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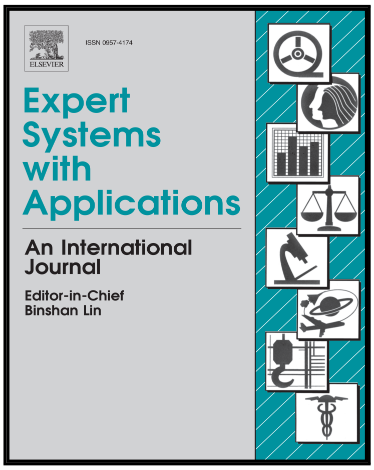

The Triangle Assessment Method: A New Procedure for Eliciting Expert Judgement

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Highlights

- We introduce a new method (MTC©) for eliciting expert opinion by comparison of trios.
- We present the axiomatic bases and theorems for the MTC©.
- We compare the MTC© and the Analytic Hierarchy Process (AHP).
- We illustrate an example of application of the MTC© to multi-criteria decision-making.

THE TRIANGLE ASSESSMENT METHOD: A NEW PROCEDURE FOR ELICITING EXPERT JUDGEMENT

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Abstract

The Analytic Hierarchy Process (AHP) is one of the most widely used Multi-Criteria Decision-Making methods worldwide. As such, it is subject to criticisms that highlight some potential weaknesses. In this study, we present a new Multi-Criteria Decision-Making method denominated the "Triangular Assessment Method" (referred to by its Spanish abbreviation, MTC©). The MTC© aims to make use of the potential of AHP while avoiding some of its drawbacks. The main characteristics and advantages of the MTC© can be summarised as follows: (i) evaluation of criteria, and of the alternative options for each criterion, in trios instead of pairs; (ii) elimination of discrete scales and values involved in judgements; (iii) a substantial reduction in the number of evaluations (trios) relative to the corresponding number of pairs which would have to be considered when applying the AHP method; (iv) consistent decision-making; (v) introduction of closed cyclical series for comparing criteria and alternatives; and (vi) the introduction of *opinion vectors* and *opinion surfaces*. This new method is recommended for supporting decision-making with large numbers of subjective criteria and/or alternatives and also for group decisions where the consensus must be evaluated. The MTC© provides a different promising perspective in decision-making and could lead to new research lines in the field of information systems.

Key Words: Decision support systems, Triangular Assessment Method, MTC©, Judgements, Group decisions, AHP, Analytic Hierarchy Process

1. Introduction

The Analytic Hierarchy Process (AHP) (Saaty, 1977, 1980) is one of the best known and most commonly used Multi-Criteria Decision-Making (MCDM) methods worldwide (Velasquez, Hester, 2013, Segura, Ray, Maroto, 2014). The method is used in several fields of science (Vaidya, Kumar, 2006, Ho, 2008, Ishizaka, Labib, 2011 and Subramanian, Ramanathan, 2012), particularly when the decision-making process involves intangible criteria (Saaty, 2013), such as environmental (e.g. Schomoldt, Kangas, Mendoza, Pesonen, 2001) and forest-related criteria (e.g. Kangas, Kangas, 2005, Kangas, Kangas, Kurttila, 2008, Diaz-Balteiro, Romero, 2008 and Feng, Wang, Yao, Ding, 2016), as well as in quality analysis (Badri, Al Qubaisi, Mohaidat, Al Dhaheri, Yang, Al Rashedi, Greer, 2016), assessment of different agricultural processes (Zhang, Su, Wu, Liang, 2015, Aydi, Abichou, Nasr, Louati, Zairi, 2016 and Yalew, van Griensven, Mul, van der Zaag, 2016) and in water resource management (Sun, Wang, Liu, Cai, Wub, Gengd, Xu, 2016 and Zhao, Jin, Zhu, Xu, Hang, Chen, Han, 2016).

Briefly, the AHP is a hierarchical decision-making process comprising the following steps (Forman, Gass, 2001): (i) structuring, i.e. definition of a decision-making hierarchy; (ii) determination of preferences on a scale established for paired comparisons between criteria, and between pairs of alternatives for each criterion; (iii) synthesis; and (iv) selection of the best alternative. Of these, determination of preferences is the most important step because it involves transformation of opinion judgement into a value or weight that is analysed in following steps. This is made possible by using a scale (Saaty, 1980) that has been developed in different studies (e.g. Dong, Xu, Li, Min, 2008, Ishizaka, Labib, 2011, Tsyganok, Kadenko, Andriichuk, 2016 and Cables, Lamata, Verdegay, 2016). The transformation involves the following axiomatic principles (Saaty, 1986): (i) the property of *reciprocal judgements* ($a_{ij} = 1/a_{ji}$), where a_{ij} represents the value of the comparison of the elements *i* and *j*, and a_{ji} is the value corresponding to the element *j* in comparison with *i*, which is fundamental for paired comparisons; (ii) homogeneity, which is characteristic of a person's ability to compare something that does not vary greatly from a common property and therefore must be organized hierarchically; (iii) the dependence of a level below the adjacent top level; and (iv) the idea that the result can only reflect the expectations of the decision maker.

The AHP has been widely discussed (e.g. Carmone, Kara, Zanakis, 1997, Zanazzi, 2003 and Schoner, Wedley, 2007). The high sensitivity of AHP to the number of criteria and alternatives is an important consideration. The number of pairs that must compared increases with the number of criteria and alternative options as a polynomial function. The number of pairs that must be assessed in AHP is expressed as follows:

$$\left[n \cdot (n-1)/2\right] + \left[n \cdot m \cdot (m-1)/2\right] \quad \text{[Eq. 1]}$$

where *n* is the number of criteria and *m* the number of alternatives (Ishizaka, Labib, 2011). Thus, for example, in a decision with 8 criteria and 5 alternatives, 108 pairs must be evaluated.

This leads to an increase in the time required for the evaluation, and the process could become lengthy and tedious, leading to loss of concentration and consistency of the final decision and thus limiting the application of AHP to decisions with a small number of criteria and alternatives (Yannou, 2002). In order to reduce the number of pairs evaluated, bifurcated schemes (clustering) can be constructed for criteria and sub-criteria at several levels, thus enabling analysis of groups (by root criteria or sub-criteria) (Millet, Harker, 1990 and Wedley, Schoner, Choo, 1993, 1996). However, when evaluation of a root criterion is highly subjective, the weight of the assessment is directly transferred to the respective sub-criteria. Other studies have considered the use of incomplete matrixes (Ishizaka, Labib, 2011 and Ureña, Chiclana, Morente-Molinera, Herrera-Viedma, 2015). However, in these methods the role of the decision maker is unclear, thus increasing the uncertainty of the use of this kind of methods, as highlighted by the question posed by Karanik, Wanderer, Gomez-Ruiz, Pelaez, (2016): "who really makes the final decision, the decision maker or the reconstruction method?" and affecting the fourth axiom of the AHP (Bana e Costa, Vansnick, 2008 and Kulakowski, 2015).

The other point that has been criticized is the axiom of *reciprocal judgements* in pairwise comparisons (i.e. the basis of the AHP method). This axiom is not always fulfilled because of the subjective nature of certain criteria and the cognitive skew that may occur in the judgement-making process, especially in complex decisions involving numerous criteria and/or alternatives (Escobar, Moreno-Jiménez, 2000). Chiao (2016) stated that the "Uncertainty is most likely an inherent condition in decision-making process" and proposed the use of fuzzy numbers instead of discrete scales. Several authors agree with this philosophy (e.g. Zhou, Xu, 2016; Dincer, Hacioglu, Tatoglu, Delen, 2016), although it has also been criticised (e.g. Zhü, 2014; Fredizzi, Krejčí, 2015).

Finally, there is also some controversy regarding the synthesis phase. Characterisation of the error or inconsistency of matrices and their sensitivity to inconsistency may be problematical. According to Saaty (1990), the inconsistency can be measured by the so-called consistency ratio (CR), and such inconsistency only occurs if the matrix that combines all of the comparisons is homogeneous. It can then either be used in the final process (consistent when CR ≤ 0.10) or be disregarded (inconsistent when CR > 0.10), although some authors report that such inconsistency is meaningless (e.g. Osei-Brison, 2006) or propose alternative approaches (Brunelli, 2016).

Regarding the consensus in the AHP, a common point highlighted in some recent studies is the use of the optimization or heuristic approach to obtain a result achieving maximum consensus (Ma, 2016, Escobar, Aguarón, Moreno-Jiménez, 2016, Aguarón, Escobar, Moreno-Jiménez, 2016 and Moreno-Jiménez, Salvador, Gargallo, Altuzarra, 2016). This consensus could be defined by reducing the Euclidean distance for the judgement weights (Blagojevic, Srdjevic, Srdjevic, Zoranovic, 2016), minimizing the inconsistency (Li, Ma, 2007 and Abel, Mikhailov, Keane, 2015) or minimizing the deviation between the intervals of fuzzy numbers (Mou, Xu, Liao, 2016). Zang (2016) proposed reconstructing the preference vector by using linear programming in fuzzy pair comparison judgements. Wang, Tong (2016) also propose using triangular fuzzy numbers. However, as mentioned above, the use of fuzzy numbers has also been criticised (i.e. Zhu, 2014 and Fedrizzi, Krejčí, 2015).

In response to the limitations of the AHP method, we present and describe a new MCDM method, denominated the "Triangular Assessment Method" (abbreviated to MTC© from the Spanish name, *Método del Triángulo de Calificaciones*) (Pérez-Rodríguez, Rojo-Alboreca, 2009, 2012a and Pérez-Rodríguez, 2013). The MTC© involves comparison of trios rather than pairs of elements, thus making use of the potential of the AHP methodology while avoiding its drawbacks. The MTC© aims to reduce the number of pairs that need to be assessed during application of the AHP method to problems involving numerous criteria and/or alternatives. In addition, the MTC© seeks to increase the representativeness of the weights assigned to the criteria and alternatives, thus attempting to mitigate any bias that may occur in the repetitive process of issuing judgements. The latter aspect is crucial, as perceiving the decision is a key initial step within a decision-making process to enable accurate judgement. If this perception is distorted, the decision will also be distorted, inaccurate and inconsistent. Numerous studies on post-perception, attention, perception and visual memory have been published in this field (e.g. He, Cavanagh, Intriligator, 1997, Bundesen, 1998, Cavanagh, 1999, Cavanagh, Labianca, Thornton, 2001, Bechara, 2003 and Zahir, 2006).

