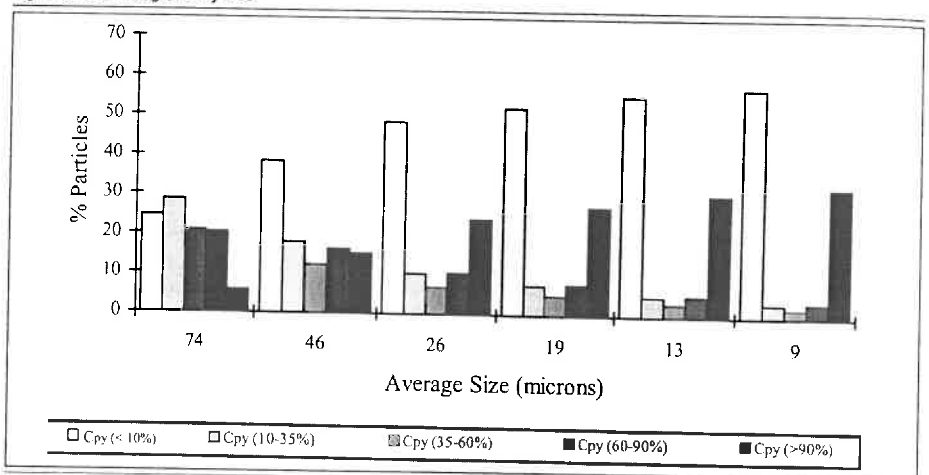
- *probability of collision, Pc_i* (which is a function of the average size s_i of the i^{th} size class)
- *probability of adhesion, Pa_j* after collision, (which is a function of particle flotability and related to the average grade g_j of the j^{th} grade class); and
- a *scale parameter, K_f* allowing for rate constants greater than unity.

Fig. 5. Grade histograms of the cyclosizer products.



Experimental values — grinding product K_{80}=40μm.

Fig. 6. Grade histograms by size.



Beta Law predictions — grinding product K_{80}=40μm.

Thus:

$$K_{i,j} = Pc_i(s_i).Pa_j(g_j).K_f \quad \text{with } Pc_i(s_i), Pa_j(g_j) \in]0,1[\quad (8)$$

For simulation purposes, the probabilities of collision and adhesion are computed with the help of a set of condensed parameters — Q_1, Q_2, Q_3 and K_f — from:

- a Beta formula for the probability of collision, normalized to 48 mesh (297 μm) which is the maximum flotable size:

$$Pc_i(s_i) = \left(\frac{s_i}{0.297}\right)^{Q_1} \cdot \left(1 - \frac{s_i}{0.297}\right)^{Q_2} \quad (9)$$

- a simple exponential formula for the probability of adhesion

$$Pa_j(g_j) = g_j^{Q_3} \quad (10)$$

Starting from a ground ore sample of known size distribution, experimental kinetic flotation tests were performed to obtain the Time-Recovery and Time-Grade curves of the cumulative concentrates.

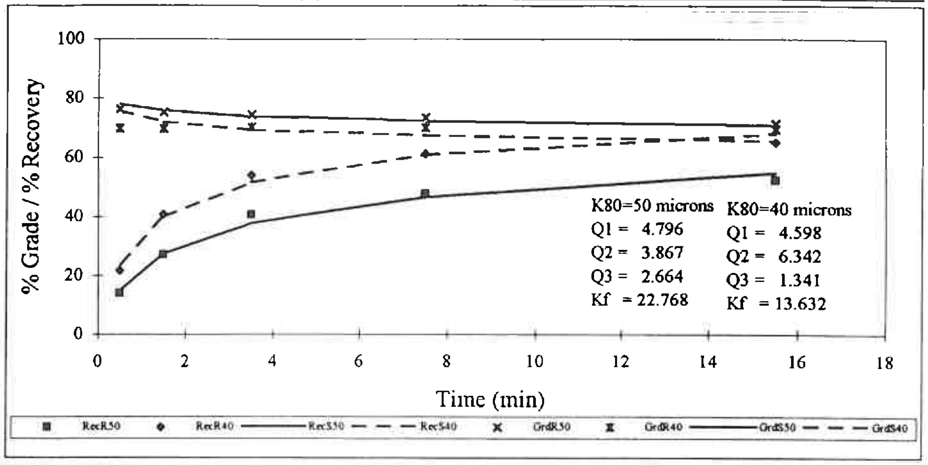
A knowledge of the feed size distribution and grade histograms by size (obtained with optical microscope and point counter) allows the use of the Liberation Beta Model to compute the size and grade joint distribution of the feed, m_{ij}.

