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Converging extended reality and Machine Learning to improve the lecturing of geometry in basic education

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ABSTRACT

Technology is constantly supporting in the innovation of the teaching-learning process. Today's students are more demanding actors when it comes to the environment they have at their disposal to learn, experiment and develop their critical thinking. The area of Mathematics has successively suffered from students' learning difficulties, whether due to lack of motivation, low abstraction ability or lack of new tools for teachers to bring innovation into the classroom and outside it. While being true that digitalization has entered schools, it often follows a basic and simple process of digital replication of approaches and materials that were previously only available on physical media. This work focuses on the use of Extended Realities, more precisely, Mixed Reality, for teaching Mathematics, and very particularly in the teaching of Geometry, through the proposition of a conceptual model that combines the use of Extended Reality and Machine Learning. The proposed model was subject to prototyping, which is presented as a form of laboratory validation as a contribution to innovate the way of how the geometry teaching-learning process is developed and to promote the integration of Extended Reality technologies into the Education Sector as practical tools, as well due to its potential use to obtain useful insights for teachers, and students, throughout the process.

Introduction

Ensuring access to technology is a key step for schools to convert to digital. However, for this conversion process to be successful, the focus mustn't be on technology but on understanding how technology can enable teaching and learning in an effective and inclusive way [1,2].

Nowadays, educational projects include physical manuals and digital materials, such as manuals in digital format, question banks, worksheets, and exams.

It is expected that the teaching and learning of mathematics, particularly geometry, will be greatly strengthened, consequently resulting in improved student performance. However, in most cases, this barely happens, often because the so-called technological support materials used are just a "blunt" digitization of what already exists in paper format - and solving exercises by only "writing on the computer", using the same paradigms, stops being interesting after a while.

A different but equally strong obstacle shared by many mathematics teachers is the lack of time to work/deepen each topic present in the curricular plan, including Geometry. At the same time, they are faced with a lack of interest on the part of students, which results in low attention and concentration during classes, as well in the subjects covered by the teacher, in the tasks to be carried out and, even more worrisome, the lack of continuity of the studies at home. According to [3], the lack of adequate educational methods is becoming increasingly evident, decreasing students' interest in learning and critical thinking.

Without resolving these basic problems, there is no prospect of improvements in terms of knowledge acquisition and application. For instance, many students do not read the statements properly and adopt a "give up" mentality simply because a geometric figure is in a position strange to them, which is a frequent scenario in Geometry exams surrounding primitives. Very often, they cannot apply the Pythagorean theorem in space, as they reveal difficulties at the level of abstraction. A study on the difficulties of applying the Pythagorean theorem can be found in [4].

Currently, a tripartite commitment between Schools, Parents, and Students is required to boost the teaching-learning process. If one fails,

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the entire teaching/learning process is compromised. According to [5], ensuring communication and supporting tools among the different actors in the school ecosystem is an essential issue to approach, and using appropriate technologies can bring these different actors closer.

Generally, Students become more interested when presented with the application of Mathematics in real-life scenarios and, more importantly, in the world of work. They need to carry out project works with a methodology that excites them - using the right tools can boost this entire process.

Technology plays a key role in shaping the future of education [6]. New technologies have undergone significant advances, making it possible today to integrate them into the teaching-learning process in a more valid way for educational practices in teaching. A wide variety of innovative equipment, programs, and serious games offer special meaning in the construction of knowledge while presenting themselves as appealing. When well chosen, used appropriately and sparingly, planned according to the characteristics/profile of the targeted students, and taking the necessary time, they are certainly a strong plus - motivating the student to learn without compromising the basic principles of each subject.

Dynamic Geometry environments can promote the teaching and learning of Geometry. It can help overcome difficulties or even avoid some of them. In fact, according to [7], the use of Dynamic Geometry Systems (DGS) has attracted special attention within mathematics education. This way of approaching the study of Geometry allows students to carry out tasks with a more exploratory and investigative approach, creating an environment where those involved can interact, discuss ideas, formulate conjectures, solve problems, and, consequently, obtain generalizations with relative ease while being able to justify the obtained results. When they become protagonists of their own learning, students develop characteristics that contribute to aspects of their social, personal, and professional lives.

One of the advantages of using new technologies in teaching Geometry is having dynamic access to complex figures that would otherwise be difficult to view from another perspective - knowledge acquisition becomes quick, easy, interactive, and accompanied by logical reasoning. In a study carried out by [8], the results indicated that the use of new technologies such as Virtual Reality (VR) and Augmented Reality (AR) improves interactivity and students' interest in teaching/learning mathematics, contributing to a more efficient understanding of various concepts, such as the study of primitives, when compared to the use of traditional teaching methods.

Empowering teachers and students through the use of appropriate technologies is vital to improve the teaching/learning paradigm. Since mathematics is often perceived as something abstract, it is hard to visualize its laws, dynamics, and applicability, so it becomes important to provide mechanisms that enable a more "physical" visualization of these same laws and applications.

However, it is equally important to the teacher, when using technology, to understand the way in which students conduct their learning process, being able to personalize their experience and draw useful insights to better respond to the needs of each student. In this way, teaching actors will be able to maintain high levels of motivation and promote autonomous learning and exploration in the students while using an immersive environment that enables critical thinking.

According to [9], immersive media presents benefits in terms of increasing motivation and expanding traditional teaching practices, involving students in different ways. Moreover, in [10] is shown that 3D virtual representation in learning can reduce the cognitive load of students with difficulties regarding mental spatial abilities, a common obstacle when learning Geometry, while students with an already high spatial ability of abstraction can better focus on others subjects. The use of 3D virtual representation also assists in developing students spatial abilities while keeping them motivated and excited.

The use of Machine Learning (ML) techniques could significantly contribute to developing better XR solutions in the educational sector due to their ability to personalize and optimize the teaching-learning process. ML models can analyze large amounts of student data, identifying patterns and gauging individual needs, allowing the creation of tailored and adaptive pedagogical approaches. ML-based tools like ChatGPT provide instant, personalized responses, promoting continuous interaction and real-time feedback. Furthermore, ML can help predict learning difficulties, allowing for more effective pedagogical interventions and improving knowledge retention and overall student performance while improving student engagement with the teaching-learning process.

This article proposes a model based on the use of Extended Reality (XR) to support the immersiveness and objectification of the not-always-objective dynamics of mathematics and ML to personalize the student experience and generate knowledge that will help the teacher to better understand and map the students' difficulties. The potential of ML is studied by [11], highlighting this technology's role in predicting students' performance.

The proposed approach - a conceptual model that combines XR and ML - supports an immersive vision of the teaching-learning process based on computational intelligence. A model prototype is also presented for teaching mathematics, particularly Geometry, in the context of basic education.

Methodology

In this work, the Action Research method was used - one of the four that make up the Applied Research methodology (action research, case study research, ethnography, and grounded theory, [12]) - which focuses mainly on four stages: planning, action, observation, reflection, and review [13]. Applied Research is a methodology used when specific research is developed to apply the results achieved to solve an existing problem.

This work began by identifying the problem: How can we explore using XR combined with ML to improve the teaching-learning process? Subsequently, an adequate study of the state of the art of technology was carried out to identify and compare the available technologies. A specific area of activity was then defined - the teaching of Geometry in Basic Education - and one was developed as a proof of concept.

This process was accompanied, from the beginning, by a basic education teacher in the area of mathematics who configured some scenarios of special interest so that appropriate and testable solutions could be prototyped.

In addition, the experience of a teacher with more than two decades of teaching in the basic education system was extremely important throughout the process of reflecting on the role that XR combined with ML could have in the teaching-learning process and the potential impact on the mechanics of the classes and the perception and stimulation of students, both in terms of understanding the content, acquiring skills and motivation to study inside and outside classes.