The main objectives of the present study were (i) to develop a transitivity-based method of capturing consistent judgements, (ii) to establish automated series of comparison criteria and alternatives and (iii) to compose the confidence intervals for each of the criteria and options under each criterion, at a given significance level.

2. The MTC© method

The MTC© was developed from two axiomatic principles involving consistent judgements for the evaluation of trios in the determination process: i) the *transitivity* and ii) *temporal variability of the opinion*. The MTC© proposes a construction of opinion surfaces and probability regions for the judgements during the synthesis phase while also introducing a stochastic approach to generating a set of possible solutions.

2.1. Determination: Consistent judgments for comparison of trios

The first axiom on which the MTC© method is based is as follows:

Axiom 1: Transitivity: *in a context of homogeneity in which items that are compared are of the same order of magnitude, if a criterion C_1 is preferred to a criterion C_2 and that same criterion C_2 is preferred to a criterion C_3, then criterion C_1 must be preferred to a criterion C_3 on the same scale on which the former two have been evaluated.*

This can be expressed mathematically as:

$$\forall C_1, C_2, C_3 \in C : C_1 R C_2 \wedge C_2 R C_3 \rightarrow C_1 R C_3$$

According to Axiom 1, when comparing any two criteria C_1 and C_2 (or two alternatives under the influence of one criterion), the importance (weight) that the decision maker gives to one element relative to the other is determined. The same applies for comparing criteria C_2 and C_3. Finally, when

C_1 and C_3 are compared, the relationship between the three criteria C_1, C_2 and C_3 becomes apparent. Thus, if criterion C_1 is preferred to C_2, and C_2 is preferred to C_3, then criterion C_1 should be preferred to C_3, so as to preserve the consistency (Saaty, 1990). In the AHP, determining the relationship between C_1 and C_3 is possible without explicitly evaluating the pair, provided the relationships $C_1 - C_2$ and $C_2 - C_3$ have already been evaluated (or more generally, provided enough evaluations have been made on pairs from which to derive the $C_1 - C_3$ relationship by means of a cascade of deductions, e.g. $C_1 - C_4$, $C_1 - C_2$, $C_2 - C_4$, $C_3 - C_4$). However, if inconsistency arises, detecting which particular pair causes the inconsistency may be complicated in the AHP. Figure 1 graphically illustrates this problem by representing the independent comparisons of pairs on an equilateral triangle formed by the three criteria (C_1, C_2 and C_3). The black spots on each side of the triangle represent the decision maker's trials corresponding to different pairs of criteria (vertices). Figure 1 shows that an error can be projected if one of the (unspecified) pairs is over- or undervalued. The difference between predictable or logical weight and the real weight obtained (in Figure 1 designed for the pair formed by criteria C_2 and C_3) may therefore have been projected on any side of the triangle, as the error distribution in the pairs that compose the triangle is not known.

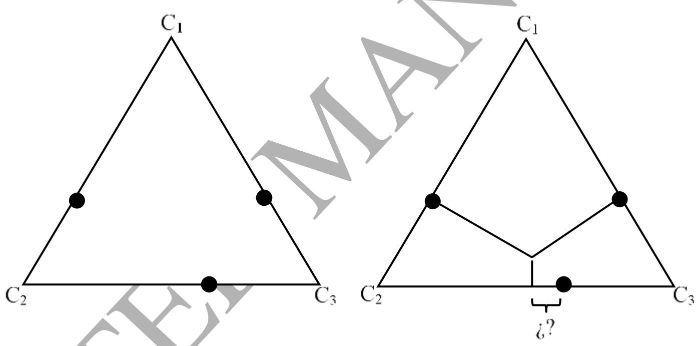

Figure 1: Graphical representation of a possible inconsistency in the pairwise evaluation of three criteria by the AHP method (the inconsistency is projected on an arbitrary side of the triangle).

A method of making consistent judgements based on Axiom 1 must therefore be established in order to prevent the incorporation of inconsistent judgements in the overall calculations. In Figure 2, the point defined as (x, y) represents the judgement of a decision maker on the basis of criteria C_1, C_2 and C_3 whose pairwise trade-offs are represented along the corresponding sides of an equilateral triangle. For example, the side $L = \left(X_{C_2} - X_{C_1}\right)$ represents the full range of magnitude with which C_1 may be preferred to C_2. Projecting the point (x, y) perpendicularly to the sides of the triangle yields the magnitude of preference or comparison (VC_{ij}) between each pair of criteria (on a continuous scale based on L graduation), which implies and strictly complies with Axiom 1.

Mathematically, as result of assessing triangle C_{123}, a vector composed of three elements (VC_{12}, VC_{23}, VC_{31}), whose sum is a constant, can be established:

$$(VC_{12}+VC_{23}+VC_{31})=\frac{2\delta}{3} \quad VC\text{ij} \in [0,\delta\,]$$ [Eq. 2]

Equation 2 is defined by bounded assessments in an arbitrary interval [0, δ], which should be established for a given decision, where the lowest value of the comparison of the pair is zero and the maximum is δ (which must be constant throughout the decision-making process and in the following possible repetitions).

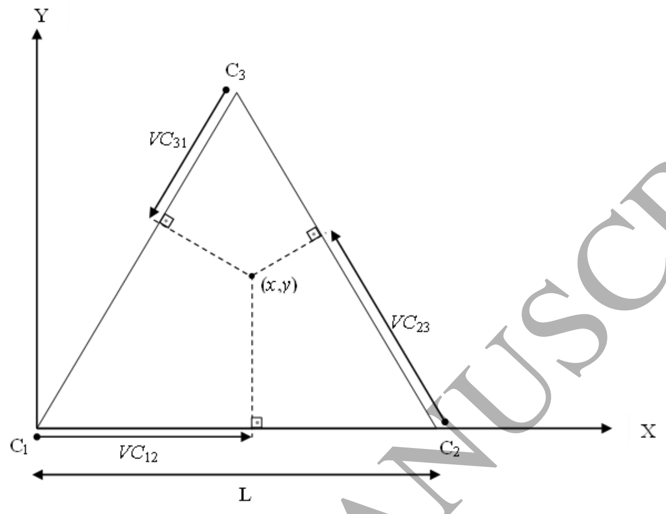

Figure 2: Triangle rating scheme, i.e. the basis of the MTC© method.

The weights assigned to pairs of criteria in the comparison triangle (VC_{ij}) are calculated simultaneously and complying with the transitivity, using Equation 3:

$$VC_{12}=\left(x/\left(x_{C_1}-x_{C_2}\right)\right)\cdot\delta$$

$$VC_{23}=\frac{\sqrt{\left(\frac{\left(x_{C_1}-x_{C_2}\right)}{2}-x_s\right)^2+(y_s)^2}}{\left(x_{C_1}-x_{C_2}\right)}\cdot\delta$$

$$VC_{31}=\frac{2\cdot\delta}{3}-\left(VC_{12}+VC_{23}\right)$$

where: $$x_s=\frac{y-\tan\left(\frac{\pi}{6}\right)\cdot x-\tan\left(\frac{\pi}{3}\right)\cdot\left(x_{C_1}-x_{C_2}\right)}{-\tan\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{6}\right)}$$

$$y_s=-\tan\left(\frac{\pi}{3}\right)\cdot x_s+\tan\left(\frac{\pi}{3}\right)\cdot\left(x_{C_1}-x_{C_2}\right)$$ [Eq. 3]

In assessing a triangle, when a decision maker assigns a greater weight to one criterion than to the other two, the point (x, y) approaches the vertex of the preferred criterion. Moreover, the evaluation of the other two criteria is equalized, for fulfilment of transitivity. Otherwise, if (*x*, *y*) is close to the centre of the triangle, there is no preference for any of the three criteria, i.e. the evaluations are equal.

However, this would become extremely complicated for four criteria, as the evaluation would be represented by a three-dimensional equilateral pyramid. The complexity would lead to loss of manageability and ease of evaluation in computer software displayed on a screen. Logically, the decisions and calculation involved in evaluating more than four criteria are even more difficult.

Once the judgement has been input in a triangle, the inconsistency of this triad can be assumed to be equal to zero (Axiom 1). However, the variability in the judgement (weight) in other comparison processes must be considered, i.e. for one pair (C_1 and C_2) the weight has been obtained by evaluating the triangle comprising criteria C_1, C_2 and C_3, but when the third vertex is changed to a different criterion C_4 (so that the triangle now comprises criteria C_4, C_1 and C_2), the weight for the pair C_1 and C_2 may be different from that obtained in the triangle initially evaluated. This could occur with a large number of criteria, because of the complexity of scaling the judgement under a group of criteria, and accurately reflects each comparison value in each triangle, as in the comparison of pairs. A second axiom is therefore assumed in the MTC© method:

Axiom 2: Temporal variability in the judgement: *in evaluation of all criteria or alternatives the decision maker is influenced by the environment and the context of the decision, which can lead to confusion and distortion in the assessments, especially in assessing criteria or alternatives with subjective opinions, which generally become more complicated as the decision-making time increases.*

According to Axiom 2, the time factor is important and must be taken into account in analysis of the process, because it may introduce some variability in the judgements of the criteria and alternatives (Chiao, 2016). Mathematically, Axiom 2 can be expressed as follows:

$$VC_{ij}(t) = \delta - VC_{ji}(t+1) \pm \varepsilon \qquad \text{[Eq. 4]}$$

where δ is the maximum value that can be established in a comparison between two criteria and ε is the associated uncertainty.

