

# Classes of model structures for state and parameter identification of vector controlled induction machines

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## Abstract

The purpose of this paper is to present a synthesis of classes of model structures for joint state and parameter identification of vector controlled induction motors for real time and normal operating conditions. Based on its classical model a set of new classes of model structures is discussed and proposed for simultaneous estimation of rotor flux components and electrical parameters.

**Keywords:** model structures, extended Kalman filter, recursive prediction error methods, online identification, induction machines.

## 1. Introduction

The induction motor (IM) is the most widely used motor in high performance electrical drive applications due to its well known advantages. Owing to the large-scale utilization of the IM in industrial drive systems, the effect of parameters variation on the control scheme has been the reason for so much research. In the recent past, a very common way for estimation of the electrical parameters of the IM was achieved by using a recursive least squares method approach based on a linear model structure obtained by linearization of the classical IM model. On the other hand, the determination of rotor flux for the control scheme was treated separately by deterministic procedures or reconstruction and [1] is a good example of this strategy. The recent trend is to implement a solution with joint online estimation of induction motor states, namely rotor fluxes, and electrical parameters, [2-6] being some examples. The extended Kalman filter (EKF) is suitable for joint state and parameter estimation of the nonlinear state-space model structure describing physically the IM. By using this framework, the authors have given new contributions in this context and presented some results in [5-8] which improve the performance of joint state and parameters identification of innovative model structures for use with the EKF and recursive prediction error method (RPEM) based approaches. This paper is a contribution for the purpose of selection of the best model structure according to specific objectives of the identification procedure. The most important aspects are presented when stator, rotor or both reference frames are used for this purpose and new identification methodologies are suggested.

## 2. Classes of model structures

A set of model structures, linear in its parameters as in [1], can be easily obtained from the classical IM model, expressed in its  $dq$  components, by elimination of rotor quantities, namely, flux and currents as well as stator flux. Furthermore, it is necessary to have very slow dynamics where the speed derivative is close to zero. As a result, this kind of linear models are not suitable for transient conditions and, furthermore, they typically need to compute the first derivative of stator voltage besides the first and second derivatives of stator current. In this procedure, if the space phasors are used instead of its orthogonal components, the transfer function (A.1a), in appendix, with complex coefficients is obtained which is referred to a general reference frame, indicated by the upper script  $g$ . These coefficients are function of the four electrical parameters shown in (A.1b) and (A.1c), namely,  $R_s, L_s', \tau_r$  and  $L_M$ , of the per-phase IM model described, for instance, in [9]. From this transfer function, two strategies can be adopted.

The first one consists of its discretization by using, for example, the bilinear transformation and then the parameters of the discrete transfer function are estimated and, finally, these estimated parameters are mapped again to the time-continuous domain, but, in general, this mapping is not bi-univocous. The authors present a solution for these cases in [10].

The second possibility consists of direct calculation of the first and second derivatives, using filters as described in [11], and rearranging the terms in order to get a linear regression, directly from the complex equation below, or by considering only its real or imaginary components, [1] being an example of this case. Using the complex notation we have:

$$\ddot{i}_s^g = -\bar{a}_1 \dot{i}_s^g - \bar{a}_0 \bar{i}_s^g + \bar{b}_1 \dot{u}_s^g + \bar{b}_0 \bar{u}_s^g \quad (1)$$

Here we have four parameters to be estimated ( $\bar{a}_1, \bar{a}_0, \bar{b}_1, \bar{b}_0$ ), provided that the rotor speed is measured, and also four electrical parameters to be calculated ( $R_s, L, \delta_r, R_{ref}$ ), from the previously estimated ones. All these parameters are described in (A.1b) and (A.1c). A recursive prediction error based approach available in MATLAB, can be used for the estimation purpose. According to [11] the derivatives can be calculated recursively by the following filters:

$$\dot{x} = dx/dt|_{t=t_k} \approx \frac{1}{T_s} \sum_{i=0}^{n-1} C_i x(t_k - iT_s), \text{ and } \ddot{x} = d^2x/dt^2|_{t=t_k} \approx \frac{1}{T_s^2} \sum_{i=0}^{n-1} D_i x(t_k - iT_s) \quad (2)$$

