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An optimisation model for the warehouse design and planning problem

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Abstract

In spite of the importance of warehouses in the field of the supply chain management, there is not a single decision model that integrates all the decisions that concerns the warehouse design and planning problem. A number of warehouse decision support models have been proposed in the literature but considerable difficulties in applying these models still remain, due to the large amount of information to be processed and to the large number of possible alternatives. In this paper we discuss a mathematical programming model aiming to support some warehouse management and inventory decisions. Our aim is to address the complexity related to the modeling of the warehouse design and planning problem. In particular a large mixed-integer nonlinear programing model (MINLP) is presented to capture the trade-offs among both inventory and warehouse costs in order to achieve global optimal design satisfying throughput requirements.

Keywords: Supply chain management, Warehouse design and planning, Mathematical modeling.

1 Introduction

In a supply chain network (see Figure 1) products need to be physically moved from one location to another. During this process, they may be stored or buffered at *warehouses* for a certain period of time for strategic or tactical reasons. Within this context, warehouses play an important role in supply chain management and may be considered a key aspect in a very demanding, competitive and uncertain market.

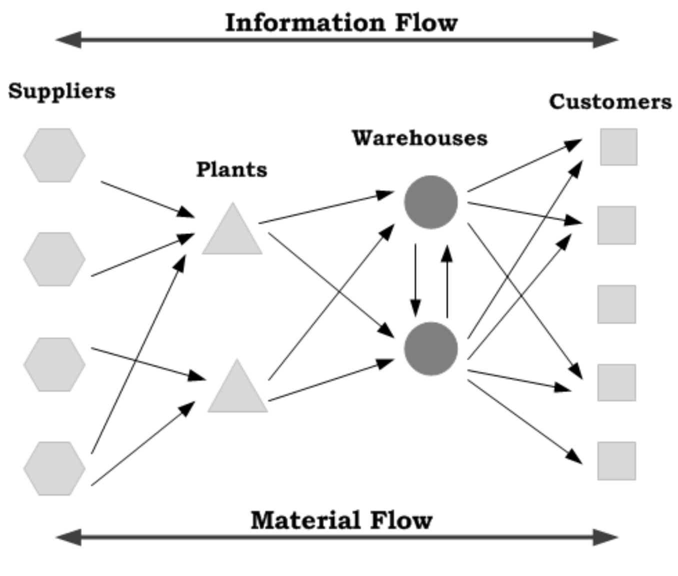

Figure 1: A typical supply chain network.

Modern supply chain principles compel organizations to reduce or eliminate inventory levels. Additionally warehouses require capital, labour, and information technologies, which are expensive resources. Although many companies examined the possibility of directly supply to customers, there are still many circumstances where this is not appropriate. So, why do we still need warehouses? According to Bartholdi and Hackman [2] there are four main reasons why warehouses are useful:

1. To consolidate products in order to reduce transportation costs and provide customer service;
2. To take advantage of economies of scale;
3. To provide value-added processing services, and
4. To reduce response time.

Thus, warehouses will continue to be an important node at the logistic network by the fact that if a warehouse cannot process the orders quickly, effectively, and accurately, then all the supply chain optimisation efforts will suffer (see Tompkins [14]).

In distribution logistic where market competition requires higher performances from the warehouses, organizations are compelled to continuously improve the design and planning of warehouses. Furthermore, the ever increasing variety of products, the constant changes in customer demands and the adoption of agile management philosophies also bring new challenges to reach flexible structures that provide quality, efficiency and effectiveness to the warehouse operations.

The primary functions of a warehouse include: (i) temporary storage of products; and (ii) providing of value-added services such as packing of products, after sales services, inspection, and assembly. To perform these functions warehouses are generally divided into different functional areas: receiving area, reserve storage area; picking or forward area, and the accumulation and shipping areas (see Figure 2).