In the case of a number of criteria *n* at the same level (more than three), the MTC© divides the decision-making process into multiple triangles, so that all the criteria are implicitly or explicitly evaluated. Thus, even if one pair has not been directly evaluated, the weight can be determined from the evaluations of the other pairs. This is possible by comparing different series of comparisons, one of which is shown in Equation 5 for a generic triangle with criteria C_a, C_b, C_c, where a, b and c represent the index of each criteria of a set $C = [C_1, C_2, C_3, \ldots, C_n]$.

$$\begin{aligned} &\mathrm{a} = i \in [1, n] \cap \mathrm{N} \\ &\begin{cases} \mathrm{b} = (i+1) - n \Leftarrow i+1 > n \\ \mathrm{b} = i+1 \Leftarrow i+1 \le n \end{cases} \\ &\begin{cases} \mathrm{c} = (i+2) - n \Leftarrow i+2 > n \\ \mathrm{c} = i+2 \Leftarrow i+2 \le n \end{cases} \end{aligned} \qquad \text{[Eq. 5]}$$

By using these series, a decision-making scheme for a criterion level is evaluated in trios in a cyclic or closed process that returns to the starting point, in which criterion C_i is compared with its nearest neighbours (C_{i-1} and C_{i+1}) and then with second level neighbours (C_{i-1}, C_{i-2}, C_{i+1} and C_{i+2}). For example, in the case of eight criteria, the following triangles are constructed: C_1-C_2-C_3; C_2-C_3-C_4; ... C_7-C_8-C_1 and C_8-C_1-C_2, i.e. eight trios of criteria are produced. The MTC© thus greatly reduces the time required for decision-making, as in the above example eight triangles are evaluated instead of the 28 pairs that would have to be compared in the AHP (calculated with Equation 1).

Continuing with the example, from the eight triangles developed in the MTC©, the results of the comparison of 8·3 = 24 pairs can be obtained, so that all the relationships needed in order to adapt this system to a square matrix of relationships are not available. However, some pairs are repeated and the triangulation is a geometric figure. If the criteria are distributed in a circle and all the relationships in the above scheme are represented, an octagon will be produced for eight criteria.

Thus, in the MTC© proposed number of comparisons in trios is

$$(n)+(n \cdot m) \quad \text{[Eq. 6]}$$

where *n* is the number of criteria and *m* the number of alternatives. In the same example (8 criteria and 5 alternatives), the number of triangles is 48, which can be considered a substantial reduction (relative to the 108 pairs in the AHP calculated with Equation 1).

Furthermore, this configuration enables the vertices of the triangles to be rotated so that the position of the pairs will differ in each triangle, thereby minimizing the effect of visual memory of pairs that are repeated. Each new repetition will therefore be independent of the previous one and will characterize the accuracy of the weighting of the pairs evaluated.

2.2. Synthesis: construction of opinion surfaces and probability regions for the judgements

The defined succession of series of triangles is used to determine how the weight assigned to a pair will vary relative to the qualification of that same pair in successive triangles. On completion of this series it can be assumed that all pairs evaluated have been rated, as the comparative cycle of triangles with zero inconsistency has been completed. In this series, and considering two contiguous evaluations of the trios, for example (C_1, C_2, C_3) and (C_2, C_3, C_4), we would obtain VC_{12}, VC_{23} and VC_{31} from the first, and VC'_{23}, VC_{34} and VC_{42} from the second.

In this case $VC_{23} = \delta - VC'_{32} + \varepsilon$, where δ is the maximum value of the comparison between criteria and ε is the error that can be committed in the evaluation of the C_2 - C_3 pair in the triangle. If $VC_{23} = \delta - VC'_{32}$ then $\varepsilon = 0$, and the same pair has thus been assigned the same weight in the two different assessments. However, the weight assigned to a pair of criteria may change in an evaluation depending on the third criterion with which it is compared (Axiom 2).

The series of proposed evaluations result in *n* vectors, which are known as *opinion vectors* (these will comprise six elements each for the example with eight criteria). These have in common that they are assessments of the same criterion relative to others. For example, taking into account that

$VC_{ij} = \delta - VC_{ji}$, the vector obtained for criterion C_1 for a set of eight criteria is (VC_{12}, VC'_{12}, VC_{13}, VC_{18}, VC_{17}, VC'_{17}), as the assessment can include the reciprocal within an evaluation of certain duration. Basically, this vector is shown in Equation 7:

$$WC_i = (VC_{i,i-2}, VC_{i,i+1}, VC'_{i,i+1}, \lfloor \delta - VC_{i-1,i} \rfloor, \lfloor \delta - VC'_{i-1,i} \rfloor, \lfloor \delta - VC_{i+2,i} \rfloor) \quad \text{[Eq. 7]}$$

with the following conditions:

$$\begin{cases} (1+2) - n \Leftarrow (i+2) > n \\ (i+1) - n \Leftarrow (i+1) > n \\ n - (1-i) \Leftarrow (i-1) \leq 0 \\ n - (2-i) \Leftarrow (i-2) \leq 0 \end{cases}$$

and where *n* is the number of criteria considered in the decision-making process.

The WC_i vector can be used statistically to obtain ranges from relationships that have been assessed only once, as these are within triangles whose sides are formed by duplicate relationships. Triangulation is used to obtain the relationships of pairs that have not been evaluated, closing the side or the relationship that is unknown with two sides that are known and that have been obtained in duplicate, as they have been repeated. It is thus possible to calculate the range of the value of the pairs that are not assessed directly, because the same relationship can be achieved with different triangles, by selecting the appropriate sides. The procedure for calculating uncertainty is graphically illustrated in Figure 3.

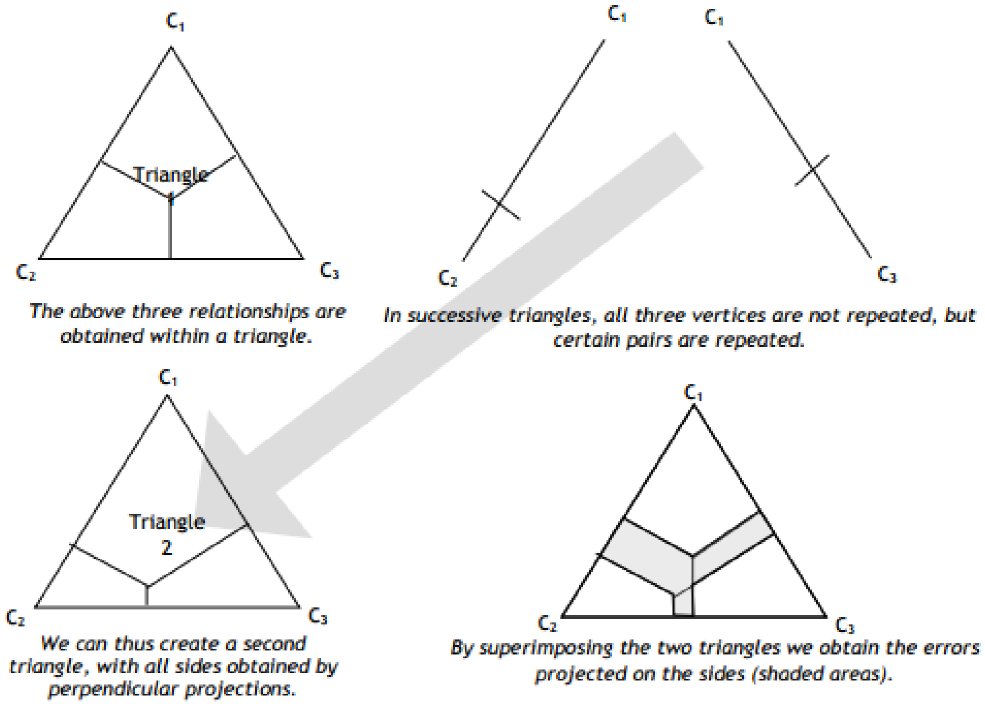

Figure 3: Procedure for determining uncertainties associated with weights in the MTC© method, in which only one repetition of two pairs is applied, and the other is estimated by constructing another triangle.

Obviously, all of the information provided in this section for evaluating criteria is equally applicable in MTC© to the comparison by trios of alternatives under each criterion (Pérez-Rodríguez, Rojo-Alboreca, 2009, 2012a and Pérez-Rodríguez, 2013).

Opinion vectors obtained by evaluating the full set of triangles show that judgements may vary according to the criteria involved, but may also vary according to the time factor. Thus, the opinion of a decision maker with respect to a criterion should not be considered a discrete value, but as a range or interval within which this opinion is likely to be included, with a certain probability. However, as noted above, opinion vectors include a small number of elements (six for the above example with eight criteria), which are not sufficient to establish a representative opinion distribution.

Therefore, all possible opinions that decision makers have about a criterion and an alternative under the influence of that criterion can be enclosed in an *opinion surface* bounded by ranges or intervals obtained. This surface represents the entire domain in which the weight of the alternative under the considered criterion can be found as a result of the repeated evaluations, and it may be a rectangle, an ellipse or a similar figure. In successive iterations, the opinion surfaces can become limiting, in the case of overlap, and their position and surface may vary. Opinion surfaces thus provide information about the influence of repetitions in the final decision, and in the case of group decisions the degree of uniformity can be measured.

Opinion surfaces illustrate the favourable opinion (areas of greater weight) or unfavourable opinion (lighter areas) of the decision maker regarding the criterion and alternative, as shown in Figure 4 where the opinion surface is established as a rectangle.