The weights  $C_i$  and  $D_i$  are determined from the Taylor's series expansion of the equation above to  $m+1$  terms, with  $m = \{1, 2, \dots, n\}$ ,  $m$  being the order of the filter and  $n$  the number of points. In this case, if the derivatives in (1) are substituted by its filter approximations of 2<sup>nd</sup>-order with 3 data samples, the first and second derivatives are given by:

$$\dot{x}_{t=t_k} = (3x(k) - 4x(k-1) + x(k-2))/(2T_s) \text{ and } \ddot{x}_{t=t_k} = (x(k) - 2x(k-1) + x(k-2))/T_s^2 \quad (3)$$

Replacing the derivatives in (1) by (3) and rearranging the terms one can get an ARMAX model structure, and use once again the tools available in MATLAB. For example, in the stator reference frame, an ARMAX model is given by:

$$\bar{i}_s^s(k) + \bar{e}_1 \bar{i}_s^s(k-1) + \bar{e}_2 \bar{i}_s^s(k-2) = \bar{f}_1 \bar{u}_s^s(k) + \bar{g}_1 (-2\bar{u}_s^s(k-1) + 0.5\bar{u}_s^s(k-2)) \quad (4)$$

Which represents a system with a single output,  $y(k) = \bar{i}_s^s(k)$ , and two inputs  $u_1(k) = \bar{u}_s^s(k)$  and  $u_2(k) = -2\bar{u}_s^s(k-1) + 0.5\bar{u}_s^s(k-2)$ . The coefficients  $\bar{e}_1, \bar{e}_0, \bar{f}_1$  and  $\bar{g}_1$  are function of  $\bar{a}_1, \bar{a}_0, \bar{b}_1, \bar{b}_0$  as well as the sampling time  $T_s$ . However, in spite of the simplicity of these model structures, the online simultaneous estimation of the main electrical parameters in steady state, under normal operating conditions, is not feasible due to the lack of persistent excitation provided by the stator signals. A different approach that is also suitable for transient conditions and joint estimation of states and parameters is obtained by considering again the well-known established  $dq$  dynamic model of the squirrel-cage induction motor, under known simplifications, in a general reference frame, and eliminating stator flux and rotor currents, but described by the time-domain state-space model structure (B.1a) and (B.1b). This can be derived from [9], for example. A time-discrete state-space model of (B.1) can next be obtained by assuming that the series expansion of the matrix exponential function  $e^{AT_s}$  is performed and the first terms are retained.  $A$  represents the state matrix in (B.1a) and  $T_s$  the sampling time. The resulting time-discrete state-space model structure can be used for rotor flux estimation, by using, for instance, a Kalman filter observer. When flux and the four electrical parameters are jointly estimated, four additional equations of the kind  $\theta(k+1) = \theta(k) + e(k)$  must be used, where  $e(t)$  is a random walk for the parameter evolution and is assumed to be white Gaussian noise. The result is an 8<sup>th</sup>-order time-discrete state-space model structure, where the state vector becomes:

$$x(k) = \begin{bmatrix} \bar{i}_{sd}^g(k) & \bar{i}_{sq}^g(k) & \psi_{rd}^g(k) & \psi_{rq}^g(k) & \delta_s(k) & \delta_r(k) & R_{ref}(k) & L(k) \end{bmatrix}^T \quad (5)$$

The parameters and flux components in (5) are described in (A.1c). Different sets of electrical parameters can be used but in any case a strong computational effort is needed. It is important to notice here that the first two lines in state equation (B.1a) contain the same information for identification purposes. So, we can use only one of them, for instance the first one. By this way, the computational effort is reduced by decreasing the model order from 4 to 3, as shown in (B.2a), or from 8 to 7 when the state vector is extended to the four electrical parameters. Since both stator currents  $dq$  components can be measured, one can reduce the model order even more by considering only the rotor flux  $dq$  components in the state vector, as in (B.3a). The state-space model composed by state equation (B.3a) and one of the equations (B.3b) or (B.3c) to form the output equation is now a reduced order model of 6<sup>th</sup>-order when a set of four electrical parameters is jointed to the state vector of (B3.a). For the first derivative of the stator current  $d$  component in the output equation (B.3b) or (B.3c), instead of Euler's formula, a better recursive approximation to the first derivative must be used by using a filter in which the number of points and its order should be selected as in [11], as explained before. Three new model structures, based on an innovative methodology, are proposed by the authors for joint flux and parameters identification.