Literature background on the use of extended reality and Machine Learning in education

The use of technology to innovate in the teaching-learning process is nothing new. In fact, according to [14], integrating information technologies as a tool capable of assisting in education is mandatory, especially in an era in which there is a notable transition in the teaching-learning process.

In a study carried out by [15], which analyzes the role of reality-enhancing technologies in the teaching and learning of mathematics, some conclusions point to the need to promote the use of these technologies by both students and teachers. In contrast, the latter needs to know more than just about the tools used, but also how they can teach through it. Likewise, in a study carried out by [16], which analyzes, using a systematic review, the different approaches to learning with Immersive Virtual Reality (IVR), educational researchers are urged to consider existing approaches and rethink the process used to design IVR-based learning tasks to achieve their pedagogical goals. Being IVR, according to [17], is a potential approach to transform traditional classrooms into immersive virtual reality scenarios that are effective in the learning process and for research purposes.

Several authors have described the use of AR in geometry teaching. According to [18], over the past decade, the use of AR has proliferated in the education sector. However, the number of articles systematically reviewing the research trends in implementing AR for learning mathematics was reduced.

In [19], a structure of a learning environment system based on AR, supported by mobile devices, is presented. This system allows students to get assistance as well as tips to solve problems in the context of geometry. While in [20], proposed a design, implementation, and evaluation of an AR-based geometry learning application where interactions are based on hand gestures. However, the model does not include using ML to understand students' difficulties and/or personalize teaching-learning experiences.

Also, in [21], AR was used to develop material as a support mechanism for teaching and learning mathematics and examining its effectiveness. While an improvement in the student's learning outcomes, it is stated that the development of AR material presents some difficulties, being necessary to solve technical problems, proceed with the improvement of certain features, and be able to provide clearer instructions to users. Similarly to the previous ones, AR and ML were not combined in this work.

While applied in primary school, [20] presented a work where AR was used to assist learning achievement and generate motivation and creativity for children. The results of this investigation showed a positive impact on the degree of students' satisfaction in their teaching-learning process, an increase in students' motivation to learn less popular subjects (e.g., such as geometry), while also being noticed an improve in their creative thinking.

In a study by [21], which analyzed the potential of AR for Teaching Mathematics, more specifically for teaching Vector Geometry, the teacher and students considered using AR in mathematics classes beneficial and fun. The use of AR supported the development of spatial imagination, a switch that can be difficult to evolve when using only traditional 2D material.

Still regarding AR, in [22] is made a preliminary exploratory study with 6-Grade Primary Students with the goal of comparing the learning of geometry through traditional methods, more precisely with the use of manipulative materials, versus AR-based applications, being used the applications *Geometry* and *Quiver*, in this study the students already had previous experience in using AR on a different context, relatively to the topics of study, it was focused 3D-geometric contents, such the identification of shapes, principally polyhedrons. Although in this study was not detected a major difference between the introduction of the topics when comparing manipulative materials and AR, the score of the group using AR was slightly higher than the control group that used only manipulative materials, and while not existing a direct impact on the score, which was justified by the fact that manipulative methods being currently one of best approaches to introduce 3D-geometric solids and their characteristics, it was detect more motivation in the group using AR technologies.

However, according to [23], although many of the teachers expressed curiosity about XR technologies, the majority never used them in their teaching practice.

Although there are several studies and applications of XR, namely AR, and VR, in the field of teaching, the approaches used do not combine the potential of XR with ML, reducing the ability to generate intelligence each time a student uses the application during their learning process. In this way, it does not take advantage of the full potential return that would come from knowing the study patterns and failures. Students use this approach when solving or experimenting with 3D educational materials. Combining ML with XR will allow students to better understand their errors. At the same time, for teachers, it will provide a personalized and manageable teaching tool with a positive impact on the readjustment of methodologies and pedagogy used in the classroom, combined with innovation.

ML can help understand the student and support the provision of adapted and personalized content, materializing an efficient recommendation system in the learning environment while considering the student's behaviors and preferences when recommending learning materials - a system that adapts to the needs and student's learning skills [24].

Proposed conceptual model

This section presents a conceptual model (see Fig. 1) based on using XR with ML and intended to contribute to innovating the teaching-learning process. The current model is applicable in any teaching area; however, the developed prototype was targeted at learning and teaching mathematics and, more specifically, at learning and teaching geometry in Portuguese basic education.

The proposed model has several components and actors, which are presented below, and where their main objectives and form of action are characterized:

3D assets database: Represents the repository of 3D assets that are used in XR applications (e.g., solid primitives).

Geometry contents & exercises: It represents the content associated with the subject that will be taught and/or the object of study to be developed inside or outside the classroom. This content will make use of the 3D assets to support XR applications.

XR models: It represents the set of scenarios that will be personalized by the teachers depending on the classroom context, homework, or free study activities to be carried out by students.

ML models: It represents a group of models that have been trained to recognize certain types of patterns obtained through data sets and algorithms that are used to weigh and learn from stored data (which are obtained from the data trail that is left by students when using XR applications and keep on an interoperable database)

XR engine: Represents the module that extracts the actions carried out by the student and learner (being a teacher or a colleague), feeding the ML engine. Being capable of generating personalized XR scenarios based on input from the ML engine.

ML engine: Represents the intelligence engine of the proposed model. It is capable of analyzing the insights and actions carried out by students when carrying out activities and interacting with the XR Model to generate new scenarios with personalized activities based on the difficulties of each student, as well as, in real-time, the generation of each new step of the activity (e.g., next question).

Students assessments: Represents the repository of assessments and results of these from each student with the finality of assisting the teacher in understanding students' difficulties.

Student interface: Represents the students' view of the system. Students will use XR Glasses (e.g., HoloLens® 2) and all available functionalities for them to use (e.g., collaborative working in the classroom, homework, free activities at home).

Teacher interface: Represents the teacher's view of the system. Teachers will use XR Glasses (e.g., HoloLens® 2), which will enable them to use the same functionalities of the students and those directed to them on an administrative/manager level.

To exemplify some functionalities to be implemented, the following use case diagram (see Fig. 2) illustrates the activities involved in classroom learning.

Thus, as illustrated, it can be seen that the system constantly tracks the interactions that will serve as the basis for machine learning insights. In classroom learning, both the student and the teacher will be able to interact, allowing different types of learning (identification of solids, areas, and volumes calculation, among others) while being able to manipulate objects in different ways (rotate solids, highlight vertices

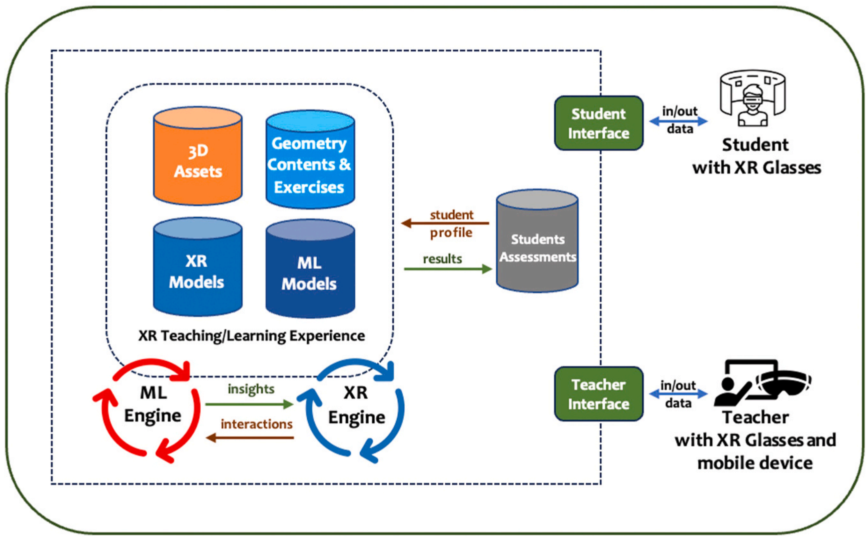

Fig. 1. Proposed conceptual model.