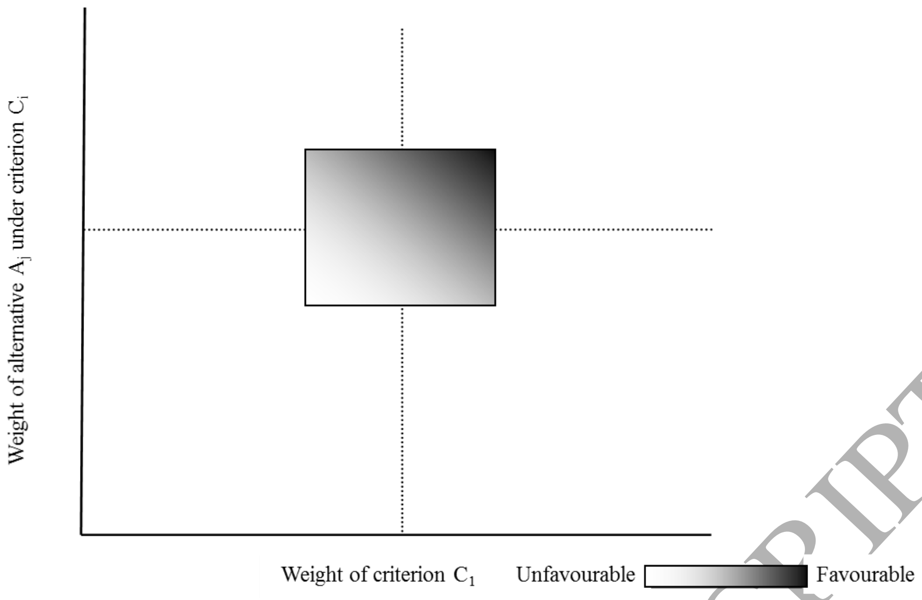

Figure 4: Zoning of favourable/unfavourable opinions from the weighting of a criterion and an alternative to the opinion surface of a decision maker in the case of a rectangle.

On the other hand, a probability of occurrence can be assigned to each point of an opinion surface, and thus it is possible to construct three dimensional *regions of probability of the opinions* (weight of the criterion, weight of the alternative and probability of occurrence), which include all possible trios of decisions. Depending on the assumed probability distribution, these regions could be parallelepipeds (if opinion surfaces have been established as rectangles), elliptic paraboloids (if opinion surfaces are ellipses), or some similar figure or mixture of the above.

From the data available in opinion vectors, different alternatives can be used to establish the dimensions, the shape and the position of the opinion surfaces on axes. However, it is difficult to establish the probability distribution at any point of the opinion surface as a result of the trio comparison process. We therefore propose the following Hypothesis I:

Hypothesis I: *All points that are part of the opinion surface have a probability of occurrence as a bivariate normal distribution characterized by a mean and a standard deviation for each of the two variables*.

Assuming that this hypothesis means that regions of opinions take the shape of elliptic paraboloids, as a result of the intersection of the two normal distributions considered, one for the criterion (on the X axis) and the other for the alternative (on the Z axis), the opinion surfaces (projection on the X and Z axes) will therefore be ellipses. Only in the case of two exactly equal normal distributions, the probability region would be a paraboloid of revolution, whose sections (projections onto the X and Z axes) would correspond to circles.

Two possible theorems can be considered within this hypothesis, as the bivariate distributions can be established for dependent or independent variables:

Theorem I: *The weight assigned to the criteria and the alternatives are independent*.

In the MTC©, criteria are assessed regardless of the evaluation of alternatives under each criterion, so that one judgement does not depend on the other, and the covariance between the two variables is thus equal to zero. In this case the probability density function is given by Equation 8:

$$f(x_1, x_2) = \exp\left[-\frac{1}{2}\left[\left(\frac{x_1-\mu_1}{\sqrt{\sigma_1}}\right)^2 + \left(\frac{x_2-\mu_2}{\sqrt{\sigma_2}}\right)^2\right]\right] \quad \text{[Eq. 8]}$$

where μ is the average and σ is the standard deviation of each variable.

Theorem II: *the weights assigned to the criteria and the alternatives are dependent*.

In this case it is assumed that the assessment of criteria and of alternatives depend on each other, so that in the bivariate analysis the covariance between the two variables must be taken into account. In this case, the probability function is given by Equation 9:

$$f(x_1, x_2) = \exp\left[-\frac{1}{2\left(1-\rho_{12}^2\right)}\left[\left(\frac{x_1-\mu_1}{\sqrt{\sigma_1}}\right)^2 + \left(\frac{x_2-\mu_2}{\sqrt{\sigma_2}}\right)^2 - 2\rho_{12}\left(\frac{x_1-\mu_1}{\sqrt{\sigma_1}}\right)\left(\frac{x_2-\mu_2}{\sqrt{\sigma_2}}\right)\right]\right] \quad \text{[Eq. 9]}$$

where μ is the average, σ is the standard deviation of each variable and ρ is the existing covariance between the two variables.

Theorem I seems reasonable and was assumed here, as evaluation of criteria in the MTC© occurs independently of the alternatives (one evaluation can be done before another, indifferently, and at different times), and there will not usually be any relationship or dependence between them.

In addition, and independently of which theorem is assumed, when *N* repetitions are made (by one or various decision makers), the probability according to the hypothesis I is the result of arithmetically summing the probabilities corresponding to the overlapping surfaces (representing consensus). Thus, the areas of greater probability of occurrence are those in which there is greater overlap or consensus between the opinion ellipses of the different decisions (Figure 5), although in this case the value that joins each ellipse will not always be the same.

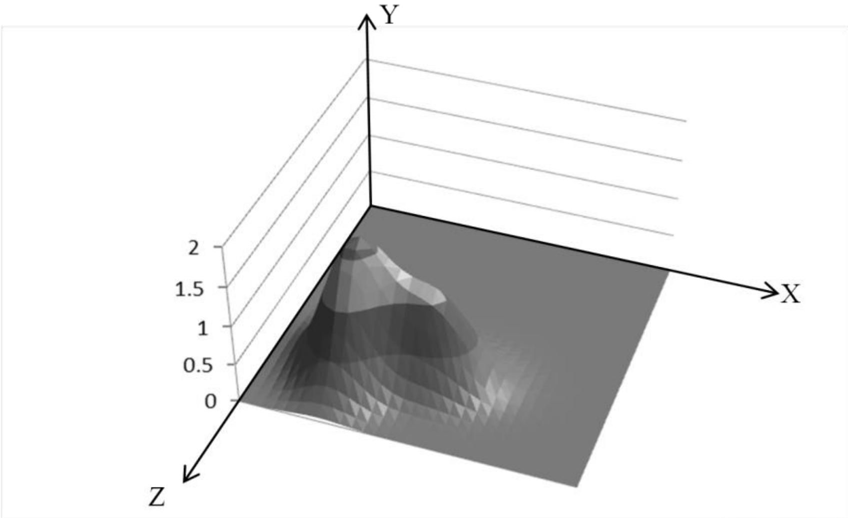

Figure 5: Example of a probability distribution of the weights assigned to a criterion and an alternative, in the case of *N* repeats by applying an independent bivariate function (Theorem I).

The main advantages of this hypothesis are that (i) it does not accept that all points in the region of opinion have the same probability of occurrence and (ii) the value of the probability varies when several decisions overlap (depending on whether they occur in areas of high or low probability within the elliptic surfaces of opinion). However, the disadvantage of the hypothesis is that it is assumed that the probability of the opinion about the criteria and alternatives follows a bivariate normal distribution, when the small number of data of the vectors of opinion does not disclose the actual distribution with certainty.

2.3. Stochastic approach to obtaining a set of possible solutions

The resolution of a decision under a hypothesis becomes a stochastic problem, in which the weights assigned to criteria and alternatives will be randomly extracted under the probability distribution established according to this hypothesis. The use of random results implies that an infinite number of satisfactory solutions can be generated, and therefore the number should be limited by applying statistical inference.

In order to generate random results under a given distribution (or region of probability of the opinions), we generated a random value between minimum probability of occurrence and the maximum cumulative probability, using the discrete values of the weights assigned to the criteria (X axis) associated with the probability of occurrence (Y axis). This yields the weights for each criterion and alternative.

For example, assuming a region of probability of opinions as shown in Figure 6, projecting it on the XY-plane yields the probability distribution of the weight assigned to that criterion according to one or more decision makers, as shown in Figure 7.

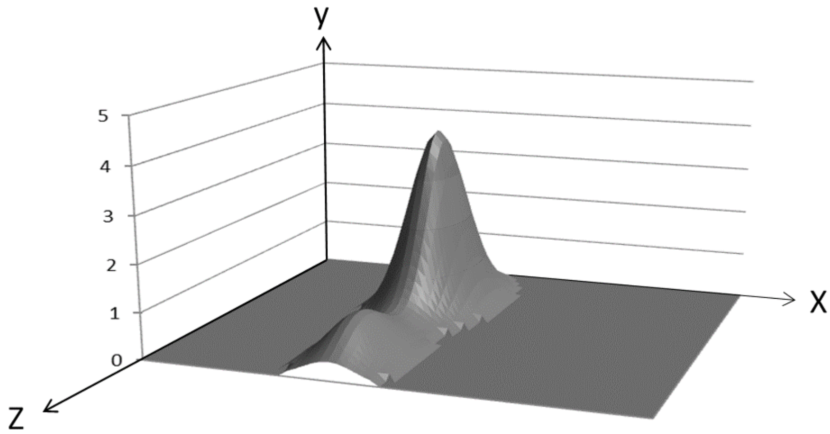

Figure 6: Example of one region of probability of opinions.

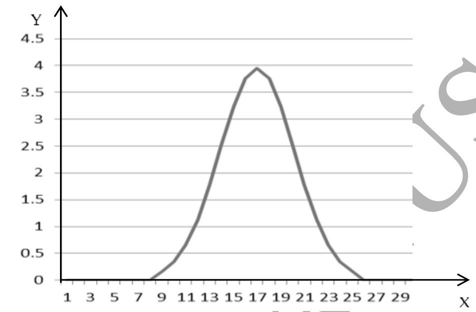

Figure 7: Projection on to the XY axis of the region of probability of the opinions of Figure 6 for a random value of the alternative.