Once this distribution is calculated, the flotation kinetic equation can be used to simulate the response of the system for different flotation residence times. This response depends on:

- parameters Q_1 and Q_2 to tune the probability of collision with particle size;
- parameter Q_3 to assess the collector selectivity and tune the probability of adhesion with particle grade; and
- parameter K_f to adjust the time scale.

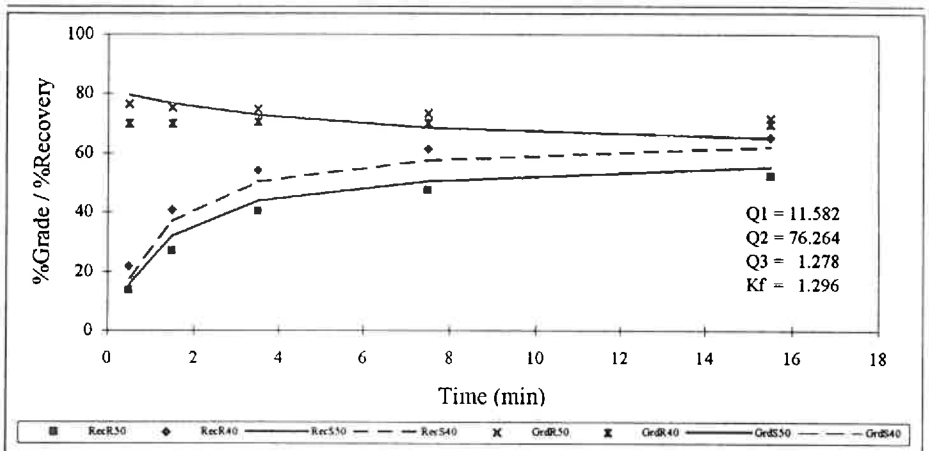
The final output of the model includes, for each time interval, the size and grade joint distributions of the concentrate and tails products, which allows the calculation of the cumulative recoveries, the cumulative grades and the size distributions of the concentrates.

Fig. 7. Fit of liberation and flotation models to batch flotation data (time-recovery and time-grade curves).



RecR50 – Real recovery – K_{80} = 50 µm; RecS50 – Simulated recovery – K_{80} = 50 µm
GrdR40 – Real grade – K_{80} = 40 µm; Grd S40 – Simulated grade – K_{80} = 40 µm

Fig. 8. Fit of flotation and liberation models to batch flotation data. Combined fitting to results from tests with K_{80} of 50 and 40.



RecR50 – Real recovery – K_{80} = 50 µm; RecS50 – Simulated recovery – K_{80} = 50 µm
GrdR40 – Real grade – K_{80} = 40 µm; GrdS40 – Simulated grade – K_{80} = 40 µm

Based on this approach, a fit of both Time-Recovery and Time-Grade curves of the cumulative flotation concentrates obtained in the batch experiments was performed. The results from flotation of two feed products with K_{80} = 50 µm and K_{80} = 40 µm are shown in Figure 7. It should be noted that other traditional kinetic models used in flotation (Fast and Slow, Rectangular Distribution and others) do not allow this type of global fitting because they are not able to predict the concentrate mean grades.

As can be seen, there is excellent agreement between the simulated and experimental grades and recoveries for the flotation of both grinding products tested. It seems that the liberation model has been sufficient at resolving the differences of flotation performance of the two tests.

To test this hypothesis, all recoveries and grades from the two flotation tests were fitted simultaneously with a single set of flotation parameters — Q_1, Q_2, Q_3 and K_f. In fact, the results shown in Figure 8 seem to corroborate the validity of the proposed hypothesis.

The decrease of the goodness of fit shown by the global adjustment (Fig. 8), when compared with the individual adjustments plotted on Figure 7, indicates that in the flotation batch data there is information that was not completely explained by the phenomenological models. The main cause of the difference may be the deviation of comminution from random breakage.

Although extreme care has been taken to ensure the reproducibility of the experiments, small differences in collector dosage, froth discharge and pulp level between the two tests may be reflected in the goodness of fit of the global adjustment.

Conclusions

The results demonstrate that mineral liberation is mostly responsible for differences in the time-recovery curves of the different feed products tested. The Beta Liberation Model was able to make use of the mineralogical data obtained by ore microscopy.

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