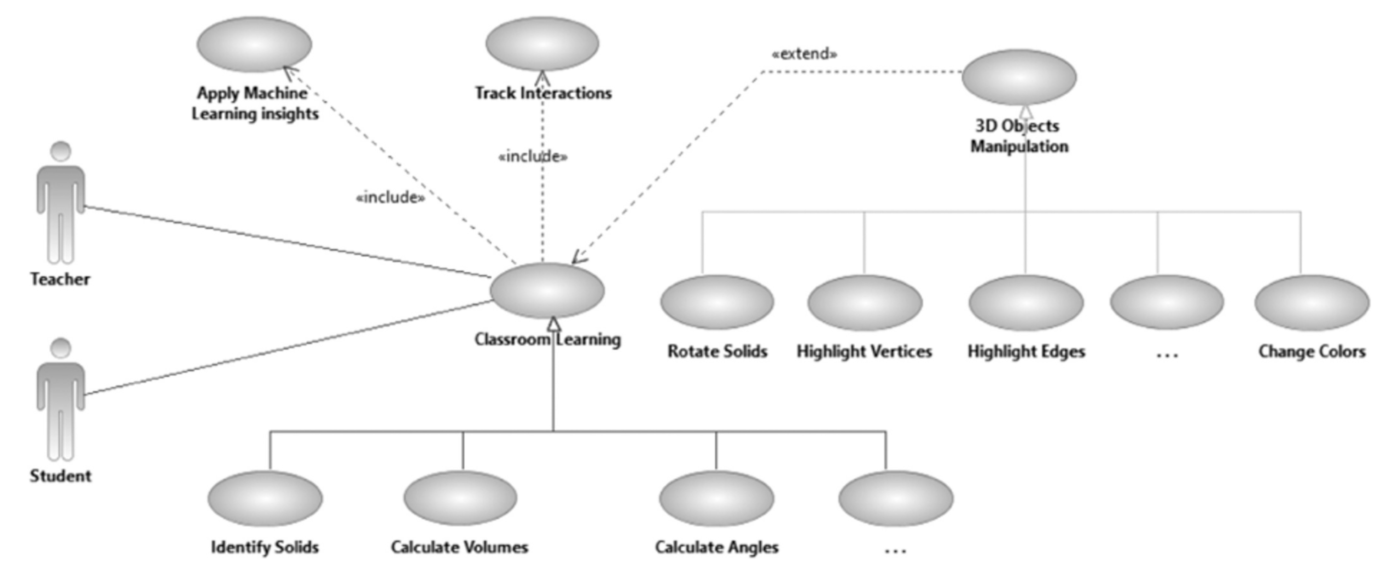

Fig. 2. Use case - classroom learning.

and/or edges, change colors, among others).

On the other hand, the system should allow the teacher to monitor the progress of students' learning, as represented in the following use case diagram (see Fig. 3).

Monitoring the evolution of learning using statistical analysis and inference will result in a better understanding of the difficulties experienced by students. It will also allow the teacher to propose new challenges and recovery plans.

The conceptual model presented supports innovation; we idealize and operationalize new teaching-learning paradigms, which are more capable of meeting students' motivations and contributing to their critical development. Likewise, it aims to contribute to the creation of collaborative environments in the classroom and to help break barriers in the difficulty of abstraction associated with mathematics, especially in the teaching of Geometry, where, most times, the 2D view of materials on paper stands as an obstacle instead of support. Finally, it aims to help motivate students to develop independent study work while providing valuable insights for the teacher at an interoperable level.

Prototype developed

This section presents an extension of the developed prototype [25] based on the conceptual model presented in the previous section being referred the technologies used, the implemented features and planned ones as well. This is an ongoing work that aims the validation of the conceptual model proposed, and to be tested in real school classes context.

Materials

Over this section will be presented the hardware and software used to develop of the current prototype.

HoloLens® 2: When developing the prototype it was considered the current available equipment for Mixed Reality (MR) in the laboratory, being this the HoloLens® 2, an Optical Head-Mounted Device (HMD), also known as Optical See-Through (OST) HMD [26], manufactured by Microsoft®. This device was released in 2019, being the successor of the HoloLens® 1 which was released in 2016, and one of the responsible for the popularization of the concept of MR, it works as a standalone HMD,

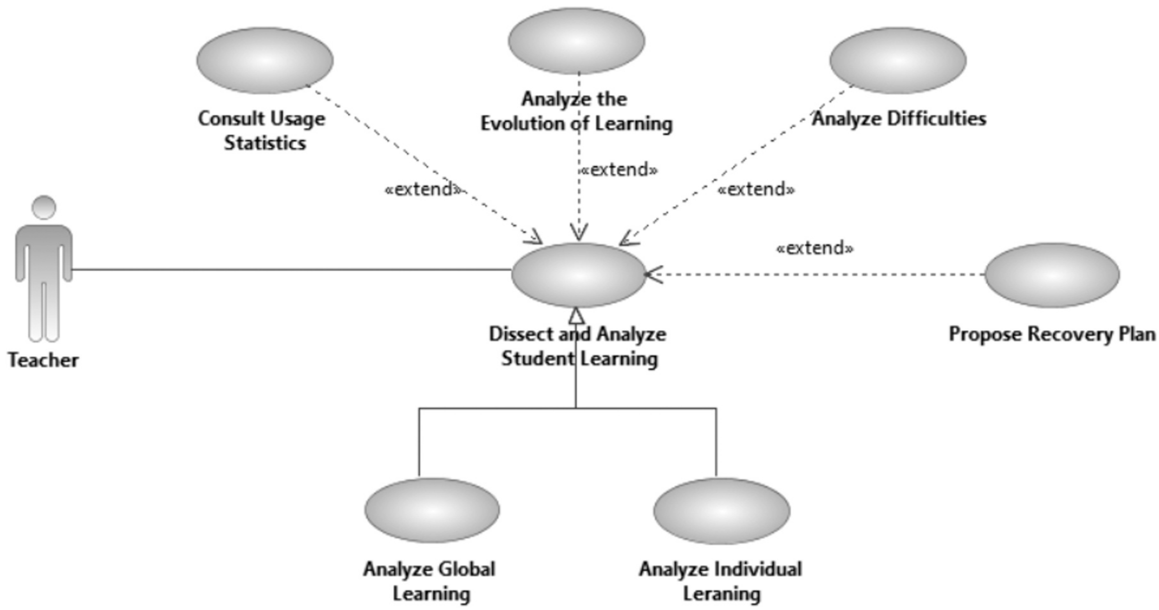

Fig. 3. Use case - analyze the evolution of learning.

this is, without the need of controllers or connection to external components such as controllers, which is possible by featuring technologies like Hand Tracking, Eye Tracking and Spatial Mapping.

On contrary to others devices like the Apple Vision Pro and the Meta Quest 3, the HoloLens® Series makes use of hologram technology to create MR experiences, which makes use of diffractive waveguides, the lenses inside the visor, and laser light projectors, while the others devices make use of Passthrough, which can be described as being a normal display, such as the ones present in VR headsets, that displays frames captured from frontal cameras and are processed (e.g., is added the 3D virtual content), and while the Passthrough technology allows to make devices cheaper it also comes with some constraints such as an inclination to latency and difficulty in reading real-world text.