On the other hand, if a weight allocated to a criterion on the basis of the previously obtained probability is chosen at random (Figure 6) and projected onto the YZ plane, the distribution of the weight of that alternative under the influence of that criterion is shown in Figure 8. If a weight based on that distribution is randomly chosen, the weight of that alternative is obtained.

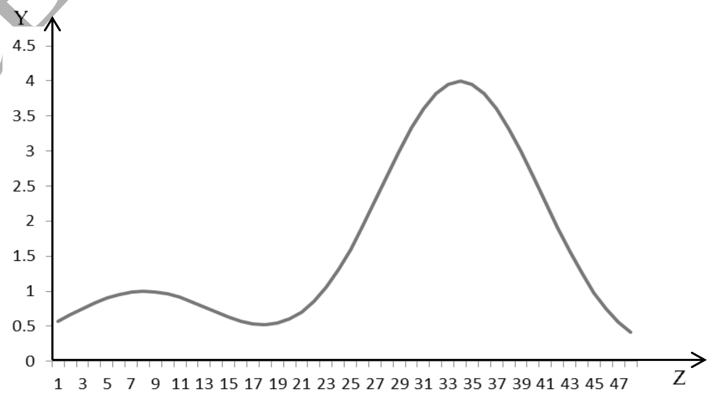

Figure 8: Projection on the YZ axis of the region of probability of the opinions of Figure 6 for a random value of the criterion.

Analytically, a decision is summarized in a set of opinions, and there are endless possibilities that lead to different results. A single iteration shows a satisfactory, possible and viable result. Nonetheless, in a stochastic context a number of iterations must be performed to achieve the result that meets the desired target. When the weights assigned to the criteria and alternatives are randomly extracted, sufficient data are obtained to complement the paired comparisons matrix proposed by Saaty (1980). Thus, a comprehensive analysis of these results identical to that used in the AHP method can be conducted: this consists of matrix multiplication of the weights assigned to alternatives and criteria to produce a hierarchy of the alternatives. The process should be repeated a number of times to yield robust data. It is therefore necessary to obtain a large number of results, which can then be analysed by statistical procedures or scores (e.g. the number of times that an alternative is chosen).

3. Comparative application of the MTC© and AHP methods

3.1. Design of an example for comparing the methods

The MTC© and AHP methods were compared by applying them to a complex example including numerous criteria and alternatives. The example was evaluated by different expert decision makers who repeated their evaluations with both methods, without discussing their decisions with each other. The decision makers also chose the order in which to apply the two methods and the number of non-consecutive repetitions.

The decision involved selection of a particular type of forestry harvesting equipment. It was assumed that a small/medium sized forest harvesting company wishes to decide whether to continue with the current harvesting method carried out by two workers with chainsaws or to acquire a self-propelled harvesting machine. In this case, 18 criteria (classified in four groups and, therefore, on two levels) and 7 alternatives were considered. As already mentioned, one of the weak points of the pairwise comparison methods is that they are very sensitive to the number of criteria and/or alternatives, as pairs are constructed in different combinations. This example was therefore designed with a large number of criteria and alternatives, to assess the influence of the two methods on performance. Moreover, the large number of criteria and/or alternatives also made the decision-making process very long.

The criteria proposed for this decision were grouped as follows: economic criteria (lower purchase price; lower maintenance costs; lower fuel consumption; lower cost of travel); environmental criteria (low erosion; less soil compaction; less impact on vegetation not belonging to the exploitation; lower CO_2 emissions); social criteria (less damage to infrastructure; less need for training; increased security/stability; less noise; increased job creation); and technical criteria (better performance; greater manageability/operability; greater flexibility of use: final cutting, thinning and other uses; ease of movement within the forest; greater cutting diameter). Different types of criteria (such as social and environmental criteria) were therefore included in addition to the usual economic and technical criteria, so that the decision was in accordance with current demands for forest sustainability.

For comparison of the methods, 7 alternative options were chosen in order to make the decision feasible, which is an essential requirement in the design of any decision. The various alternatives differed in several aspects (such as the size of the machinery and whether it was new or used, specifically designed for forestry or not) and also included the existing harvesting method (two workers with chainsaws). Thus, the following 7 alternatives were proposed: Sampo Rosenlew 1046Pro; Sampo Rosenlew 1066; Valmet 911.1; Valtra 130 Tractor with Monra 600 harvester head; JCB Excavator 210 with Kesla 20RHS harvester head; Ponsse Ergo H73A; and two workers with chainsaws. A dossier with clear and concise information about the alternatives was compiled and given to each decision maker in lieu of verbal instructions/explanations.

To carry out the decision according to the AHP method, MPC© 2.0 software (Pérez-Rodríguez, Rojo-Alboreca, 2012b) was used. To facilitate the application of the MTC© method, a prototype was developed specifically for this case in Microsoft© Excel 2013, supplemented with scheduled forms with Visual Basic for applications.

3.2. AHP results

Time was a decisive factor in this method, and most decision makers only repeated the process once with AHP, arguing reasons such as monotony or fatigue due to the long duration of the evaluation, which required consideration of 564 pairs. Therefore, all except decision maker 1 made a single repetition and invested between 40 minutes and almost two hours, i.e. an average of 6.9 seconds for each pair of criteria evaluated.

The weights obtained indicated that the opinions of the different decision makers were very variable. Figure 9 shows the weights assigned to the root criteria by different decision makers; for example, the weights assigned to the technical criteria varied enormously, from 0.25 to almost 0.60, although the weights assigned by all decision makers except one ranged from 0.45 and 0.60 for that group of criteria.

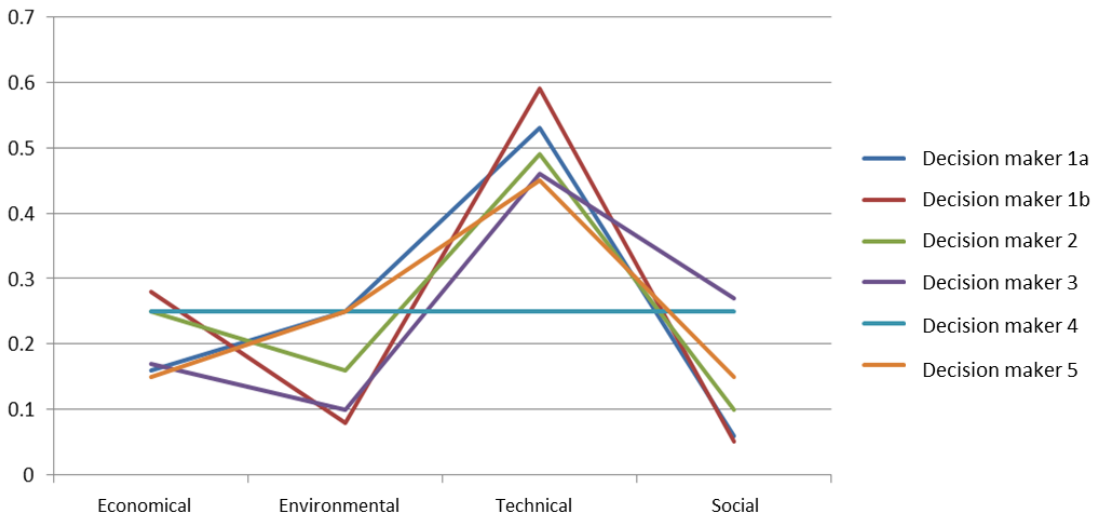

Figure 9: Weights for the root criteria according to the opinions of the different decision makers using the AHP method.

All the evaluations were inconsistent, to a greater or lesser degree, and the consistency ratio (CR) (Saaty, 1990) was higher than 30% in some cases. The inconsistency displayed by decision maker 1 was reduced slightly in the second assessment, from 24.5% in root criteria tree of decisions in the first assessment to 17.5% in the second, although the consistency ratio remained very high.

Overall, the alternative assigned the greatest weight by all decision makers was the existing option, i.e. workers with chainsaws (Figure 10). Nonetheless, the great variability in the results should be noted (Table 1).

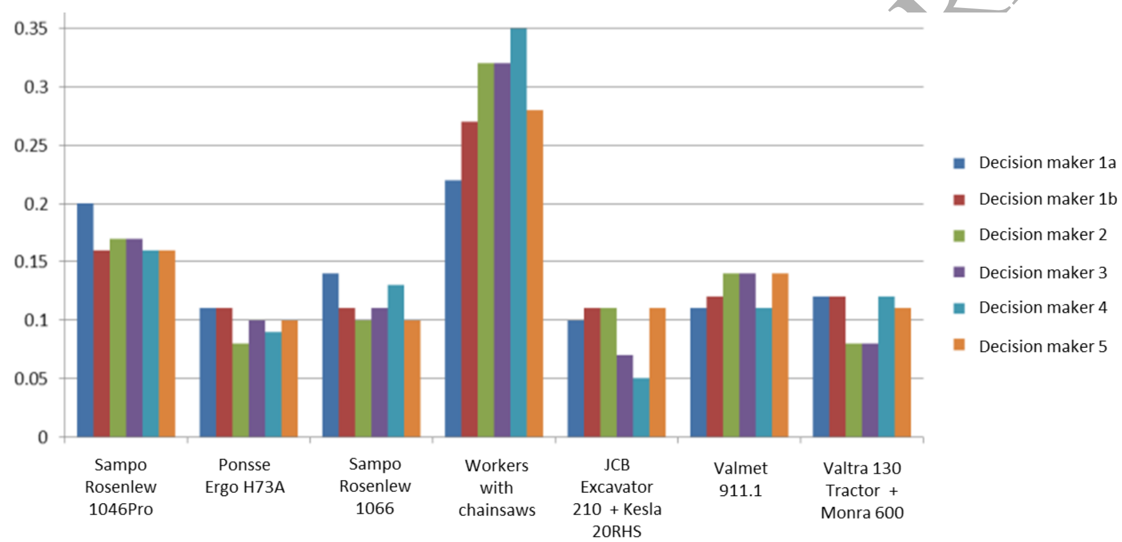

Figure 10: Global results of the weight assigned to each alternative in the AHP method.

