A Maple interface for computing variational symmetries in optimal control

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The concept of variational symmetry entered into optimal control in the seventies of the twentieth century. Variational symmetries, which keep an optimal control problem invariant, are very useful in optimal control, but unfortunately their study is not easy, requiring lengthy and cumbersome calculations. Recently there has been an interest in the application of Computer Algebra Systems to the study of control systems, and collections of symbolical tools are being developed to help on the analysis and solution of complex problems. The first computer algebra package for computing the variational symmetries in the calculus of variations, and respective Noether's first integrals, was given by the authors in [1]; then extended to the more general setting of optimal control [2] and, more recently, upgraded in [3] with the introduction of new capacities, by means of several optional parameters, and improvements of efficiency.

Here we provide a graphical user interface to our computer algebra package [3]. This application is named `octool` and was created with `Maplet` technology, the graphical programming language of the `Maple` 10 system. With this interface users can, in a point-and-click environment, interact with all the symbolical tools of the package and deal with concrete problems of optimal control: (i) with the procedure `Symmetry`, to obtain the variational symmetries; (ii) with the procedure `Noether`, to obtain the correspondent conservation laws; and (iii) with the `PMP` (Pontryagin Maximum Principle), to try to obtain the Pontryagin extremals or, alternatively, the equations of the Hamiltonian system, stationary condition or the Hamiltonian. We refer the reader to [2, 3] for a general overview on these Maple procedures. The Maplet `octool` allow us to quickly investigate the problems, without learning all the optional parameters of the Maple procedures [1]–[3]. Moreover, it permits additional algebraic manipulations. The complete Maple package, with the new procedure `octool`, can be freely obtained from `http://www.mat.ua.pt/delfim/maple.htm` together with many practical examples and an online help database for the Maple system.

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[3] Gouveia, P.D.F., Torres, D.F.M., Rocha, E.A.M., Symbolic Computation of Variational Symmetries in Optimal Control, *Control & Cybernetics* (Accepted. To appear, presumably, on No. 3/2006). E-Print: `http://pam.pisharp.org/handle/2052/113`.