Unity: Also known as Unity3D or Unity Engine, is a Game Engine used primarily for game development, being highly recognized among the XR Developers Community duo to his low learning curve and ease-of-use when compared with others game engines like Unreal or CryEngine, as well the big community and support when developing XR applications, it is also a primary suggestion by most of the pioneers of this technology like Meta®, Microsoft® and VIVE®, being currently used the version 2022.3.

Visual studio: It is an Integrated Development Environment (IDE) owned by Microsoft® and commonly used in parallel with Unity to write scripts in C#, an object-oriented language programming used by Unity. It's also a required tool to build an application package to install an application in HoloLens® 2, this process starts however in Unity by building an Universal Windows Platform (UWP) application which is then used to create the application package, being defined the minimum platform version as 10.0.10240.0 and the targeted Software Development Kit (SDK) as 10.0.22621.0.

MRTK3: The Mixed Reality Toolkit is a framework/toolkit originally created and maintained by Microsoft® alone, with the goal to provide the necessary tools to developers when creating MR applications for HoloLens® 2 in Unity [27], the current version is the MRTK3 and while available as public preview since June 2022, it was declared as generally available in September 2023, being also transferred to an independent open-source organization, which has Microsoft, Qualcomm and Magic Leap as steering committee members. While initially focused on the HoloLens® devices, applications developed with MRTK2 and principally with MRTK3 can also be built to be used in other devices that support MR like Magic Leap 2, as well the more recent and accessible Meta® Quest 3.

Mechanics implemented

In this section will be described the implementation of features present on the prototype which are not directly detected by the user.

3D solid architecture: Geometry is the branch of mathematics concerned with the study of solids/shapes, edges, faces and vertices, as such these elements are the very basic necessity to be replicated in MR being also necessary the possibility of treating each component as individual element. These shapes are mainly divided between polygons which are two-dimensional (2D) shapes, such as squares and triangles, and polyhedrons, which are three-dimensional shapes with flat polygonal face(s).

While polygons can be considered simple shapes to simulate in MR, similarly to how simple they are already with the current approaches, the same does not applies to polyhedrons, since XR technologies enables a true 3D approach to these shapes, bringing the 2D view to a new dimension as well new expectations, similarly to the use of physical models made industrially, by hand or thought the new technologies like 3D printing [28], however without the implicated and potentially cumulative costs. As such when it comes to polyhedrons, being a cube or a triangular prism for example, the *GameObject* representing the solid is mounted by defining each component individually to then enable an increased control over the shape as well the implementation of different scenarios (see Fig. 4).

The general structure of the *GameObject* consists on a *Handler* script which will store references for the remaining *Hanlers* responsible for handling aspects such as vertices and faces, this script is stored in the very base of the hierarchy of the *GameObject* which will also stores the component *Rigidbody*, being this component responsible to implement physics into the object, such as making it subjective to gravity or collisions with others objects.

The second *GameObject* in the hierarchy, named as *Solid*, stores all scripts necessary to allow being manipulated by the user, such as moving it, change scale or rotation, being this scripts provided by the MRTK3 framework. Relatively to the *GameObjectBody*, it stores the *PhysicsHandler* scripts, this script handles the very basic physical aspects of the solid, by storing references to the *RigidBody*, as well a general solid *Collider* and individual *Colliders* for each face of the solid, being these present as children *GameObjects* of the *OcclusionBounds GameObject*.

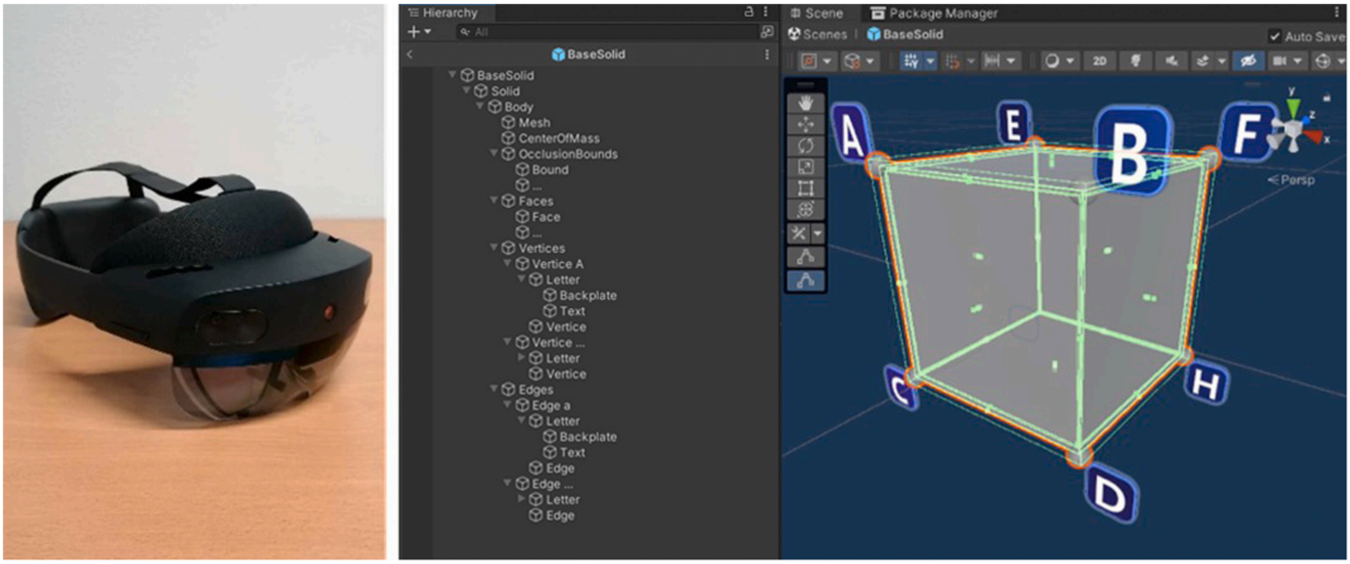

Fig. 4. HoloLens® 2 and prototype polyhedron GameObject architecture with example in unity editor.

While a *Rigidbody* simulates physics, the *Colliders* are mainly responsible of defining the shape of the objects, commonly these components would be able to be located in the same *GameObject*, however during the development of the prototype it was necessary to separate these components due to compatibility issues with others aspects of the solid, such as between the *RigidBody* and the object manipulation when enabled gravity, which resulted on the solid falling, but such was not being interpreted by the object manipulations scripts, creating discrepancy.

Among the remaining *GameObjects* are *Faces*, *Vertices* and *Edges*, each one stores a *Manager* script, which has references for individual *Handlers*, per example, in a Cube, the Edges Manager will have the references of twelve Edge Handlers, similarly, in a Dodecahedron, the *Manager* will have the references of the thirty *Handlers*, enabling this way a easier manipulation across many components when it comes to general modifications, such as show markers for all vertices, as well for individual manipulation, like changing one face color,

The remaining *GameObjects*, *Mesh* and *CenterOfMass*, have specific use cases, *Mesh* is consists on a semi-transparent version of the solid, to enable scenarios like to highlight the edges, while the *CenterOfMass* is for more complexes solids that are expected to be added in future developments.

Relatively to the components which define the solid, such as vertices and edges, each one is defined individually and store a *Handler* script. A vertex consists on a small sphere, while the edges make use of a feature of Unity called *Line Renderer* being used the vertices to define the start and end of the line. Lately, for the definition of the faces, as well to have the complete solid as individual component, it was used polygons, while some are provided as primitives by Unity, most are not present, being acquired the asset Deluxe Primitives by Reactorcore Games and Ultimate Procedural Primitives by KANIYONIKA from the Unity Asset Store.