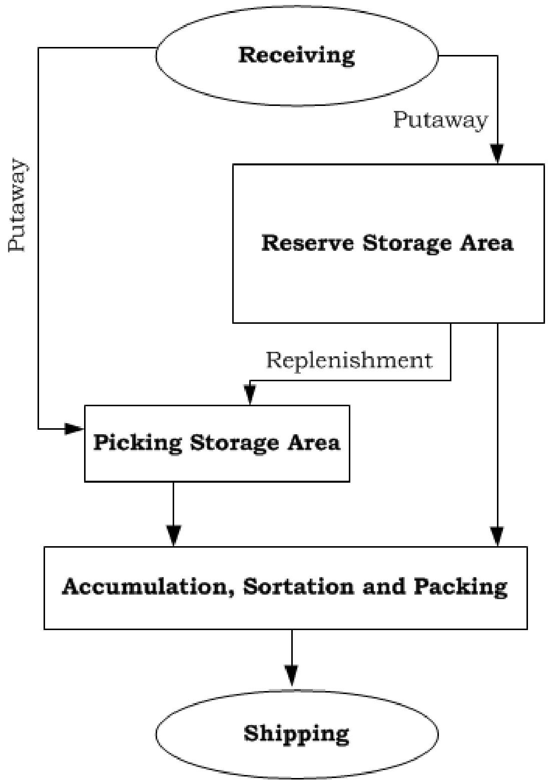

Figure 2: Typical warehouse functions and flows.

At the receiving area products are unloaded and inspected to verify any quantity and quality inconsistency. Afterwards, products are transferred to a storage zone. We can distinguish two types of storage areas: the reserve storage area and the forward or picking area. The reserve area is the place where the products stay until they are required by costumers' orders. The picking area is a relatively small area, typically used to store fast moving products. Most of the flows between these areas are the result of replenishment processes. Order picking is one of the most important functions in most warehouses. Stock Keeping Units (SKU) are retrieved from their storage positions based on customers' orders and moved to the accumulation and sorting area or directly to the shipment area. The picked units are then grouped by customer's order, packaged and stacked on the right unit load and transferred to the shipping area.

The design and planning of a warehouse is a very complex problem due to the large number of interrelated decisions. Some major decisions involved in the warehouse design and operation problem are illustrated in Figure 3 (see Gu et al. [5]).

Warehouse design decisions typically run from a functional description, through a technical specifica-

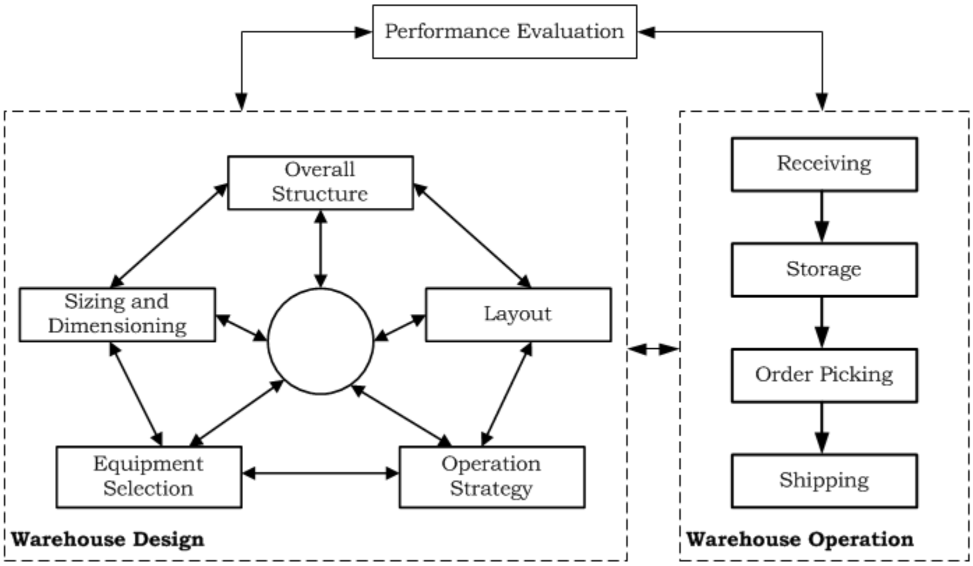

Figure 3: A framework for warehouse design and operation problems (adapted from Gu et al. [5]).

tion, to equipment selection and determination of the layout. The overall structure decision determines the material flow patterns within the warehouse, the specification of the functional areas and the flows between the areas. Sizing and dimensioning decisions determine the total size of the warehouse as well as the space allocation among functional areas. Layout definition is the detailed configuration within a functional area and equipment decisions define an automation level for the warehouse and identify equipment types. Finally operating policies refer to storage, picking and routing decisions.