Table 1: Variability of the results obtained (in %) with the AHP method.

	Weight	**Stand. Dev.**
Two workers with chainsaws	29.80%	5.02%
Sampo Rosenlew 1046 Pro	17.20%	1.64%
Valmet 911.1 (2003)	12.80%	1.64%
Sampo Rosenlew 1066	11.60%	1.82%
Valtra Tractor + Monra 600	10.20%	2.05%
Ponsse Ergo H73A	9.60%	1.14%
Excavator + Kesla RHS20	8.80%	2.68%

3.3. MTC© results

In the MTC©, each of the decision makers carried out a single evaluation, investing between 20 and 50 minutes to evaluate the 146 triangles generated in this example. This represented an average of 15 seconds for each triangle evaluated.

Each decision maker considered the proposed alternatives, according to his/her own opinions and experience, seeking the most satisfactory alternative and also considering the nature of the criteria used and the uncertainty of the context. Obviously, the decision makers who participated in the evaluation did not necessarily agree or have all points in common.

With respect to group decisions, with the MTC© it is possible to seek consensus from all decision makers analytically. In this case, the results obtained by grouping all decisions are shown in Figure 11.

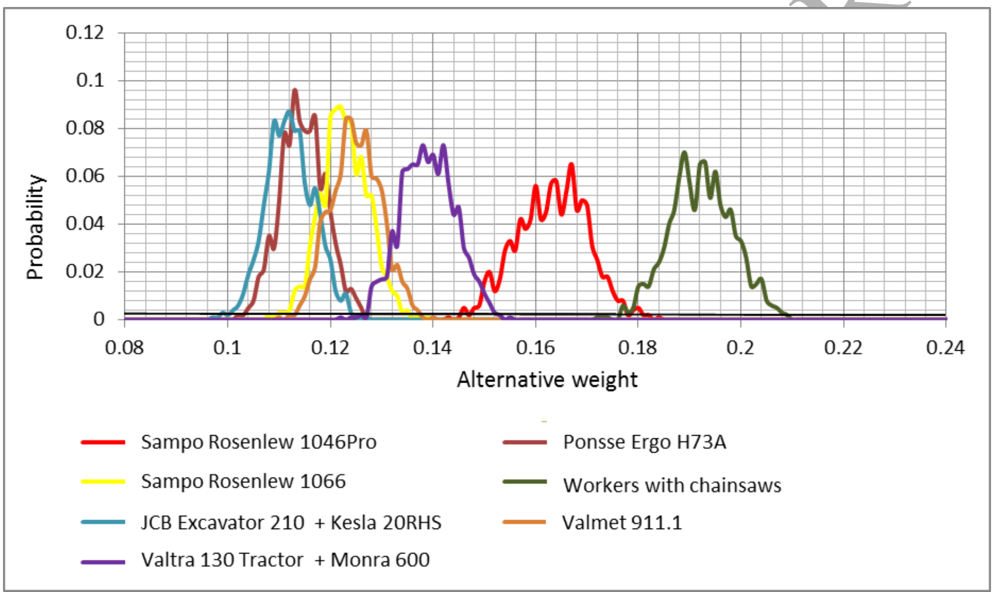

Figure 11: Result of the probability of the weight assigned to each alternative under a group simulation of the probability associated with each decision maker in the MTC© method.

With respect to the criteria, the results obtained in the evaluation group are shown in Figure 12. The range of uncertainty of all criteria varied widely, and some had a low weight and low uncertainty (such as environmental root criterion) and others had a high level of uncertainty (such as less need for training).

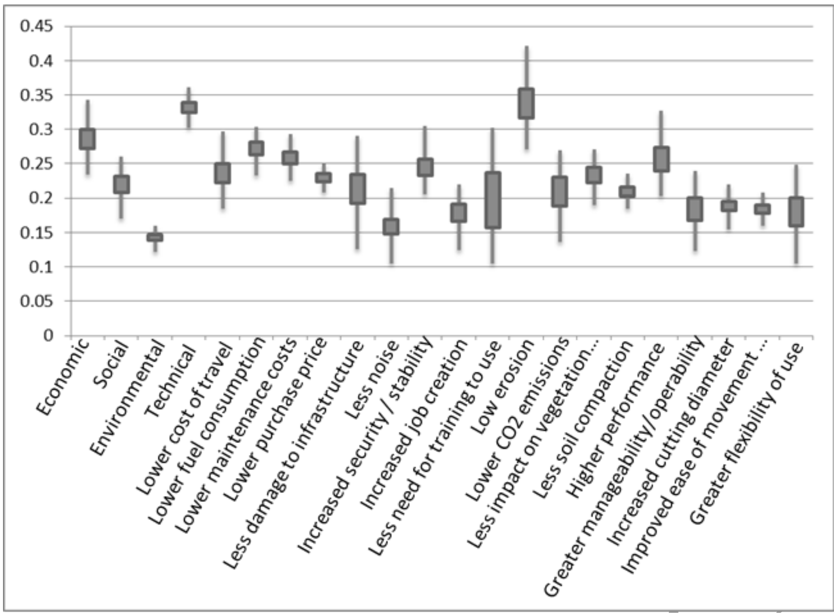

Figure 12: Ranges of uncertainty in the group decision for the weight assigned to each criterion with the MTC© method.

In the simulated procedure, the number of times that an alternative came first or last in the hierarchy was counted. According to this, the workers with chainsaws came first in 98.2% of the simulations, which was a very high proportion relative to the other alternatives. However, a hierarchy of alternatives is often required, in case the first choice is not available for any reason. The hierarchy obtained by MTC© in one thousand iterations is shown in Table 2.

Table 2: Hierarchy obtained by one thousand iterations with the MTC© simulation.

	Weight	**Stand. Dev.**
Workers with chainsaws	19.27%	0.63%
Sampo Rosenlew 1046Pro	16.37%	0.69%
Valtra T130 Tractor + Monra 600	13.96%	0.53%
Valmet 911.1 (2003)	12.54%	0.48%
Sampo Rosenlew 1066	12.33%	0.47%
Ponsse Ergo A73A	11.51%	0.44%
Excavator + processor head Kesla RH20	11.26%	0.47%

4. Discussion

4.1. Determination: Number of judgements

The number of pairs considered in AHP is important for producing enough data to assess in the synthesis phase and thus yield representative results. However, the number of possible pairs can be very large, making the method tedious and impractical (Yannou, 2002). The number of pairs that must been compared in the AHP method increases with the number of criteria and alternative options as a polynomial function (Equation 1). Several lines of study have pursued the goal of decreasing the number of comparisons in different ways, e.g. by using clustering (Millet, Harker, 1990) or incomplete matrix (Ureña, Chiclana, Morente-Molinera, Herrera-Viedma, 2015). The current trend is to apply complex linear programming approaches (i.e. Wang, Tong, 2016), which could render the decision-making process opaque (Bana e Costa, Vansnick, 2008) contrary to the transparent principles of AHP (Saaty, 2005). The MTC© method focuses on the determination phase by using triangles instead of pairs and reducing the number of elements that must be evaluated (Equation 6) relative to the AHP (Figure 13). For this reason, MTC© seems a more appropriate method for decisions involving large numbers of criteria and/or alternatives.

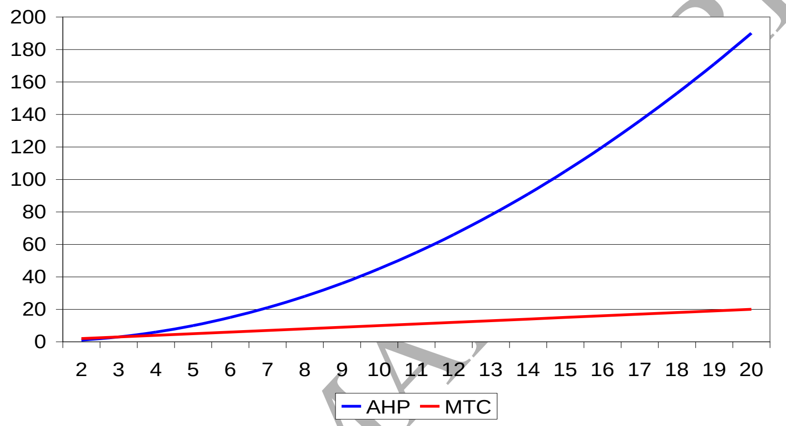

Figure 13: Comparing the number of pairs of decisions considered in AHP with the number of triangles considered in MTC©, depending on the number of elements involved in the decision.

The reduction in the number of evaluations does not mean a reduction in the time required for each evaluation. As the example shows, each decision maker spent on average twice as long to evaluate the criteria and alternatives when using the AHP than with the MTC©. This is because the MTC© involves far fewer assessments (in the example, 146 triangles) than in the AHP (564 couples, 386% more in this case). This is true despite the much longer time required to evaluate a triangle in the MTC© (15 seconds in the example, because of the greater complexity) in comparison with the time required to assess one pair in AHP (6.9 seconds, 46% less). For decisions involving a large number of criteria and/or alternatives, the AHP method takes a long time, which may lead to loss of concentration, increase the number of inconsistencies and decrease the value of the results (Kurtilla, Pesonen, Kangas, Kajanus, 2000). However, it is clearly much easier to compare a pair of elements than a trio, as indicated by the average time required for each evaluation with the two methods. Nonetheless, the greater difficulty involved in the MTC© requires greater attention from decision makers.

4.2. Determination: transitive axiom

Is debatable whether the clearly and logically transitive relationship occurs in the AHP method of pairwise comparisons, because inaccurate evaluation of one of the pairs may be made for various reasons (inappropriate use of the scale, error in the evaluation, unclear opinion about the pair being compared, etc.), thus generating an illogical unforeseeable result and inconsistency (Saaty, 1980, 1990). The MTC© does not work with discrete weights, but with ranges representing the uncertainties, which are generated by repeated iterations. It can therefore be assumed that the ranges cover the variation in weights assigned to each of the criteria/alternatives under the bounded uncertainty.