The first one consists of a first estimator that uses the EKF to find good estimates of rotor parameters and the rotor flux components using (B.3), followed by a recursive prediction error based estimator that can be found in [12], to obtain the remaining stator parameters, in a boot-strap manner as described in general terms in [13] and, for this case, is represented in fig. 1, where:

$$\hat{x}(k, \hat{\theta}_{t_{k-1}}) = \begin{bmatrix} \psi_{rd}^r(k) & \psi_{rq}^r(k) & \delta_r(k) & L_M(k) \end{bmatrix} = \begin{bmatrix} \hat{x}_1(k) & \hat{x}_2(k) & \hat{x}_3(k) & \hat{x}_4(k) \end{bmatrix}, \quad (6)$$

$$\text{and } \hat{\theta}(k) = \begin{bmatrix} R_s(k) & L_s(k) \end{bmatrix} = \begin{bmatrix} \hat{\theta}_1(k) & \hat{\theta}_2(k) \end{bmatrix} \quad (7)$$

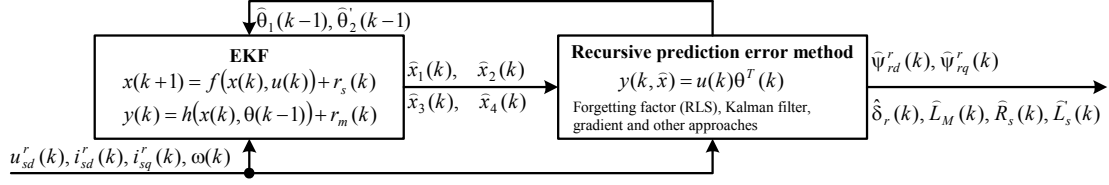


Fig.1 – A boot-strap methodology for states and parameters identification in the rotor frame.

The results of the application of this boot-strap methodology in real time operation can be found in [8], where the model structure, used in the EKF, is formed by a nonlinear discrete state-space 4<sup>th</sup>-order model (two fluxes and two parameters) obtained by discretization of the state equation in (B.3a) in the rotor reference frame where  $\omega_g = \omega$ , and the output equation in (B.3c), using (2) for the current first derivative. For the recursive prediction error based method, a linear regression model structure is derived from (B.3c), in the rotor reference frame, as follows:

$$u_{sd}^r + \omega \widehat{\Psi}_{rq}^r + \widehat{\delta}_r \widehat{\Psi}_{rd}^r - \widehat{L}_M \widehat{\delta}_r i_{sd}^r = R_s i_{sd}^r + L_s' (i_{sd}^r - \omega i_{sq}^r), \text{ or} \quad (8a)$$

$$u_{sd}^r + \omega \widehat{\Psi}_{rq}^r - \dot{\widehat{\Psi}}_{rd}^r = R_s i_{sd}^r + L_s' (i_{sd}^r - \omega i_{sq}^r) \Leftrightarrow y(k, \widehat{x}) = \theta_1 u_1 + \theta_2 u_2 = \widehat{y}(k, \theta) \quad (8b)$$

The second new methodology proposed by the authors is similar to the one described above and is represented in fig. 2, but for the recursive prediction error based method, the linear regression model structure is derived from the stator voltage equation, expressed in the stator reference frame, described in (9) that can be rewritten as a general linear regression  $y(k, \widehat{x}) = \theta(k)u(k)$ , as shown in (10). The application results, in real time operation, of the methodology shown in fig. 2 can be found in [7].

