MATHEMATICAL INVESTIGATIONS IN EARLY YEARS — FIFTH GRADE STUDENTS

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This poster reports on a learning experience aiming to present and analyze the work produced by 5th grade students in solving a mathematical task with investigative characteristics.

Investigations

— Mathematical tasks, according to their type and nature, can provide different ways of *understanding* or *doing* mathematics[7].

— Mathematical investigations appear as an expression of a non-routine work, referring to complex mathematical processes and involving a strongly problematic activity[5].

— The investigations may provide a divergent activity that encourages students to be curious, to search for alternative strategies, to consider what would happen if certain conditions change or to generalize the situation[4].

— The investigations require a similar work to the one produced by mathematicians: towards a particular situation that must be answered, the students have to ask questions, to make and test conjectures, to justify these conjectures based on mathematical arguments and to validate the results[8], valuing the practice of argumentation[1,6].

The learning experience

— This experience is part of a broader study[7], involving an experienced 5th grade mathematics teacher, aiming to know how teachers integrate investigation tasks in the (current) curriculum development and how they reflect upon their teaching practices.

— The learning experience focused on the work of one lesson by the twenty-five 5th grade students of the teacher, who proposed them to discover, record and validate "interesting relationships" in the Pascal triangle.

O triângulo de Pascal contém números dispostos desta maneira.

Pascal's triangle contains numbers arranged thus.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Encontra relações interessantes no triângulo de Pascal.

Find interesting relationships in triangle.

— The study followed a qualitative and interpretative approach[2,3].

— The **data collection** was supported by (i) teacher's written productions, (ii) students' written productions while solving the tasks, and (iii) field notes recorded by the author in the classroom.

— The **data analysis** process involved a floating approach to the work developed by the students and followed by its systematization[2].

Evidences

— Working in pairs facilitated the communication process of presentation and validation of the conjectures "with the partner". It was often heard comments like:

> *"this can not be, in this line it does not happen", "and if we did ... no it's not possible!", "you may be right, but it does not seem to me", "this is very easy, we must try another discovery", "teacher, which of us is right?"…*

— The pairs recorded a significant number of conjectures: some of them quite obvious, much based on direct observation (like A or E), and others more sophisticated, involving connections between mathematical concepts (like B, C, D or F), but, in all cases, with mathematical meaning to those who had established it[7].

— Generally, the students were able to present and defend their reasoning and productions. Also, to a lesser extent, they were able to listen and understand the opinions of the others. But, as they showed a good general behaviour, the collective validation (or refutation) environment of the conjectures has not been negatively *contaminated* by their postures[1].

Some conjectures presented by the students
Aspects of the collective validation processes

A (not validated)
Manuel

Nas diagonal 2 existem as potencias de Base 1

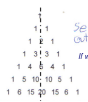
In the diagonal 2, there are the powers with base 1.

Manuel had just begun to explain his reasoning, Telma already argued that it could not be because "1^2 is not equal to 2, is equal to 1, the power base 1 is always equal to 1 because $1^2 = 1 \times 1 = 1$; $1^3 = 1 \times 1 \times 1 = 1$". Realizing his mistake, Manuel did not argue accepting Telma's justification.

E
Berta, Rosa

As diagonais 2 têm a sequência dos números naturais: 1, 2, 3, 4...

The diagonals 2 have the sequence of natural numbers: 1, 2, 3, 4…

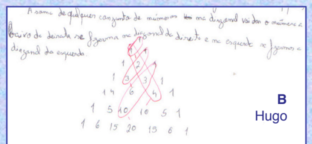


D
Paula, Teresa

Se fizermos um risco no meio de um lado e do outro é igual.

If we make a trace in the middle, it is equal to one side and the other.

The written record is not perfect. The oral explanation of the students — indicating the axis of symmetry and the numbers, "this one and this one, the values are the same on both sides" — was more explicit and contributed to the almost immediate collective validation.



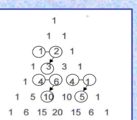
B
Hugo

The sum of any set of numbers on the diagonal line will give the number below on the right if we do on the right diagonal and on the left if we do on the left diagonal.

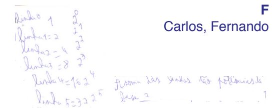
Both discussion and validation of this conjecture were not very easy, because many students did not understand it immediately. Hugo's explanation helped to clarify his statement, using the triangle and noting the set of numbers with a colour...

A soma de os números que vêm por cima dá o número em baixo.

The sum of the numbers that are above gives the number below…



C
Célia, Nuno



F
Carlos, Fernando

The sums of the lines are powers of 2.

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The exploration of relations between numbers of the Pascal triangle allowed the students to follow forms of investigative work, whose characteristics produced a stronger sense and meaning to students' learning[6], developing their ability to communicate, argue and generalize.