Despite the importance of warehousing there is not a comprehensive systematic method for the warehouse design problem. Baker and Canessa [1] explored the literature on the overall methodology of warehouse design, together with the literature on tools and techniques used for specific areas of analysis. The output was a general framework of steps, with tools and techniques that can be of value to warehouse practitioners. Also Hassan [8] presented a framework for the design of a warehouse. The proposed framework accounts for several factors and operations of warehousing such as:

1. Specification of warehouse type and purpose;
2. Analysis and forecasting of the demand;
3. Definition of operating policies;
4. Establishment of inventory levels;
5. Class formation;
6. Definition of functional areas and general layout;
7. Storage partition;
8. Selection of equipment for handling and storage;
9. Design of aisles;
10. Determination of space requirements;
11. Location and number of Input/Output points;
12. Location and number of docks;
13. Arrangement of storage;
14. Zone formation.

Even though interrelated these decisions are dealt independently in a pyramidal top-down approach. Strategic decisions create limits to decisions taken at the tactical and operational levels and tactical decisions limits operational decisions. Also decisions taken at each level are handled independently and sequentially (see Van den Berg [15]).

The majority of warehouse decision models available in scientific literature addresses isolated or simplified problems. However, most of the real problems are unfortunately not well-defined and often cannot be reduced to multiple isolated sub-problems. Therefore, warehouse design often requires a mixture of analytical skills and creativity. Anyhow, research aiming an integration of various decisions models and methods is badly needed in order to develop a methodology for systematic warehouse design (see Rouwenhorst et al. [12]).

Furthermore, as the literature review in the next section shows, most research efforts have been dedicated to warehouse operations decisions instead of design decisions. This is not surprising since design decisions models are more difficult to develop and treat analytically once they require the integration of several and complex issues.

In this paper we present a warehouse and inventory mathematical model that jointly integrates issues concerning:

- The size of the warehouse and the size of the functional areas;
- The external storage additional capacity, if needed;
- The assignment and allocation of SKU to storage areas;
- The replenishment quantities and reorder points of the products to be stored.

Our aim is to test an integrated approach that takes into account inventory decisions and some warehouse design decisions.

In the Section 2 of this paper we will present a brief literature review on warehouse design and planning issues. The purpose of this section is not limited to the specific studied problem but also covers other important topics in warehouse literature. The Section 3 will present the proposed warehouse design and planning model formulation and a description of the methodology used to solve the model. Computational results of a preliminary study will be presented and summarized in Section 4, and finally some conclusions and future work directions are reported in Section 5.

2 Literature Review

2.1 Warehouse Design and Planning

Warehouse design can be defined as a structured approach of decision making at distinct decision levels in an attempt to meet a number of well-defined performance criteria. At each level, multiple decisions are interrelated and therefore it is necessary to cluster relevant problems that are to be solved simultaneously. According to Rouwenhorst et al. [12] a warehouse design problem is a "coherent cluster of decisions" and they define decisions to be coherent when a sequential optimization does not guarantee a globally optimal solution.

The design of a warehouse includes a large number of interrelated decisions involving warehouse processes, warehouse resources and the organization of the warehouse (see Heragu [9]). Rouwenhorst et al. [12] classified the management decisions concerning warehousing into strategic decisions, tactical decisions and operational decisions. Strategic decisions are long term decisions and always mean high investments. The two main issues involved are concerned with the design of the process flow and with the selection of the types of the warehousing systems. Tactical management decisions are medium term decisions based on the outcomes of the strategic decisions. The tactical decisions have a lower impact than the strategic decisions, but still require some investments and should therefore not be reconsidered too often. At the operational level, processes have to be carried out within the constraints set by the strategic and tactical decisions made at the higher levels. In this level, the concern includes the operational policies such as storage policies and picking and routing operations.