In relation to the *Colliders* used, in the Unity Engine, there are four primary types of 3D colliders: *Box Collider* (cube), *Sphere Collider*, *Capsule Collider*, and *Mesh Collider*. Additionally, there is the *Terrain Collider*, though it is not included as a general option due to its specific focus on games. Among these, the *Capsule Collider* is not viable, as it is commonly employed in specific contexts such as 3D characters or environmental props like trees. The *Sphere Collider* has a very specific use, that is, for Spheres. Which leaves the *Box Collider* and *Mesh Collider* as the available choices.

Between the two, the *Mesh Collider* demands significantly more hardware resources, as it relies on a *Mesh* made up of numerous triangles to represent the object's shape. Which can become a concern for standalone XR devices, due to them having limited hardware capabilities. However, the *Mesh Collider* offers a higher degree of accuracy, as it conforms precisely to the model's surface *Mesh* (or "skin"). For example, when applied to a 3D model of a hand, a *Mesh Collider* would closely follow the contours of the fingers, palm, and joints. In contrast, using *Box Colliders* would require creating multiple *Colliders*, approximately 15 separate colliders to represent the palm and individual finger joints, if used a high abstraction. While this approach would improve performance, it would come at the cost of precision in interactions, such as when the hand holds an object, as well as requiring more time to setup, being this type of "trade" and choice between performance, time and accuracy common in game development.

Other advantage of *Box Colliders* over *Mesh Colliders* is that we can handle each *Box Colliders* separately, while the *Mesh Collider* works as one, that is, if used 15 *Box Colliders* to represent a Hand, it is possible to detect with "precision" which part of the Hand touched (collided) with an object, which would not be possible with a *Mesh Collider*.

Based on this, *Box Colliders* are used to define the *Colliders* of the Faces whenever feasible due to their lower performance overhead. Mesh Colliders are generally reserved for more complex geometries, such as polygons (e.g., Triangle) or polyhedrons (e.g., Triangular Prism). For example, when working with a triangular prism, is used three flatten *Box Colliders* for the Rectangular faces and two *Mesh Colliders* for the Triangular faces. While the choice of using flat *Box Colliders* for the Rectangular faces my seem worse than using *Mesh Colliders*, which would roughly translate in two triangles, it does provides better performance due to how *Box Colliders* collisions are calculated in the engine when comparing with *Mesh Colliders*, being already optimized the detection of collisions with *Box Colliders* in 3D environments, while in 2D environments the most optimized are *Box Collider 2D* which work as Squares. On the other hand, the previously mentioned "general solid *Collider*, is a *Box Collider* that covers all the shape volume, which is used for manipulation in the MR environment, being suggested in the documentation of the MRTK3 framework the use of *Box Collider* to handle the manipulation of any kind of 3D model.

This architecture can then be replicated to construct the remaining shapes, however is not an automatic process, requiring the adaptation of the data (e.g., vertices, faces, …) to each different polyhedron shape.

Object pooling: Is a technique commonly used in Game Development to minimize the use of *Instantiate* and *Destroy* methods from Unity, being these methods used to create and destroy *GameObjects* during runtime, which are not performance friendly methods [29] due to involve a lot of background processes, such as memory allocation/deallocation and updating stored references to components and scripts present on the *GameObject*, the use of these methods also make the game/application susceptible to Garbage Collector (GC) spikes. While this may not be a priority concern when developing Desktop Games or even for platforms such as PlayStation, it does become a concern for devices like the HoloLens Series and standalone VR/MR headsets, in which is important to make use of good practices that optimize the application/game performance and lower the devices resources consumption.

Since it is expected the use of multiple objects for temporaries scenarios (e.g., in an exercise to calculate how many smaller cubes fit on a larger cube), the implementation of object pooling was considered an important mechanic to be implemented.

User interactions

The following section will present the current available interactions.

Objects creation and interaction: The creation of objects is done through drag and drop of the objects shown in the interface *Create Shape* (see Fig. 5) into the world space. This interface is currently divided between four options, the creation of 2D shapes (Polygons), 3D shapes (Polyhedrons), others elements, such as cylinders and spheres, and utilities, which include holographic surfaces and light sources.

Through the *Hand Menu*, the user can open the *Home Panel*, from which access's the *Create Shape*, but also change the mode of interaction with the shapes, being the default one *Manipulate*, which enables the user to move, scale a rotate the shapes. In *Edit* mode, when the user selects an object, it opens a new panel to edit/personalize the selected object, while the *Interact* mode is planned to provide interactions such as activating animations showing the shape net, which is "how it would look" if opened out flatly.

Shape personalization: Consists in empowering the user, being a teacher or student, by enabling him to modify the solid in a way to better highlight a concept (see Fig. 6).

Change color: Enables the user to highlight specific components of the solid or even differentiate them, being a basic concept present both in school manuals or when using physical solid representations, but without limitations or constraints to presets. This enables in one moment highlight one face of the solid, and in the next moment the respective vertices or just a pair of them.

Edit vertices: Allows the user to modify the representation of the vertices, while they are defined through simple and immaterial 3D vectors, the user can choose to display visual markers (spheres), or letters (in Uppercase) as well display only specific ones.

Edit edges: Provides a group of possibilities to manipulate the edges, which includes the same options as the vertices, this is, the visualization of letters (in Lowercase) and visual markers (*Line Renderer*), as well enabling Occlusion, which consists on differentiating edges which are visible to the user from those who are not, and enabling Outline, being this the highlighting of the outer border of the solid.

Physics: Allows changing solid characteristics such as how it interacts with gravity, and mass, as well with other solids, which can be used for explaining geometric concepts such as cube number.

When it comes to gravity, by the default the *GameObject* would fall for eternity, and while, when enable, the HoloLens® 2 is able to detect the user surroundings through the use of Spatial Mapping, it can limit the user workspace, for example, if using the device in the outside, being the used the holographic tables (see Fig. 7), this mechanic however requires dynamic modification of the components provided by the MRTK, since the inherited handlers of the object collide with the table, as if providing a fake sense of volume.

Playground manipulation: Refers to the basic ability of the user to create more solids at his own choice and to manipulate them both in terms of position, scale, and rotation in the digital world, as well to create composite solids. Depending on how the solid physics is configured those same solids can collide between them or be inserted inside each other to assist for example at calculating volumes.

Learning assistance: This is the first stage of integrating and testing ML in the prototype, by making use of the Large Language Model (LLM) ChatGPT, more specifically through the use of the OpenAI API and the model "gpt-4o-mini" is made a request to provide a simple, and oriented to a Basic School student, explanation of the solid (see Fig. 8).

In the current stage of exploring the tool is being used the general API, however is possible the creation of *Assistants*, which are "customizable" versions of the ChatGPT, at which can be attributed roles like *"You are a Basic School Teacher"* or *"You have 10 years old"*, as well empower it through the exposition of features of the application, as per example, by allowing ChatGPT to modify the solid while explaining specific topics.

To simplify and optimize the process of integration of the OpenAI

Fig. 5. Hand menu and examples of interface.

