EGID3 PROJECT: PRIOR KNOWLEDGE OF FUTURE TEACHERS IN GEOMETRY

Marcela Seabra[1], Paula Maria Barros[2], Manuel Vara Pires[3], Cristina Martins[3]

[1]Escola Superior de Educação, Instituto Politécnico de Bragança (PORTUGAL)
[2]Escola Superior de Tecnologia e Gestão, Instituto Politécnico de Bragança (PORTUGAL)
[3]Research Centre in Basic Education (CIEB), Instituto Politécnico de Bragança (PORTUGAL)

Abstract

In this paper we intend to present and discuss knowledge about geometric concepts and procedures developed by students, future teachers, during their non-higher education path. The identification of this knowledge is made in the context of the research project "EGID3: Geometry teaching, investing in diagnosis, difficulties and challenges", developed in the context of the Geometry curricular unit of the Undergraduate Degree in Basic Education of a Portuguese higher education institution, involving the teacher (first author) and the students of her class in a reflexive and collaborative research context about their own professional practice. The project aims, among other objectives, (i) to investigate students' perceptions about Geometry and its teaching; and (ii) to verify the contribution of diagnostic assessment practices to the learning of Geometry. For the construction (or consolidation) of new learning it is crucial to build upon the knowledge that students have been previously developing, in order to enhance more significant learning. For a student to achieve significant learning, he needs to mobilise and restructure his previous knowledge in order to establish connections between what he has already mastered and the new content to be learned. The valorisation of a student's previous knowledge allows the teacher to map and elaborate planning strategies in order to take this knowledge into account. In this sense, the data collection involved, in the first lesson of the curricular unit, a questionnaire of ten questions of open nature, involving geometric concepts and procedures that the students had worked in their basic education. Subsequently, a content analysis of the answers was performed. The results reveal that there are concepts in which students feel more secure (geometric figure, rectangle). They also reveal that many of them use a not very rigorous language when communicating their ideas about some concepts and procedures (angle).

Keywords: Prior knowledge, future teachers, geometric concepts, geometric procedures.

1 INTRODUCTION

The study of Geometry is fundamental, as it contributes to the "development of geometric reasoning, which includes the ability to visualize, to formulate conjectures, to argue and to demonstrate" of students, as Santos and Oliveira [1] point out. Also, for the NCTM [2], it offers powerful tools to represent and solve problems in all areas of mathematics, in other school subjects and in everyday applications and, therefore, De Villiers [3] considers natural that it occupies a place of great relevance in education in general. In the same sense, Rodrigues and Branco [4] consider it essential to provide quality training to teachers [and future teachers] in this field.

The EGID3 project - Geometry teaching, investing in diagnosis, difficulties, and challenges - had its genesis in the concerns and interest of researchers with the teaching and learning of Geometry, particularly in the context of teacher education. In this project we intended to investigate the perceptions of students attending the Geometry curricular unit of the Undergraduate Degree in Basic Education, concerning geometry and its teaching, to diagnose their difficulties in geometric concepts and to reflect on the contribution to their learning of a pedagogical intervention focused on an exploratory type of teaching, using diversified tasks.

According to Barrantes and Blanco [5], the teaching model that students have experienced in non-higher education influences their conceptions about several aspects of mathematics and its teaching and learning. For this reason, during the initial training of educators and teachers, it is necessary to consider and value the students' conceptions, because, along with the knowledge acquired in the meantime, they will contribute decisively to their way(s) of being a teacher [5].

In this article, it is our aim to account for the prior knowledge of future teachers at the beginning of their initial training in four geometric concepts - geometric figure, polygon, angle and rectangle. At the beginning of initial teacher training for early stages, and much due to the multiplicity and complexity of disciplinary knowledge involved, there is a recognition that future teachers have some gaps in the specific knowledge of each discipline. For example, Couto and Vale [6] report difficulties in the acquisition and use of elementary geometric concepts and procedures, such as the identification of properties associated with triangles and quadrilaterals, in the classification of polygons or in the use of specific mathematical language. Also Menezes et al. [7] refer that the knowledge of elementary topics of geometry of future teachers of three institutions reveals some gaps, especially in the classification of geometric figures and in the recognition of properties in triangles and quadrilaterals, although they also recognize that, throughout the training, these difficulties were attenuated.

2 METHODOLOGY

We regard this work as an investigation of our own professional practice, of a reflective and collaborative nature, in the sense defined by Ponte [8]. The study has as participants the teacher and the twenty-six students of a class of the Undergraduate Degree in Basic Education in the context of the Geometry curricular unit. Voluntary and anonymous participation in the study was ensured, as well as the guarantee that the data collected were confidential and exclusively intended for scientific purposes.