$$u_{sd}^s(k) = R_s i_{sd}^s(k) + L_s' (k) \dot{i}_{sd}^s(t_k) + \dot{\widehat{\Psi}}_{rd}^s(t_k) \quad (9)$$

$$y(k, \widehat{x}) = u_{sd}^s(k) - \dot{\widehat{\Psi}}_{rd}^s(t_k) \text{ and } \widehat{y}(k, \theta) = \theta(k)u(k) = -R_s i_{sd}^s(k) + L_s' (k) \dot{i}_{sd}^s(t_k) \quad (10)$$

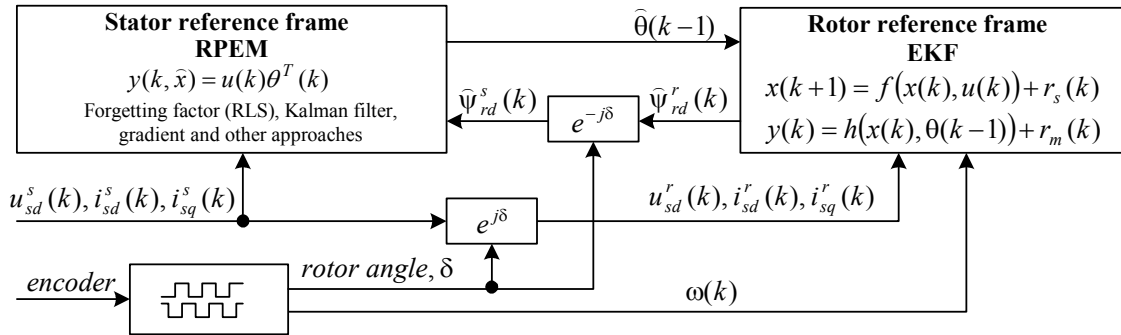


Fig.2 – The boot-strap methodology using both rotor and stator reference frame.

Taking into account that stator resistance has slow a dynamics when compared with the others electrical parameters and can be measured or even estimated from time to time, its value can be assumed to be known and in this case the robustness of the above-proposed identification procedures can be improved.

A new alternative model structure to (10) can be applied that avoids the need of first derivative calculation of the estimated rotor flux. So, the third new methodology is based on the reconstruction of the stator flux in the stator reference frame, as in (11) and

then, the calculated stator flux is converted into the rotor reference frame and the linear regression (12) is obtained.

$$\phi_{sd}^s(t) = \int (u_{sd}^s(t) - R_s i_{sd}^s(t)) dt \quad (11)$$

$$\phi_{sd}^r(t) - \psi_{rd}^r = L_s' i_{sd}^r(t) \Leftrightarrow y(t) = \theta(k)u(k) \quad (12)$$

By this way the stator transient inductance,  $L_s' = \theta$ , can be estimated with the same methodology used in fig. 1. However, this strategy has a drawback that is the implementation of a practical integrator. Finally, the derivative of the estimated flux in (8b) and (10) can not be a problem since the flux noise is tunable by the EKF.

## 5. Conclusions

A new set of model structures for joint flux and parameters identification has been presented in this paper. New methodologies were proposed, which are an innovation introduced by the authors for the presented model structures which use only rotor or both rotor and stator reference frames. In the rotor reference frame the sampling time can be longer and the computational effort is consequently reduced. The sampling frequency can be less than 5 kHz in the rotor reference frame but should be higher in stator one. On the other hand, the stator reference frame is a more natural one for stator's parameters estimation, namely the transient inductance and, furthermore, the convergence time is smaller when compared to rotor the reference frame. This is, however, the most natural one for rotor parameters estimation and mainly for rotor flux. Moreover, the resulting model structure (B.3a) becomes very simple because its matrices are diagonal since  $\omega_g = \omega$ . With this natural simplification provided by the rotor frame, the discretization process is much easier mainly for higher order approximations in the expansion of the matrix exponential function,  $e^{AT_s}$ . Anyway, according to the authors' experience, the main conclusion should be stated as follows: the online joint states and parameters estimation, in a vector control scheme, under normal operating conditions, depends slightly on the estimation method itself, but is strongly dependent of three factors: the signals persistency, and consequently the dynamic conditions of the machine; the selected model structures; and the methodology used in the identification procedure. This paper has just presented some solutions for the identified difficulties. Rotor states and parameters should be estimated in the rotor reference frame and this is performed with a good robustness. Other parameters can be estimated separately by using the proposed boot-strap methodology. Hence, the independency that is available by this way enables a supervisor algorithm to choose when and which parameters should be estimated tacking into account the dynamic conditions. Finally, the states must be always estimated by the EKF at every iteration for the vector control scheme.