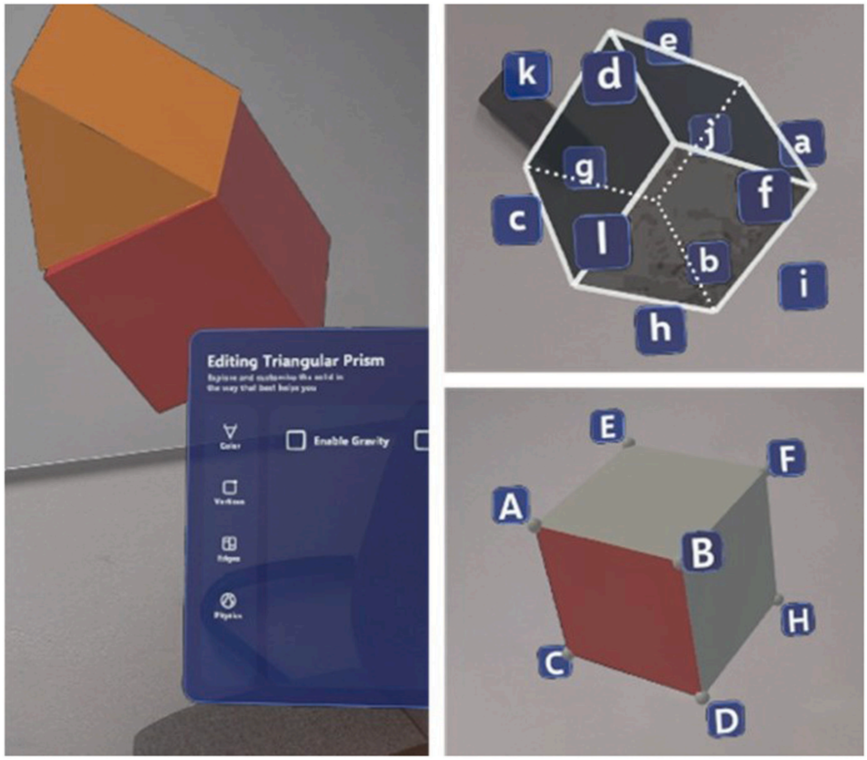

Fig. 6. Polyhedron personalization use-cases.

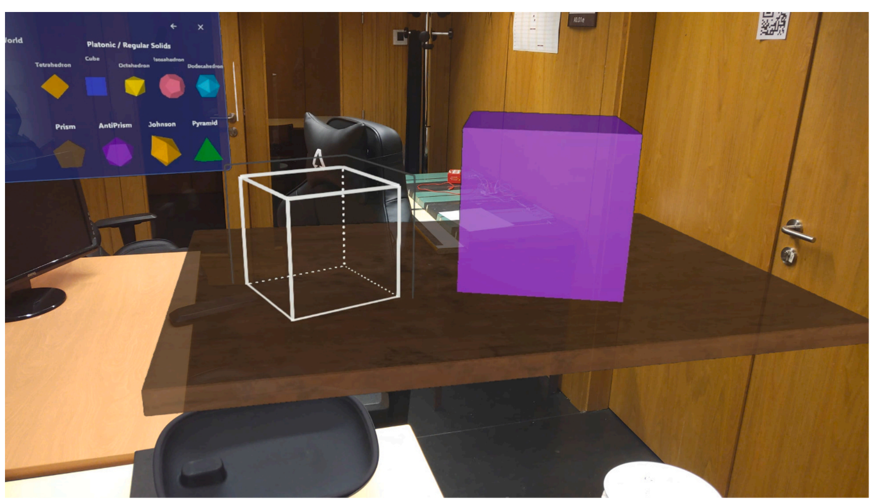

Fig. 7. Holographic table for scenarios involving gravity.

API in the project is being used the MIT Licensed, unofficial package, “srcnalt/OpenAI-Unity” by Sercan (Sarge) Altundas, however the API from OpenAI works similarly to others API’s being mainly used HTTP POST Requests containing the message as “Body”. These requests - the interaction Student/LLM - are then made by predefined interactions for example, through the use of buttons, as shown in Fig. 8, where is used the button “Learn” to ask ChatGPT about the selected solid, though it is possible to make use of voice commands for this level of interaction by using “Voice Recognition”, which is available in devices like the HoloLens 2, these would still need to be predefined at the Development Phase. In an advanced level it can be used Voice-To-Speech and Text-To-Speech services to create a more practical and personal interaction, similar to what is proposed in the project from *Replica Studios* where is being created *Smart NPC’s*, however, this approach brings new costs and has the potential to aggravate some of the already identified problems of the OpenAI API.

The use of OpenAI API however brings some challenges and problems, as principal problem stands the monetary cost per request done to the API, referenced as *Tokens* in the service, per example, the model used for the prototype costs $2,50 (+ − 2, 28€) for one million of *Input Tokens* while costing $10 (+ − 9, 11€) for the equivalent quantity of *Output Tokens*, these values are then changed based on the ChatGPT model being used. However, these *Tokens* are not directly related to the request/response, but the content of the message itself, for example, if made a query asking “What is a cube?”, the query will consume five *Input Tokens*, equivalent to the division of the message into an array like [“What”, “is”, “a”, “cube”, “?”], to which ChatGPT may response like “A cube is a three-dimensional shape with six square faces, twelve edges,

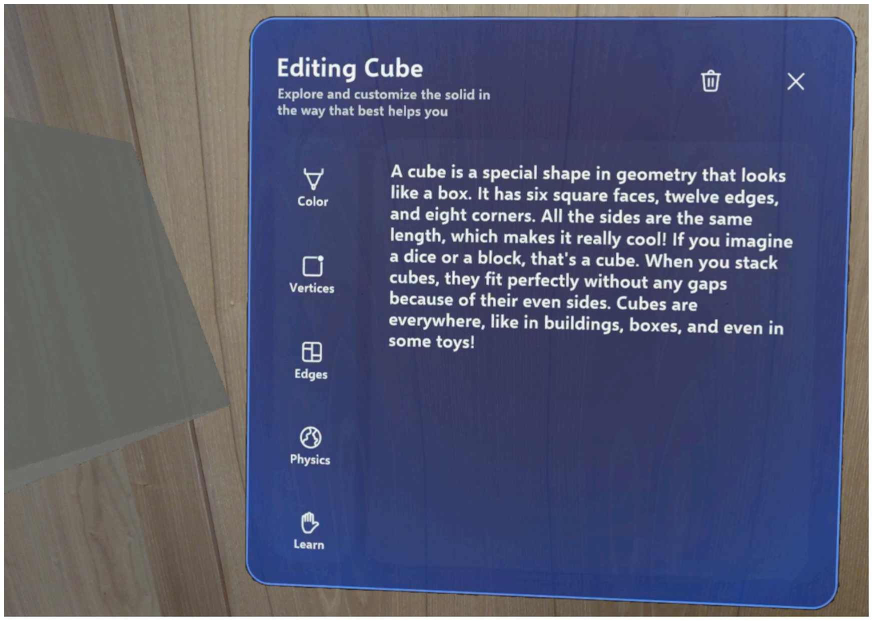

Fig. 8. Example of using OpenAI API in the learning process.

and eight vertices.", consuming then 19 *Output Tokens*, resulting on a total of 24 *Tokens* used on a single basic interaction.

It is also necessary to note that the division is not done by words but by a process named *tokenization*, which is based on Byte Pair Encoding (BPE), this process tends to isolate common words as one *Token*, however symbols and compound/rare words (e.g., basketball, snowman, floccinaucinihilipilification) can result in two (e.g., ["basket", "ball"]) or more *Tokens* (e.g., ["Fl", "occ", "in", "auc", "ini", "hil", "ip", "il", "if", "ication"]). With this knowledge, it is possible to the developer to try to reduce the use of *Input Tokens* in the application during the development phase, however the consumption of *Output Tokens* requires some manipulation, per example, when creating the *Assistant*, aside to attribute to him a "personality", it is also possible to define a limit of words to be used when answering to queries (e.g., "You don't use more than 10 words per answer."), which however don't cover the use of numbers (e.g., 4) or symbols.