After determining warehouse location and its size, layout decisions must include areas definition and the dimension that should be allocated to each functional area. The forward-reserve problem (FRP) is the problem of assigning products to the functional areas. In this problem the critical decision concerns the choice of products that will be stored in the forward area. Van den Berg et al. [15] proposed a binary programming model to solve the FRP in the case of unit load replenishment, and presented efficient heuristics that provide tight performances guaranties. These replenishments can occur during busy or idle picking periods. The objective was to minimize the number of urgent or concurrent replenishments of the forward area during the busy periods. Although addressing this problem is a strategic decision problem, it is strongly associated upon some tactical problems such as how the items will be distributed among the functional areas. However, the approach usually adopted is to solve the problems sequentially by generating multiples alternatives for the functional area size problem and then determine how the products can be allocated for each of the alternatives.

Gray et al. [6] developed an integrated approach for the design and operation of a typical order-consolidation warehouse. This approach included warehouse layout definition, equipment and technology selection, product location, zoning, picker routing, pick generation list and order batching. Due to the complexity of the overall problem, they developed a multi-stage hierarchical decision approach. This hierarchical approach used a sequence of coordinated mathematical models to evaluate the major economic trade-offs and to reduce the decision space to a few number of alternatives. They also used simulation technique for validation and fine tuning of the resulting warehouse design and operating policies.

Heragu et al. [9] developed a mathematical model and a heuristic algorithm that jointly determines the size of the functional areas and the allocation of the product in a way that minimizes the total material handling and storage costs. The proposed model uses real data readily available to a warehouse manager and considers realistic constraints.

Geraldes et al. [3] adapted the mixed-integer programming model proposed by Heragu et al. [9] to tackle the storage allocation and assignment problems during the redesign process of a Portuguese company warehouse.

Liu [10] applied clustering techniques to extract the correlated information from the customer orders, and then stock locations were optimised. The author proposed a binary programming model to group products or customers and simulations results demonstrated the potential benefits of the clustering technique to solve the stock location problem.

More recently Strack and Pochet [13] presented a robust approach that integrates aspects such as: (i) the size of the functional areas; (ii) the assignment and allocation of products to storage locations in the warehouse; (iii) the replenishment decision in the inventory management. This is probably the most integrated decision model found in this area, nevertheless still assumes fixed and known capacity for the warehouse.

2.2 Inventory Decisions

The adoption of new management philosophies compels companies to eliminate or reduce inventory levels. In addition to warehouse management decisions, an appropriate inventory policy may result in a reduction of the total warehousing costs and can also improve the efficiency of the operating policies within the warehouse. The aim of inventory management is to minimize total operating costs satisfying customer service requirements (see Ghiani et al. [4]). To accomplish this, an optimal ordering policy must answer questions, for each SKU, such as when to order and how much to order.

Two different inventory policies arise (see Hadley and Whitin [7]): continuous review policy and the periodic review policy. The first policy implies that the stock level will be monitored continuously. Whenever the inventory on hand decreases to a predetermined level, referred to as the *reorder point*, a new order is placed to replenish the inventory level. The placed order is a fixed quantity that minimizes the total inventory costs and is normally called the *economic order quantity*. In the second policy, the inventory level is checked at specific fixed time periods and an order decision is made to complete the stock to a desired upper limit. In this system the inventory level is not monitored at all during the time interval between orders, so it has the advantage of little or no required record keeping. The disadvantage is less direct control. Such system also requires that a new order quantity must be determined each time a periodic order is made. The operating costs taken into account in both inventory policies are the acquisition cost, the holding cost and the shortage cost.

These basic policies can be adapted to take into account special situations such as single or multi-item models with or without a constraint on the total storage space, deterministic or stochastic demands, lost sales, etc. For more details and examples see Ghiani et al. [4] and Nahmias [11].

3 Mathematical Programming Model

Some fundamental questions occur during the design and planning of a warehouse: (i)

1. How much inventory should be kept in the warehouse?
2. How frequently and at what time should the inventory be replenished?
3. What should be the size of the warehouse?
4. What should be the size of the functional areas?
5. Which product must be stored within each functional area and how much must be stored?
6. Etc.

The first two decisions lead to the traditional inventory management problem and the others are some of the decisions concerning warehouse design and planning. Among these decisions the size of the warehouse is probably one of the most important aspects because once warehouse size is determined, it will act as a constraint that may last for a long period of time.

In this section we present a mathematical programming model that integrates both warehouse management decisions and inventory decisions in a way that minimizes the expected inventory and warehouse costs.