## Appendix

$$\frac{\bar{i}_s^g}{\bar{u}_s^g} = \frac{\bar{b}_1 s + \bar{b}_0}{s^2 + \bar{a}_1 s + \bar{a}_0} \quad (A.1a)$$

$$\begin{cases} \bar{b}_1 = L & \bar{b}_0 = L(\delta_r + j(\omega_g - \omega)) & \bar{a}_1 = \delta_r + L(R_s + R_{ref}) + j(2\omega_g - \omega) \\ \bar{a}_0 = LR_s \delta_r - \omega_g(\omega_g - \omega) + j((\delta_r + LR_{ref})\omega_g + LR_s(\omega_g - \omega)) \end{cases} \quad (A.1b)$$

$$\tau_r = \frac{L_r}{R_r}, \quad L_s' = L_s - L_M, \quad L_M = \frac{L_m^2}{L_r}, \quad R_{ref} = \frac{L_M}{\tau_r}, \quad L = \frac{1}{L_s'}, \quad \delta_r = \frac{1}{\tau_r} \quad (A.1c)$$

$$\begin{bmatrix} \dot{i}_{sd}^g \\ \dot{i}_{sq}^g \\ \dot{\psi}_{rd}^g \\ \dot{\psi}_{rq}^g \end{bmatrix} = \begin{bmatrix} -a_1 & \omega_g & a_2 & \omega a_3 \\ -\omega_g & -a_1 & -\omega a_3 & a_2 \\ a_4 & 0 & -a_5 & \omega_g - \omega \\ 0 & a_4 & \omega - \omega_g & -a_5 \end{bmatrix} \begin{bmatrix} i_{sd}^g \\ i_{sq}^g \\ \psi_{rd}^g \\ \psi_{rq}^g \end{bmatrix} + \begin{bmatrix} a_3 & 0 \\ 0 & a_3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{sd}^g \\ u_{sq}^g \end{bmatrix} \quad (\text{B.1a})$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{sd}^g & i_{sq}^g & \psi_{rd}^g & \psi_{rq}^g \end{bmatrix}^T \quad (\text{B.1b})$$

$$\psi_{rd}^g(t) = (L_m / L_r) \phi_{rd}^g \quad \psi_{rq}^g(t) = (L_m / L_r) \phi_{rq}^g \quad (\text{B.1c})$$

$$a_1 = \delta_s + R_{ref} L, \quad \delta_s = R_s / L_s, \quad a_2 = L \delta_r, \quad a_3 = L, \quad a_4 = R_{ref}, \quad a_5 = \delta_r \quad (\text{B.1d})$$

$$\begin{bmatrix} \dot{i}_{sd}^g \\ \dot{\psi}_{rd}^g \\ \dot{\psi}_{rq}^g \end{bmatrix} = \begin{bmatrix} -a_1 & a_2 & \omega a_3 \\ a_4 & -a_5 & \omega_g - \omega \\ 0 & \omega - \omega_g & -a_5 \end{bmatrix} \begin{bmatrix} i_{sd}^g \\ \psi_{rd}^g \\ \psi_{rq}^g \end{bmatrix} + \begin{bmatrix} a_3 & \omega_g \\ 0 & 0 \\ 0 & a_4 \end{bmatrix} \begin{bmatrix} u_{sd}^g \\ i_{sq}^g \end{bmatrix} \quad (\text{B.2a})$$

$$y(t) = i_{sd}^g(t) = [1 \quad 0 \quad 0] \begin{bmatrix} i_{sd}^g & \psi_{rd}^g & \psi_{rq}^g \end{bmatrix}^T \quad (\text{B.2b})$$