Aside from the monetary problem, the second principal problem is a common problem across Web API's which resides on the API availability, which may be affected by the status of the server, like if it is down or overflowed by requests, as well by being dependent of the quality of connection of the device to the Internet.

The effectiveness of using ChatGPT will be evaluated according to the objectives defined by the teacher, which align with the programmatic objectives, being measured the results achieved when ChatGPT is used or not by the students, during formal assessment scenarios and/or other class activities initially planned by the teacher. The motivational aspects generated by the use of ChatGPT - which is already present in Higher Education [30] - combined with new interactive technologies like AR, will also be evaluated, namely the commitment and motivation demonstrated by the students, as well as their feedback.

Regarding the role of the teacher, it will assess his perception regarding the level of complementarity that ChatGPT provides relative to the traditional teaching-learning process and how the later contributes to reducing the time spent in the process of explaining subjects, which may take more time than what was planned by the teacher, resulting on less time for the next subjects and in the progressive lack of interest and attention of the student, caused by his difficulties in learning new subjects which are dependent on the previous ones, these scenarios, in many occasions are connected to the teaching methodology applied by the teacher - which he is using during years - but is affected by the level of understanding of the student, as well his personality and personal interests.

Ongoing and future development

Aside from further improvements, concerning interactions available to the user and based on the necessities noted during the initial tests, is planned to implement the possibility of personalizing the faces to for example only being visible a set of them and the possibility of clipping 3D primitives.

Moreover, advanced features planned are the possibility of streaming the user view to a desktop screen using Mixed Reality Capture (MRC) and WebRTC, enabling scenarios where the teacher may present concepts during classes while projecting the 3D solids for the class. Another feature is the development of shared experiences in a way that two or more users may interact with the same 3D world similarly to Multiplayer Games by using technologies like Photon Unity Networking 2 (PUN 2) or Fish-Net: Networking Evolved. The integration of these features not only promote shared experiences among students, but also allows teachers to follow/visualize the students progress and course of actions during their learning process, which stands as a necessary requisite for the relationship teacher/student, even more in complicated subjects such as Mathematic [31]. On the other hand, to improve single user experiences is being studied possibilities to introduce gamification into the learning process, with the goal of generating motivation [32,33] but also to promote social interaction between students and collaborative learning [34].

Gamification can be also be used by the teacher as another method to evaluate the student [35], while also providing self-validation to the student, and by being software based, it does not have constraints such as language barriers or shock of personalities between students and teachers, which are some of the identified problems in the current education system [36], while also allows the students to take their own time without the classroom environment pressure

Finally, the ML engine is being implemented and will be improved after experimentation in a real scenario (i.e., school classes) and, being defined objectives, in partnership with basic school teachers, considered most relevant to help personalize the proposed exercises and which will help to understand the reasons that tend to characterize the difficulties in understanding and learning, of each student.

Regarding data collection, the strategy includes the extraction of data captured from real sources, such as school assessments, practical activities carried out by students, both when alone and in group study sessions, as well during classes, ensuring the diversity of data to cover then a wider range of content and difficulties that students are facing during the lecturing geometry, based on the history of results obtained regarding this subject.

This approach allows us to create exercise scenarios based on expected more suitable exercises for teaching and self-study tasks.

At the same time, the continuous use of this application, and its use across different schools, will allow a progressive collection of data, converging on optimized and customized learning pathways for the future tasks that the application will propose.

Conclusion

Technology has supported and encouraged the introduction of innovation in education, promoting new methodologies inside and outside the classroom and innovating the concepts of school manuals and study support material.

Among the technologies that are most promising, XR can enhance essential factors in the teaching-learning process, such as increasing levels of motivation, capacity for abstraction, and development of critical thinking.

Likewise, using XR to create cooperative teaching-learning environments allows the construction of a new paradigm of interaction between teachers and students, especially in the classroom.

Regarding XR technologies, MR presents as the next step after the already existent solutions in AR and VR, this technology enables the user, being it a student or a teacher, to explore Geometry with "its own hands" in accord with the discovery-based learning process that is closely linked to the history of Geometry study field [37], by providing 3D visualization and manipulation, while also allowing the use of 3D animations or multiplayer experiences.

The combination of XR with ML raises the potential of XR, allowing the acquisition of insights that can be decisive in understanding, on the one hand, students' difficulties in a personalized way and, on the other, the definition of more appropriate strategies inside and outside the classroom.

This paper proposes a conceptual model based on using XR and ML to contribute to creating new and innovative teaching-learning strategies. To validate the presented model, a prototype was developed based on MR to assist in teaching Geometry.

The presented prototype is on continuous testing and complementation through the addition of features based on insights acquired from user tests in real-life scenarios, focused on single-user experience, being also continuous analysed and studied the ability of adaptation of the user to the use of XR technologies, as well the User Experience (UX) through the prototype, being adapted the User Interface (UI) in accord to identified problems, due to the already identified difficulty concerning users when they are using 2D interfaces in a MR context when compared to, for example, web applications [38], being this a problem already identified during the validation phase, in which the Basic School teacher took some time in adapting to the technology.

This work aims to obtain better recommendation systems applied to teaching scenarios, promoting a more effective self-learning in the process of acquisition of skills, at the same time as it promotes better support for the teaching-learning process, working to improve the abstraction capacity and the level of student motivation. Finally, it aims to provide teachers with mechanisms that are more appropriate to the reality that their students already experience (e.g. use of XR) in recreational environments (e.g. entertainment games) that can stimulate interest in studying.

CRediT authorship contribution statement

João Pedro Gomes: Writing – review & editing, Writing – original draft, Investigation, Conceptualization. **Vítor Mendonça:** Writing – review & editing, Writing – original draft, Investigation, Conceptualization. **Sílvia Coelho:** Validation, Supervision, Conceptualization. **André Moreira:** Writing – review & editing, Writing – original draft, Software, Investigation. **Carlos Rompante Cunha:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] K. McKnight, K. O'Malley, R. Ruzic, M. Horsley, J. Franey, K. Bassett, Teaching in a digital age: how educators use technology to improve student learning, J. Res. Technol. Educ. 48 (2016) 194–211, https://doi.org/10.1080/15391523.2016.1175856.

[2] N. Dahal, N.K. Manandhar, L. Luitel, B.C. Luitel, B.P. Pant, I.M. Shrestha, Ict tools for remote teaching and learning mathematics: a proposal for autonomy and engagements, Adv. Mob. Learn. Educ. Res. 2 (2022) 289–296.

[3] L. Georgieva, A. Nikulin, The art of education: creative thinking and video games, Balk. J. Philos. 15 (2023) 179–186, https://doi.org/10.5840/bjp202315222.

[4] R. Sari, D. Wutsqa, Analysis of student's error in resolving the pythagoras problems, volume 1320, 2019.10.1088/1742-6596/1320/1/012056.

[5] C. Cunha, J. Gomes, V. Mendonça, Bringing together high school actors using mobile applications: A conceptual model proposal, Proceedings of the 34Th International Business Information Management Association Conference, International Business Information Management Association, 11564-11570 (2019).

[6] A. Haleem, M. Javaid, M. Qadri, R. Suman, Understanding the role of digital technologies in education: a review, Sustain. Oper. Comput. 3 (2022) 275–285, https://doi.org/10.1016/j.susoc.2022.05.004.

[7] G. Bozkurt, C. Uygan, Lesson hiccups during the development of teaching schemes: a novice technology-using mathematics teacher's professional instrumental genesis of dynamic geometry, ZDM - Math. Educ. 52 (2020) 1349–1363, https://doi.org/10.1007/s11858-020-01184-4.