3.1 Model assumptions

This work considers a general warehouse configuration that include the following four functional areas: receiving, reserve, forward or picking and shipping. Thus, the following pattern flows are possible:

1. Receiving → Reserve → Shipping
2. Receiving → Reserve → Forward → Shipping
3. Receiving → Forward → Shipping

Flow 1 refers to a pattern that characterises a typical warehouse operation. Products are stored in a reserve area and picking operation is performed as required. Usually, it is assumed that only those products that remain for long periods of time or product quantities used for replenishment of the forward area will be allocated in this area.

Flow 2 is also a typical warehouse operation. Products with this pattern flow are initially stored in the reserve area and then moved to the forward area. This pattern flow is considered for fast picking operations, order consolidation or even to perform value-added operations.

Flow 3 refers to products that go directly to the forward area. This pattern flow is usually seen when there is a need to consolidate large orders.

The next section presents a mathematical model that determines the inventory parameters, the size of the warehouse, the flow to which each product must be assigned and, as result, the size of the functional areas within the warehouse. In developing the model we also assume:

- An inventory control policy based on continuous review policy (reorder point system);
- The forward storage area will be handled through a dedicate storage policy and the reserve area will assume a random storage policy;
- The inventory costs are known;
- The warehouse operation costs are known;
- Customers demand rates are known;
- It is possible to rent external storage area, if necessary.

3.2 Model formulation

The following notation, adapted from Strack and Pochet [13], is used:

Parameters:

i	: Product number ($i = 1, ..., I$)
j	: Number of locations in the forward area ($j = 1, ..., J$)
$CostRepA$	: Cost of advanced replenishment
$CostRepC$	: Cost of concurrent replenishment
$CostR$	: Reception cost for the reserve area
$CostF$	: Reception cost for the forward area
$PickF$	: Picking cost in the forward area
$PickR$	: Picking cost in the reserve area
$CostCapaW$	: Capacity cost of the private warehouse
$CostCapaFW$	: Private warehouse capacity fixed cost
$CostCapaS$	: External capacity cost
$CostCar$	: Inventory carrying cost
$CostAcqu_i$	: Acquisition cost of product i
$CostShort$	: Shortage cost
α_i	: Number of units of product i that can be stored in a single location of the forward area
u_{ij}	: Increase in the expected number of replenishment if an additional location in the forward area is allocated for product i
$E(U_i)$	: Expected value of the demand of product i
$E(p_i)$	: Expected value of the number of picks of product i
L	: Supply lead time
μ_i^L	: Average demand of product i during L
σ_i^L	: Standard deviation of demand of product i during L
d_i^L	: Demand of product i during L
M	: Large positive number

Decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if product } i \text{ has a Flow 2 pattern with at least} \\ & j \text{ locations allocated in the forward area} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if product } i \text{ has a Flow 3 pattern} \\ 0 & \text{otherwise} \end{cases}$$

$$z_i = \begin{cases} 1 & \text{if product } i \text{ has a Flow 1 pattern} \\ 0 & \text{otherwise} \end{cases}$$

$$w = \begin{cases} 1 & \text{if we have a private warehouse} \\ 0 & \text{otherwise} \end{cases}$$

$CapaW$	: Capacity of the private warehouse
$CapaF$	: Capacity of the forward area
$CapaR$	: Capacity of the reserve
$CapaS$	: External additional storage capacity
Q_i	: Replenishment quantity of product i
r_i	: Reorder point of product i

The general formulation of the model can be stated as:

$$
\begin{aligned}
\textit{minimize} \quad & \sum_{i=1}^{I} CostRepA \times x_{i1} + \sum_{i=1}^{I}\sum_{j=1}^{J} CostRepC \times u_{ij} \times x_{ij} \\
& + \sum_{i=1}^{I} CostR \times \frac{E(U_i)}{Q_i} \times (z_i + x_{i1}) + \sum_{i=1}^{I} CostF \times \frac{E(U_i)}{Q_i} \times y_i \\
& + \sum_{i=1}^{I} PickF \times E(p_i) \times (x_{i1} + y_i) + \sum_{i=1}^{I} PickR \times E(p_i) \times z_i \\
& + CapaW \times w \times (CostCapaW + CostCapaFW) + CostCapaS \times CapaS \\
& + \sum_{i=1}^{I} Costcar \times \left(\frac{Q_i}{2} + r_i - \mu_i^L\right) + \sum_{i=1}^{I} CAcqu_i \times E(U_i) \\
& + \sum_{i=1}^{I} CostShort \times \frac{E(U_i)}{Q_i} \times \int_{r_i}^{\infty} (d_i^L - r_i) f(d_i^L)\, \mathrm{d}d_i^L \,,
\end{aligned} \tag{1}
$$