4.3. Synthesis: data variability

Regarding the inconsistencies, it is difficult to understand the uncertainty as it is an indicator of the matrix level (Osei-Brison, 2006). The MTC© does not work with this indicator, as when weights are randomly generated and fed back into the AHP matrix calculation process, the matrix is completely homogeneous, and therefore there are no inconsistencies. In the MTC©, the trio evaluation process is based on a closed cycle of comparisons, returning to the beginning of the process once the comparison is completed. Thus, the variation in the weighting of the criteria or alternatives is determined by the change in the context of the evaluation (i.e. changing any vertex of the triangle), as well as by extracting different opinions relative to the same criteria in different time periods, because the subjective opinion is intrinsically associated with variability (Chiao, 2016).

The system of making judgements could potentially be improved by comparing criteria or alternatives in trios instead of in pairs; this is the basis of MTC© method implemented by Axiom 1. Thus, the decision maker must make a judgement about three elements (and then to three pairs) at the same time. Although the assessment is more complex, the inconsistency in the trio of pairs compared can be assumed to be zero (by strict transitivity). This triad of pairs can be regarded as an equilateral triangle, as shown in Figure 2, in which each vertex represents a criterion (e.g. criteria C_1, C_2 and C_3), with all possible combinations for comparison appearing on the surface. Indeed, although it may be more difficult to assess a trio rather than a pair, we believe that this implies the need for greater attention in the decision and a consequent reduction in the number of inconsistencies. Moreover, the relationship between three criteria is relatively easy to establish by considering an equilateral triangle, helped by concentric lines and rectilinear lines perpendicular to each side of the triangle (coming together in the centre), as shown in Figure 14.

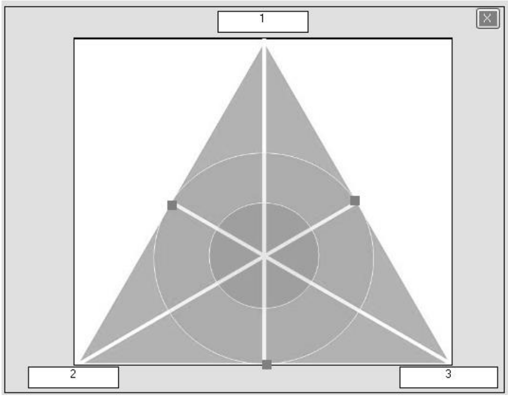

Figure 14: MTC© triangle with reference lines and circles to facilitate its evaluation.

In analysing vector data, various authors (e.g. Mateu-Figueras, Martín-Fernández, Pawlowsky-Glahn, Barceló-Vidal, 2003) have reported that the complexity of the statistical analysis of compositional vector data is due to the fact that the sum of the components is constant. However, this is logical and necessary (as outlined in Axiom 1), and this complexity must be taken into account in the statistical analysis of these vectors. In the example used to compare the two methods, the same result was obtained (the workers with chainsaws). This indicates that both methods can be applied with some degree of certainty, at least in some cases, although the benefits and drawbacks of each must be highlighted.

On the other hand, human opinion is difficult to translate to a discrete value, which may be resolved by the use of fuzzy approaches. The proposed method simplifies this procedure and uses triads of simultaneous judgements thus allowing transitive relationships between these. Although direct judgement in the triangle is more difficult for the decision maker, the results are more representative. In this context, we must take into account that we are working with human opinions and the judgement process could be affected by many factors, such as e.g. emotion (George, Dane, 2016). This intrinsic property generates sound in the models and variability, leading to uncertainly (Chiao, 2016).

5. Future research lines

We suggest the following future research lines in order to improve the method: (i) study of the user-triangle iteration and how it could be improved to obtain representative judgements from the decision maker; (ii) analysis of the distribution of the opinion judgements -how this varies over time and how one criterion is affected by the others; (iii) study of the different series to show the triangles in the opinion/ judgement procedure; (iv) analysis of the different consensus additions; and (v) development of a simple tool to apply the method and study of the performance of such a tool in an unbiased context.

6. Conclusions

The MTC© method involving decision-making based on the comparison of trios (triangles) rather than pairs of criteria yields the following advantages over the classical AHP method: (i) capture of consistent judgements, which satisfies the axiom of transitivity, avoiding the disadvantages of common non-reciprocal judgment in AHP; (ii) increased concentration by the decision maker, thereby reducing the influence of the environment and minimizing the cognitive bias that may occur in the repetitive process of the decision; (iii) elimination of scales and discrete values in making judgments. MTC© triangles represent a continuous surface that combines all possible evaluations, and only trendsetters graduated with lines and circles that delimit the boundaries and magnitudes of selection between vertices are needed to help decision makers to make decisions jointly for the three criteria at the same time, especially in the case of trios that are difficult to assess; (iv) considerably fewer trios must be evaluated in the MTC© than the number of pairs to be evaluated in the AHP, especially when a large number of criteria and/or alternatives are involved. In cases of very complex decisions, the MTC© therefore seems a better method than the AHP; (v) capacity to determine the variation in the weights assigned to the criteria or alternatives by changing the context in which the evaluation is made (i.e. changing a vertex of the triangle); and (vi) the MTC© can produce (a) *opinion vectors* from the evaluations of a series of triangles that include the same criterion, (b) elliptical *opinion surfaces*, delimited by the ranges or intervals obtained for each criterion and alternative, and (c) *probability regions for the opinions* in three dimensions (shaped elliptic paraboloids), in which the probability of occurrence is assigned to each point as a bivariate normal distribution. In successive repetitions, the opinion surfaces and the probability regions will overlap, thus providing information about how the repetitions affect the time required to reach the decision, and in the case of group decisions, enabling evaluation of the degree of uniformity in the decisions.

In conclusion, because of its simplicity when assessing pairs, the AHP method is most useful for relatively simple (although complex) decisions, i.e. decisions that do not involve very large numbers of criteria and/or alternatives; the MTC© is a recommended alternative method for decisions involving large numbers of criteria and/or alternatives and also for group decisions. This is true despite the greater difficulty and time required to evaluate a triangle in the MTC© relative to assessment of one pair in the AHP, precisely because this implies the need for greater attention in the decision-making process and a consequent reduction in inconsistent evaluations.

The evaluation of trios of criteria (or alternatives) located at the vertices of triangles has the additional advantage of making the scale of comparison unnecessary as the decision maker is asked to make a judgement according to the trend on a surface, thus avoiding the previously mentioned disadvantages that occasionally arise with the scale in the AHP method.

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8. References

Abel, E., Mikhailov, L., & Keane, J. (2015). Group aggregation of pairwise comparisons using multi-objective optimization. *Information Sciences 322, 257-275.*

Aguarón, J., Escobar, M.T., & Moreno-Jiménez, J.M. (2016). The precise consistency consensus matrix in a local AHP-group decision making context. *Annals of Operations Research, 45(1), 245–259.*

Aydi A., Abichou T., Nasr, I.H., Louati, M., & Zairi, M. (2016). Assessment of land suitability for olive mill wastewater disposal site selection by integrating fuzzy logic, AHP, and WLC in a GIS. *Environmental Monitoring and Assessment, 188:59.*

Badri, M., Qubaisi, A.A., Mohaidat, J., Dhaheri, H.A., Yang G., Rashedi, A.A., & Greer, K. (2016). An analytic hierarchy process for school quality and inspection: Model development and application. *International Journal of Educational Management, 30(3), 437 – 459.*

Bana e Costa, C.A., & Vansnick, J.C. (2008). A critical analysis of the eigenvalue method used to derive priorities in AHP. *European Journal of Operational Research, 187, 1422–1428.*

Bechara, A. (2003). Risky Business: Emotion, Decision-Making and Addiction. *Journal of Gambling Studies, 19, 23-51*.

Blagojevic, B., Srdjevic, B., Srdjevic, Z., & Zoranovic, T. (2016). Heuristic aggregation of individual judgments in AHP group decision making using simulated annealing algorithm. *Information Science, 330, 260-273.*

Brunelli, M. (2016). Recent advances on inconsistency indices for pairwise comparisons—A Commentary. *Fundamenta Informaticae, 144(3-4), 321-332.*

Bundesen, C. A. (1998). A computational theory of visual attention. *Philosophical Transactions of the Royal Society B: Biological Sciences, 353, 1271-1281.*

Cables, E., Lamata, M. T., & Verdegay, J. L. (2016). Relation between AHP and operators based on different scales. *Fuzzy Logic and Information Fusion, 339, 155-167.*

Carmone, F. J., Kara, A., & Zanakis, S. (1997). A Monte Carlo investigation of incomplete pairwise comparison matrices in AHP. *European Journal of Operational Research, 102, 538-553.*

Cavanagh, P. (1999). Attention: Exporting vision to the mind. *Series on Biophysics and Biocybernetics, 8, 129-143.*

Cavanagh, P., Labianca, A. T., & Thornton, I. M. (2001). Attention-based visual routines: Sprites. *Cognition, 80, 47-60.*

Chiao, K.P. (2016). The multi-criteria group decision making methodology using type 2 fuzzy linguistic judgments. *Applied Soft Computing, 49, 189–211.*

Diaz-Balteiro, L., & Romero, C. (2008). Making forestry decisions with multiple criteria: A review and an assessment. *Forest Ecology and Management, 255, 3222-3241*.