[8] E. Demitriadou, K.-E. Stavroulia, A. Lanitis, Comparative evaluation of virtual and augmented reality for teaching mathematics in primary education, Educ. Inf. Technol. 25 (2020) 381–401, https://doi.org/10.1007/s10639-019-09973-5.

[9] E. Erturk, G.-B. Reynolds, The expanding role of immersive media in education, 2020, 191-194.

[10] Y. Rahmawati, H. Dianhar, F. Arifin, Analysing students' spatial abilities in chemistry learning using 3d virtual representation, Educ. Sci. 11 (2021), https://doi.org/10.3390/educsci11040185.

[11] B. Albreiki, N. Zaki, H. Alashwal, A systematic literature review of student' performance prediction using machine learning techniques, Educ. Sci. 11 (2021), https://doi.org/10.3390/educsci11090552.

[12] R.N. Rapoport, Three dilemmas in action research:with special reference to the tavistock experience, Hum. Relat. 23 (1970) 499–513, https://doi.org/10.1177/001872677002300601.

[13] S. Kemmis, R. McTaggart. The Action Research Reader: Deakin University, 3rd edition, Deakin University press, Australia, 1988.

[14] O.T. Laseinde, D. Dada, Enhancing teaching and learning in stem labs: the development of an android-based virtual reality platform, Mater. Today Proc. (2023).

[15] D. Buentello-Montoya, M. Lomelí-Plascencia, L. Medina-Herrera, The role of reality enhancing technologies in teaching and learning of mathematics, Comput. Electr. Eng. 94 (2021), https://doi.org/10.1016/j.compeleceng.2021.107287.

[16] M. Won, D. Ungu, H. Matovu, D. Treagust, C.-C. Tsai, J. Park, M. Mocerino, R. Tasker, Diverse approaches to learning with immersive virtual reality identified from a systematic review, Comput. Educ. 195 (2023), https://doi.org/10.1016/j.compedu.2022.104701.

[17] L. Hasenbein, P. Stark, U. Trautwein, A. Queiroz, J. Bailenson, J.-U. Hahn, R. Göllner, Learning with simulated virtual classmates: effects of social-related configurations on students' visual attention and learning experiences in an immersive virtual reality classroom, Comput. Hum. Behav. 133 (2022), https://doi.org/10.1016/j.chb.2022.107282.

[18] N. Ahmad, S. Junaini, Augmented reality for learning mathematics: a systematic literature review, Int. J. Emerg. Technol. Learn. 15 (2020) 106–122, https://doi.org/10.3991/ijet.v15i16.14961.

[19] S. Gargrish, A. Mantri, D. Kaur, Augmented reality-based learning environment to enhance teaching-learning experience in geometry education, volume 172, 2020, 1039-1046.10.1016/j.procs.2020.05.152.

[20] H.-Q. Le, J.-I. Kim, An augmented reality application with hand gestures for learning 3d geometry, 2017, 34-41.10.1109/BIGCOMP.2017.7881712.

[21] T. Koparan, H. Dinar, E. Koparan, Z. Haldan, Integrating augmented reality into mathematics teaching and learning and examining its effectiveness, Think. Skills Creat. 47 (2023), https://doi.org/10.1016/j.tsc.2023.101245.

[22] M. Flores-Bascuñana, P.D. Diago, R. Villena-Taranilla, D.F. Yáñez, On augmented reality for the learning of 3d-geometric contents: a preliminary exploratory study with 6-grade primary students, Educ. Sci. 10 (2020), https://doi.org/10.3390/educsci10010004.

[23] M. Schwaiger, M. Krajnčan, M. Vukovič, Jenko, D. Doz, Educators' opinions about vr/ar/xr: an exploratory study, Educ. Inf. Technol. (2024), https://doi.org/10.1007/s10639-024-12808-7.

[24] H. Oubalahcen, L. Tamym, M. Driss El Ouadghiri, The use of ai in e-learning recommender systems: A comprehensive survey, volume 224, 2023, 437-442.10.1016/j.procs.2023.09.061.

[25] C.R. Cunha, A. Moreira, S. Coelho, V. Mendonça, J.P. Gomes, Empowering the teaching and learning of geometry in basic education by combining extended reality and machine learning, Lecture Notes in Networks and Systems 988 LNNS (2024)98-109.10.1007/978-3-031-60224-5_11.

[26] M. Doughty, N. Ghugre, G. Wright, Augmenting performance: a systematic review of optical see-through head-mounted displays in surgery, J. Imaging 8 (2022), https://doi.org/10.3390/jimaging8070203.

[27] M. Franzo, S. Pascucci, M. Serrao, F. Marinozzi, F. Bini, Breakthrough in occupational therapy with mixed-reality exergaming for cerebellar ataxia patients, 2023 IEEE International Symposium on Medical Measurements and Applications, MeMeA 2023 - Conference Proceedings(2023). 10.1109/MeMeA57477.2023.10171955.

[28] O. Özeren, E.B. Özeren, S.M. Top, B.S. Qurraie, Learning-by-doing using 3d printers: digital fabrication studio experience in architectural education, J. Eng. Res. 11 (2023) 1–6, https://doi.org/10.1016/j.jer.2023.100135.

[29] C.R. Cunha, A. Moreira, L. Pires, P.O. Fernandes, Intangible approaches to improve individual health indicators and empower caregivers, Signals Commun. Technol. Part F1293 (2024) 177–192, https://doi.org/10.1007/978-3-031-34601-9_11.

[30] A.S. Almogren, W.M. Al-Rahmi, N.A. Dahri, Exploring factors influencing the acceptance of chatgpt in higher education: a smart education perspective, Heliyon 10 (2024), https://doi.org/10.1016/j.heliyon.2024.e31887.

[31] X. Yao, A. Manouchehri, Teacher interventions for advancing students' mathematical understanding, Educ. Sci. 10 (2020) 1–21, https://doi.org/10.3390/educsci10060164.

[32] M.M. Tenório, F.A.F. Reinaldo, L.A. Góis, R.P. Lopes, G. dos Santos Junior, Elements of gamification in virtual learning environments: a systematic review, Adv. Intell. Syst. Comput. 716 (2018) 86–96, https://doi.org/10.1007/978-3-319-73204-6_12.

[33] R.P. Lopes, C. Mesquita, M. de la Cruz Del Río-Rama, J. Álvarez García, Collaborative learning experiences for the development of higher-order thinking, Espacios 39 (2018).

[34] M.K. Othman, S.K. Ching, Gamifying science education: how board games enhances engagement, motivate and develop social interaction, and learning, Educ. Inf. Technol. (2024), https://doi.org/10.1007/s10639-024-12818-5.

[35] Z. Zhang, J. Crawford, Efl learners' motivation in a gamified formative assessment: the case of quizizz, Educ. Inf. Technol. 29 (2024) 6217–6239, https://doi.org/10.1007/s10639-023-12034-7.

[36] S. Scheider, S. Rosenfeld, S. Bink, N. Lecina, Educational inequality due to lack of validity: a methodological critique of the dutch school system, International, J. Educ. Res. 117 (2023) 102097, https://doi.org/10.1016/j.ijer.2022.102097.

[37] S. Jablonski, M. Ludwig, Teaching and learning of geometry—a literature review on current developments in theory and practice, Educ. Sci. 13 (2023), https://doi.org/10.3390/educsci13070682.

[38] Y. Xing, C. Fahy, J. Shell, Assessing web 2d user interface experiences in mixed reality, Heliyon 10 (2024) e31916, https://doi.org/10.1016/j.heliyon.2024.e31916.