subject to:

$$x_{ij} \le x_{ij-1} \quad \forall_{ij} : j \ge 2\,, \tag{2}$$

$$x_{i1} + y_i + z_i = 1 \quad \forall_i\,, \tag{3}$$

$$\sum_{i=1}^{I}\left[\left(\sum_{j=1}^{J} x_{ij}\right) + \left(\frac{Q_i + r_i - \mu_i^L}{\alpha_i}\right) \times y_i\right] \le CapaF\,, \tag{4}$$

$$\sum_{i=1}^{I}\left[\left(\frac{Q_i}{2} + r_i - \mu_i^L\right) \times (z_i + x_{i1}) - \sum_{j=1}^{J} \alpha_i x_{ij}\right] \le CapaR + CapaS\,, \tag{5}$$

$$CapaF + CapaR \le CapaW\,, \tag{6}$$

$$CapaW \le Mw\,, \tag{7}$$

$$CapaW \ge w\,, \tag{8}$$

$$Q_i, r_i \ge 0\,, \tag{9}$$

$$CapaW, CapaF, CapaR, CapaS \ge 0\,, \tag{10}$$

$$x_{ij}, y_i, z_i, w \in \{0, 1\}\,. \tag{11}$$

The objective function (1) minimizes the warehouses operating costs and inventory costs per period. Concerning the inventory costs we have taken into account: carrying cost, acquisition cost and shortage cost. The warehouse costs are composed by the reception costs, picking costs, the additional external storage capacity cost (from a public warehouse) and the costs of the owned (private) warehouse. Constraints (2) are sequencing constraints that specify that a j^{th} location can only be assigned to product i if $j-1$ locations have already be assigned. In addition, constraint (3) ensures that each product is assigned to only one of the three flow patterns of the warehouse. Constraints (4)-(5) ensure that the space for the forward and reserve areas are met, and constraint (6) guarantees that the total area in the warehouse is not exceeded. Constraints (7)-(8) serve to include the costs of the private warehouse. Finally, a set of variables must be nonnegative (9)-(10) and another is considered binary (11). Comparatively to the original model, proposed by Strack and Pochet [13], this formulation allows to determine the size and type (own or rented) of storage space required. Depending on which is least expensive a warehouse manager may use only one type or adopt a mixed strategy - both owned and rented storage space. More decision variables ($w, CapaW, CapaF, CapaR$) and warehouse costs ($CostCapaW, CostCapaFW$) were added since the size of the warehouse and of the functional areas are now unknown, and new constraints (6)-(8) were considered.

3.3 Model analysis and methodology

The above model considers inventory and warehouse decisions since it integrates both issues supporting decision makers defining warehouse design and planning. It is a mixed-integer nonlinear programming model (MINLP) with a large number of variables when real cases are considered. To demonstrate the complexity involved in solving the model to optimality, LINGO 12.0 solver was used on an Intel Core 2Duo 1.4 GHz CPU and 3GB RAM. For a randomly generated instance with only 5 products (SKU), a very small size instance, we only were capable of find local optimums within three hours of CPU time. Given the complexity of solving this model, such as Strack and Pochet [13], we used a sequential solution procedure. We decompose our model in two: (i) an inventory management sub model and (ii) a warehouse management sub model. These two sub models are solved sequentially. First we solve the inventory sub model and then the optimal values of the inventory variables are fixed and used to solve the warehouse management sub model.