Dincer, H., Hacioglu, U., Tatoglu, E., & Delen, D. (2016). A fuzzy-hybrid analytic model to assess investors' perceptions for industry selection. *Decision Support Systems, 86, 24-34.*

Dong, Y., Xu, Y., Li, H., & Min, D. (2008). A comparative study of the numerical scales and the prioritization methods in AHP. *European Journal of Operational Research, 186(1), 229–242.*

Escobar, M. T., & Moreno-Jiménez, J. M. (2000). Reciprocal distributions in the analytic hierarchy process. *European Journal of Operational Research, 123, 154–174.*

Escobar, M.T., Aguarón, J., & Moreno-Jiménez, J.M. (2016). Some extensions of the precise consistency consensus matrix. *Decision Support Systems, 74, 67-77.*

Fedrizzi, M., & Krejčí, J. (2015). A Note on the Paper "Fuzzy Analytic Hierarchy Process: Fallacy of the Popular Methods". *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 23, 965.*

Feng, J. Wang, J. Yao, S., & Ding, L. (2016). Dynamic assessment of forest resources quality at the provincial level using AHP and cluster analysis. *Computers and Electronics in Agriculture, 124, 184–193.*

Forman, E. H., & Gass, S. I. (2001). The Analytic Hierarchy Process, An exposition. *Operations Research, 49, 469-486.*

George, J.M., & Dane, E. (2016). Affect, emotion, and decision making. *Organizational Behavior and Human Decision Processes, 136, 47-55.*

He, S., Cavanagh, P., & Intriligator, J. (1997). Attentional resolution. *Trends in Cognitive Science, 3, 115-121.*

Ho, W. (2008). Integrated analytic hierarchy process and its applications – A literature review. *European Journal of Operational Research, 186, 211–228.*

Ishizaka, A., & Labib, A. (2011). Review of the main developments in the analytic hierarchy process. *Expert Systems with Applications, 38, 14336–14345.*

Kangas, A., Kangas, J., & Kurttila, M. (2008). *Decision Support for Forest Management*. Berlin: Springer Science.

Kangas, J., & Kangas, A. (2005). Multiple criteria decision support in forest management. The approach, methods applied, and experiences gained. *Forest Ecology and Management, 207, 133-143.*

Karanik, M., Wanderer, L., Gomez-Ruiz, J.A., & Pelaez, J.I. (2016). Reconstruction methods for AHP pairwise matrices: How reliable are they? *Applied Mathematics and Computation, 279, 103–124.*

Kułakowski K. (2015). Notes on order preservation and consistency in AHP. *Notes on order preservation and consistency in AHP, 245(1), 333-337.*

Kurtilla, M., Pesonen, M., Kangas, J., & Kajanus, M. (2000). Utilizing the analytic hierarchy process (AHP) in SWOT analysis - a hybrid method and its application to forest certification case. *Forest Policy and Economics, 1*, 41-52.

Li, H.L., & Ma, L.C. (2007). Detecting and adjusting ordinal and cardinal inconsistencies through a graphical and optimal approach in AHP models. *Computers & Operations Research, 34(3), 780-798.*

Ma L.C. (2016). A new group ranking approach for ordinal preferences based on group maximum consensus sequences. *European Journal of Operational Research, 251, 171–181.*

Mateu-Figueras, G., Martín-Fernández, J. A., Pawlowsky-Glahn, V., & Barceló-Vidal, C. (2003). El problema del análisis estadístico de datos composicionales. In *Actas del 27th Congreso Nacional de Estadística e Investigación Operativa* (pp. 480-488).

Millet, I., & Harker, P. (1990). Globally effective questioning in the Analytic Hierarchy Process. *European Journal of Operational Research, 48(1), 88-97.*

Moreno-Jiménez, J.M., Salvador, M., Gargallo, P., & Altuzarra, A. (2016). Systemic decision making in AHP: a Bayesian approach. *Annals of Operations Research, 45(1), 261–284.*

Mou, Q., Xu, Z., & Liao, H. (2016). An intuitionistic fuzzy multiplicative best-worst method for multi-criteria group decision making. *Information Sciences, 374, 224-239.*

Osei-Brison, K. M. (2006). An action learning approach for assessing the consistency of pairwise comparison data. *European Journal of Operational Research, 174, 234-244*

Pérez-Rodríguez, F. (2013). El método del triángulo de calificaciones (MTC): Una nueva metodología de apoyo en toma de decisiones complejas. *PhD Thesis. University of Santiago de Compostela, Spain.*

Pérez-Rodríguez, F., & Rojo-Alboreca, A. (2009). El método del triángulo de calificaciones (MTC): Una alternativa para la toma de decisiones dentro del método AHP. In: *Actas del 5º Congreso Forestal Español* (Ref.:5CFE01-001). Pontevedra: Sociedad Española de Ciencias Forestales.

Pérez-Rodríguez, F., & Rojo-Alboreca, A. (2012a). A Triangular Assessment Method. In *Proceedings of Conf-IRM 2012*. http://aisel.aisnet.org/confirm2012/19. Accessed 16.05.03.

Pérez-Rodríguez, F., & Rojo-Alboreca, A. (2012b). Forestry application of the AHP by use of MPC© software. *Forest Systems, 21, 418-425.*

Saaty T.L. (2005). The analytic hierarchy and analytic network processes for the measurement of intangible criteria and for decision-making, Process: What the AHP is and what it is not. In J. Figueira, S. Greco, M. Ehgott (Eds.*), Multiple Criteria Decision Analysis: State of the Art Surveys*. (pp. 345–407). Springer, New York.

Saaty, T.L. (1977). A scaling method for priorities in hierarchical structures. *Journal of Mathematical Psychology, 15, 234-281.*

Saaty, T.L. (1980). *The Analytic Hierarchy Process. Planning priority setting, resource allocation*. New York: McGraw-Hill.

Saaty, T.L. (1986). Axiomatic Foundation of the Analytic Hierarchy Process. *Management Science, 32, 841-855.*

Saaty, T.L. (1990). *Decision making for Leaders. The Analytic Hierarchy Process for decision in a complex World*. Pittsburgh: University of Pittsburgh, RWS Publications.

Saaty, T.L. (2013). On the measurement of intangibles. A principal eigenvector approach to relative measurement derived from paired comparisons. *Notices of the American Mathematical Society, 60(2), 192–208.*

Schomoldt, D. L., Kangas, J., Mendoza, G. A., & Pesonen, M. (2001). *The Analytic Hierarchy Process in natural resource and environmental decision making*. London: Kluwer Academic Publishers.

Schoner, B., & Wedley, W. (2007). Ambiguous Criteria Weights in AHP: Consequences and Solutions. *Decision Science, 20, 468-475.*

Segura, M., Ray D., & Maroto, C. (2014). Decision support systems for forest management: A comparative analysis and assessment. *Computers and Electronics in Agriculture, 101, 55-67.*

Subramanian, N., & Ramanathan, R. (2012). A review of applications of Analytic Hierarchy Process in operations management. *International Journal of Production Economics, 138, 215-241.*

Sun, S., Wang, Y., Liu, J., Cai, H., Wub, P., Gengd, Q., & Xu, L. (2016). Sustainability assessment of regional water resources under the DPSIR framework. *Journal of Hydrology, 532, 140-148.*

Tsyganok, V.V., Kadenko, S.V., & Andriichuk, O.V. (2016). Usage of scales with different number of grades for pair comparisons in decision support systems. *International Journal of the Analytic Hierarchy Process 8 (1).*

Ureña, R., Chiclana, F., Morente-Molinera, J.A., & Herrera-Viedma, E. (2015). Managing incomplete preference relations in decision making: A review and future trends. *Information Sciences, 302, 14–32.*

Vaidya, O. S., & Kumar, S. (2006). Analytic hierarchy process: An overview of applications. *European Journal of Operational Research, 169, 1–29.*

Velasquez, M., & Hester, P. T. (2013). An Analysis of Multi-Criteria Decision Making Methods. *International Journal of Operations Research, 2, 56-66.*

Wang, Z.J., & Tong, X. (2016). Consistency analysis and group decision making based on triangular fuzzy additive reciprocal preference relations. *Information Sciences, 361–362, 29–47.*

Wedley, W.C., Schoner, B., & Choo E.U. (1996). Clusters, Criteria and Calibration—a Reply to Hauser and Peniwati. *Journal of Multi-criteria decision analysis, 5, 267-270.*

Wedley, W.C., Schoner, B., & Choo, E. U. (1993). Clustering, dependence and ratio scales in AHP: rank reversals and incorrect priorities with a single criterion. *Journal of Multi-Criteria Decision Analysis, 2, 145-1 58.*

Yalew, S.G., van Griensven, A., Mul, M.L., & van der Zaag, P. (2016). Land suitability analysis for agriculture in the Abbay basin using remote sensing, GIS and AHP techniques. *Modeling Earth Systems and Environment, 2016, 2:101.*

Yannou, B. (2002). Towards a web-based collaborative weighting method in project. In *IEEE International Conference on Systems, Man and Cybernetics 6.*

Zahir, S. (2006). Eliciting ratio preferences for the Analytic Hierarchy Process with visual interfaces: a new mode of preference measurement. *International Journal of Information Technology & Decision Making, 05*, 245.

Zanazzi, J. L. (2003). Anomalías y supervivencia en el método de toma de decisiones de Saaty. In L. A. Godoy (Ed.). *Problemas del Conocimiento en Ingeniería y Geología* (pp. 148-170). Córdoba: Editorial Universitas.

Zhang H. (2016). A goal programming model of obtaining the priority weights from an interval preference relation. *Information Sciences, 354, 197–210.*

Zhang, J., Su, Y. Wu, J., & Liang H. (2015). GIS based land suitability assessment for tobacco production using AHP and fuzzy set in Shandong province of China. *Computers and Electronics in Agriculture 114:202–211*

Zhao, J., Jin, J., Zhu, J., Xu, J., Hang, Q, Chen, Y., & Han, D. (2016). Water Resources Risk Assessment Model based on the Subjective and Objective Combination Weighting Methods. *Water Resources Management 30 (9), 3027-3042.*

Zhou, W., & Xu, Z. (2016). Asymmetric hesitant fuzzy sigmoid preference relations in the analytic hierarchy process. *Information Sciences, 358-359, 191-207*

Zhü, K. (2014). Fuzzy analytic hierarchy process: Fallacy of the popular methods. *European Journal of Operational Research, 236(1), 209-217.*