$$\begin{bmatrix} \dot{\psi}_{rd}^g \\ \dot{\psi}_{rq}^g \end{bmatrix} = \begin{bmatrix} -a_5 & \omega_g - \omega \\ \omega - \omega_g & -a_5 \end{bmatrix} \begin{bmatrix} \psi_{rd}^g \\ \psi_{rq}^g \end{bmatrix} + \begin{bmatrix} a_4 & 0 \\ 0 & a_4 \end{bmatrix} \begin{bmatrix} i_{sd}^g \\ i_{sq}^g \end{bmatrix} \quad (\text{B.3a})$$

$$\dot{i}_{sd}^g(t) - \omega_g(t) i_{sq}^g(t) = -a_1 i_{sd}^g(t) + a_2 \psi_{rd}^g(t) + \omega a_3 \psi_{rq}^g(t) + a_3 u_{sd}^g(t) \quad (\text{B.3b})$$

$$u_{sd}^g(t) = -\delta_r \psi_{rd}^g(t) - \omega(t) \psi_{rq}^g(t) + (R_s + R_{ref}) i_{sd}^g(t) + L_s' (i_{sd}^g - \omega_g(t) i_{sq}^g(t)) \quad (\text{B.3c})$$

## References

- [1] J. Stepfan, M. Bodson, J. Chiasson *Real-Time Estimation of the Parameters and Fluxes of Induction Machines*, IEEE Trans. Ind. Appl., vol. 30, n.º 3, May/June 1994, pp. 746-759.
- [2] L. Loron, G. Laliberté, *Application of the Extended Kalman Filter to Parameters Estimation of Induction Motors*, in Proc. EPE, Brighton, 1993 pp. 85-90.
- [3] T. Du, P. Vas, F. Stronach, *Design and Application of Extended Observers for Joint State and Parameter Estimation in High-Performance AC Drives*, IEE Electr. Power Appl., vol. 142, n.º 2, pp. 71-78, March 1995.
- [4] J. W. Finch, D. J. Atkinson, P. P. Acarnley, "Full-Order Estimator for Induction Motor States and Parameters", *IEE Electr. Power Appl.*, vol. 145, n.º 3, pp. 169-179, May 1998.
- [5] V. Leite, R. Araújo, D. Freitas, *Flux and Parameters Identification of Vector-Controlled Induction Motor in the Rotor Reference Frame*, in Proc. AMC02, Maribor, Slovenia, July 3-5, 2002, pp. 263-268.
- [6] V. Leite, R. Araújo, D. Freitas, *A Real-time Estimator of Electrical Parameters for Vector Controlled Induction Motor using a Reduced Order Extended Kalman Filter*, in Proc. EPE-PEMC02, Dubrovnic & Cavtat, Croatia, July 9-11, 2002, full paper paper T11-029 on CD-ROM.
- [7] V. Leite, R. Araújo, D. Freitas, *A boot-strap estimator for joint flux and parameters online identification for vector controlled induction motor drives*, 2003 IEEE International Electric Machines and Drives Conference – IEMDC 2003, in press.
- [8] V. Leite, R. Araújo, D. Freitas, *A New Online Identification Methodology for Flux and Parameters Estimation of Vector Controlled Induction Motors*, 10th European Conference on Power Electronics and Applications – EPE 2003, in press.
- [9] P. Vas, *Parameter Estimation, Condition Monitoring, and Diagnosis of Electrical Machines*, Oxford Science Publications, 1993.
- [10] R. Araújo, V. Leite, D. Freitas, *Estimation of Physical Parameters of an Induction Motor using an Indirect Method*, in Proc. IEEE-ISIEC2002, L'Aquila, Italy, July 8-11, 2002, pp. 535-540.
- [11] A. J. L. Harrison, D. P. Stoten, *Generalized Finite Difference Methods for Optimal Estimation of Derivatives in real-Time Control Problems*, Proc. Instn Mech Engrs, vol. 209, pp 67-78, 1995.
- [12] L. Ljung, *System Identification, Theory for the User*, Prentice Hall, 1999.
- [13] G. C. Goodwin, K. S. Sin, *Adaptive Filtering Prediction and Control*, Prentice-Hall, 1984.