3.3.1 Inventory management sub model

To obtain the inventory management sub model we have to eliminate costs and constraints related to the warehouse management problem. We obtain a mixed-integer nonlinear programming model under inventory and storage constraints defined as:

$$
\begin{aligned}
minimize \quad & \sum_{i=1}^{I} Costcar \times \left(\frac{Q_i}{2} + r_i - \mu_i^L \right) + \sum_{i=1}^{I} CAcqu_i \times E(U_i) \\
& + \sum_{i=1}^{I} CostShort \times \frac{E(U_i)}{Q_i} \times \int_{r_i}^{\infty} (d_i^L - r_i) f(d_i^L) \, \mathrm{d}d_i^L \\
& + \sum_{i=1}^{I} CostRecp \times \frac{E(U_i)}{Q_i} + CostCapaS \times CapaS \\
& + CapaW \times w \times (CostCapaW + CostCapaFW) \, ,
\end{aligned} \tag{12}
$$

subject to:

$$
\sum_{i=1}^{I} (Q_i + r_i - \mu_i^L) \leq CapaW + CapaS \, , \tag{13}
$$

$$
CapaW \leq Mw \, , \tag{14}
$$

$$
CapaW \geq w \, , \tag{15}
$$

$$
Q_i, r_i \geq 0 \, , \tag{16}
$$

$$
CapaW, CapaS \geq 0 \, , \tag{17}
$$

$$
w \in \{0, 1\} \, . \tag{18}
$$

To render this model independent of the warehouse decision variables an approximation and a relaxation were performed. First the objective function (12) was approximated using $CostRecp$ as the reception cost which is independent on the pattern flow taken by the products (defined as an average of the reserve and forward reception costs). Secondly, both reserve and forward areas were relaxed into one global capacity constraint (13) for the entire warehouse. The objective function and the global capacity constraint so obtained are independent of the flow patterns taken by the different products. Nevertheless, the inventory variables will be dependent of the storage capacity and, in a certain way, we can consider that this sub model integrates an inventory decision and a warehouse size decision.

3.3.2 Warehouse management sub model

Warehouse management sub model is obtained fixing the inventory variables (Q_i) and the capacity warehouse variable ($CapaW$), based on the solution of the inventory sub model and eliminating costs and constraints related with the inventory problem. The resulting model is a mixed-integer problem defined as follows:

$$\begin{aligned}
minimize \quad & \sum_{i=1}^{I} CostRepA \times x_{i1} + \sum_{i=1}^{I}\sum_{j=1}^{J} CostRepC \times u_{ij} \times x_{ij} \\
& + \sum_{i=1}^{I} CostR \times \frac{E(U_i)}{Q_i} \times (z_i + x_{i1}) + \sum_{i=1}^{I} CostF \times \frac{E(U_i)}{Q_i} \times y_i \\
& + \sum_{i=1}^{I} PickF \times E(p_i) \times (x_{i1} + y_i) + \sum_{i=1}^{I} PickR \times E(p_i) \times z_i \\
& + CostCapaS \times CapaS
\end{aligned} \tag{19}$$

subject to:

$$x_{ij} \leq x_{ij-1} \quad \forall_{ij} : j \geq 2\,, \tag{20}$$

$$x_{i1} + y_i + z_i = 1 \quad \forall_i\,, \tag{21}$$

$$\sum_{i=1}^{I}\left[\left(\sum_{j=1}^{J} x_{ij}\right) + \left(\frac{Q_i + r_i - \mu_i^L}{\alpha_i}\right) \times y_i\right] \leq CapaF\,, \tag{22}$$

$$\sum_{i=1}^{I}\left[\left(\frac{Q_i}{2} + r_i - \mu_i^L\right) \times (z_i + x_{i1}) - \sum_{j=1}^{J} \alpha_i x_{ij}\right] \leq CapaR + CapaS\,, \tag{23}$$

$$CapaF + CapaR \leq CapaW\,, \tag{24}$$

$$CapaF, CapaR, CapaS \geq 0\,, \tag{25}$$

$$x_{ij}, y_i, z_i \in \{0, 1\}\,. \tag{26}$$

In the warehouse sub model the flow pattern variables and the size of both reserve and forward functional areas will be optimized and the optimal value of the external additional storage capacity ($CapaS$) is re-optimised. The mixed-integer model will be solved using a Branch-and-Bound procedure.

4 Computational results

In this section, numerical results of a preliminary study are presented. Table 1 shows parameter values used to generate the testing problems. Instances for different scenarios (see Table 2) were randomly generated to assess the behaviour of the model when the number of products increases.

Table 1: Parameter values for the numerical examples.

Parameter	Value
$CostRepA$	5
$CostRepC$	20
$CostR$	5
$CostF$	8
$PickF$	3
$PickR$	10
$CostRecp$	5
$CostCar$	3
$CostShort$	50
$CostCapaS$	20
$CostCapaW$	3
$CostCapaFW$	10
$E(U_i)$	Uniform $[1, 50]$
$E(p_i)$	Uniform $[1, 5]$
d_i^L	$N(\mu_i^L, \sigma_i^L)$

Table 2: Analysed scenarios.

Scenario	I	II	III	IV	V
SKU [units]	10	100	500	1000	5000

Table 3: Inventory sub model computational results.

	Scenario				
	I	II	III	IV	V
Total variables	13	103	503	1003	5003
Nonlinear variables[a]	12	102	502	1002	5002
Iterations	203	647	1762	4020	16021
CPU time $[mm:ss]$	00 : 03	00 : 11	00 : 41	01 : 20	14 : 01
State	Global Opt.	Global Opt.	Global Opt.	Global Opt.	Global Opt.

[a] Variables involved in the nonlinear relationships of the model.

Table 4: Warehouse management sub model computational results.

	Scenario				
	I	II	III	IV	V[b]
Total variables	43	2203	38503	152003	3760003
Binary variables	40	2200	38500	152000	3760000
No. of constraints	24	2004	37504	150004	3750004
Iterations	6	20	849	2798	–
CPU time $[mm:ss]$	00 : 02	00 : 05	05 : 03	11 : 03	–
State	Global Opt.	Global Opt.	Global Opt.	Global Opt.	–

[b] Due to the size of the generator matrix, the computer did not have sufficient memory.

The computational results for the different testing cases are shown in Table 3 and Table 4. As it can be seen, with exception of Scenario V it was possible to solve to optimality all the others four scenarios in a very satisfactory time. Nevertheless, the computational time of LINGO solver rises as the problem sizes increases. For large instances (Scenario V) the number of variables and constraints of the warehouse sub model increases considerably. Consequently using the branch-and-bound algorithm takes significant computational time and memory. In our tests it was not possible to obtain the details for Scenario V due to the insufficient memory of the used computer. It appears that more sophisticated techniques can be explored, e.g. decomposition techniques, to circumvent the memory problem encountered with the branch-and-bound algorithm and solve very large instances.

5 Conclusions and future work

Most of the times warehouse design and planning decisions are taken independently. In fact, having a single decision model capable of integrate several decisions is a very complex task due to the enormous amount of existing alternatives, and to the existence of various and often conflicting objectives through and out of the warehouse. In this work a mathematical model for the warehouse design and planning problem was extended. The proposed model jointly integrates: (i) the size of the warehouse, including the size of both reserve and forward functional areas; (ii) the external storage additional capacity, if needed; (iii) the assignment and allocation of SKU to storage areas and; (iv) the replenishment quantities and reorder points of products to be stored.

Due to the complexity of the obtained analytical model, an optimal global solution was definitely difficult to achieve. For this reason our global model was decompose in two sub models, which were solved using a hierarchical sequential approach. First a nonlinear inventory model was solved taking into account the inventory variables and a global capacity constraint for the warehouse. Secondly, a warehouse management sub model was obtained fixing the solution of the inventory sub model. This sub model allowed us to determine the pattern flow for each product, the sizes of the functional areas and to re-optimise the value of the external additional storage capacity.

Computational results of the preliminary study suggest that it is possible to solve to optimality both sub models. Nevertheless, one must note that the computational time rises considerably as the problem sizes increases.

Even though the presented model integrates important decisions concerning the design and planning of a warehouse, many other decisions were not included, such us the storage policy, the picking and routing strategies, etc. However gathering in a single model several decisions leads us to very complex models difficult to treat and analytically solve. For that reason we believe that simulation technique can be used to validate the models and to incorporate dynamic aspects not yet included. For example, we can use the solution of the analytical model and then simulation can be used to introduce demand fluctuations or operational decisions related with the storage, picking or routing policies.

In summary, despite some advances in integrated approaches, further research focusing integrated models where different processes in the warehouse are jointly considered (and its corresponding dynamic nature), is still required. Given the prevalence of warehouses in the supply chain networks we believe that such research achievements can have a significant impact in the supply chain performance.

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