The study follows a qualitative methodology, considered by Amado [9] as very appropriate for this kind of studies, having as main purpose, at this stage, to interpret information about the students' prior knowledge in Geometry. Data were collected using a questionnaire that asked for the students' written opinion about geometric concepts and procedures developed during their non-higher education. In this paper, data regarding four categories, associated with four concepts, are presented and discussed: (i) What is, for you, a geometric figure?, (ii) What is a polygon?, (iii) How would you explain to a colleague what an angle is?; and (iv) Present a definition of rectangle. The analysis and treatment of the data focused on the content analysis of the students' answers. For Amado, Costa and Crusoe [10], this type of analysis, "in addition to a rigorous and objective representation of the contents or elements of the messages (speech, text, article, etc.) through its coding and classification by categories and subcategories", allows "the advance (fruitful, systematic, verifiable and to some extent replicable) towards the capture of its full meaning".

The reading of all answers, their comparison and discussion led to the specification of subcategories defined *a posteriori*. The unit of analysis was defined as the sentence or sentence excerpt, so different indicators could be found in the same answer.

3 RESULTS

This section presents the main results concerning the knowledge revealed by students about the concepts of geometric figure (2D), polygon, angle, and rectangle, supported by the answers given in the questionnaire.

3.1 Concept of geometric figure

Concerning the concept of geometric figure, based on the analysis of the students' answers (two of them did not answer), five subcategories emerged, with the association to: (i) constituent elements of figures; (ii) concrete types of figures; (iii) shapes of the surroundings; (iv) geometric solids; and (v) their geometric representation, which, jointly, with students' answers (Evidences) and the number of references (No.), can be seen in Table 1.

Table 1. Concept of geometric figure

Subcategories	*Evidence (examples)*	*No.*
Association with elements of the figures	A geometric figure is a figure with its own characteristics that can be measured. (S4 - Student 4) Figures formed through lines and points. (S8) A geometric figure has x sides and x vertices. (S12) A geometric figure is a figure that we refer to by the right term depending on its shape or the number of sides. (S13) For me, a geometric figure is a set of lines that when joined together form angles of various measures. (S23) A geometric figure consists of vertices, angles, line segments (...). (S25)	12
Association with concrete types of figures	A geometric figure are (sic) triangles, rectangles, squares and others that are represented only in two dimensions. (S15) A rectangle, a triangle, square, pentagon, etc. (S18) It is a figure such as the square, circle, triangle, rectangle, hexagon among others. (S24)	5
Association with forms of the surrounding environment	It is the shape of the things around us. (S3) It's the shapes of objects. (S10)	3
Association with geometric solids	These are objects such as a cube, a cone, a pyramid that contain angles and points that connect them. (S1)	2
Association with geometric representation	It is a geometric representation of a given figure, they can have different shapes. (S9)	2

With regard to the first subcategory, Association with the constituent elements of figures, students mention words like points, lines, sides, vertices or surface in their definition. In some cases, a connection of the concept of geometric figure to the concepts of polygon or polygonal line seems to be implicit, insofar as the focus of the definition is the existence of a certain number of sides or vertices. The reference to line segments, straight lines or lines (which in the context seem to be more interpreted as straight lines): "...consists of vertices, angles, line segments" (S25) or "...has x sides and x vertices" (S12).

This more immediate association of geometric figure/ polygon is to some extent also evidenced in the second subcategory, Association with concrete types of figures, in which of the five students who presented examples of geometric figures only two made explicit reference to non-polygons, in this case the circle: "It is a figure like the square, circle, triangle, rectangle, hexagon among others" (S24). This tendency may be due to the fact that, at the basic schooling level, an increased importance is given to polygons and their classification, compared to other non-polygonal geometric figures, and therefore they constitute the examples most remembered by students. The need to resort to concrete types also highlights difficulties in mathematically expressing their ideas about the concepts.

In the third subcategory, Association with shapes of the surrounding environment, students refer to the shapes of objects or the surrounding environment, not being very clear in their references: "It's the shape of things around us (S3)". Thus, some of them (mainly S3 and S10) may also be making the figure-solid association, like the students in the fourth subcategory, Association with geometric solids, who explicitly mention solids as corresponding to geometric figures.

There is also a fifth subcategory, Association to geometric representation, in which students explicitly refer to the figure as a representation.

3.2 Concept of polygon

Regarding the concept of polygon, the analysis of the students' answers led to the definition of the three subcategories evidenced in Table 2, with the association to: (i) figures bounded by line segments; (ii) three-dimensional objects; and (iii) only geometric figures. It should be noted that there were only nineteen answers in this category: four students did not answer and another three assumed they did not know what a polygon was.

Table 2. Concept of polygon

Subcategories	*Evidence (examples)*	*No.*
Association with figures delimited by line segments	It is the junction of several points joined by straight lines forming a figure. (S1) It is a figure bounded by lines. (S2) It is a figure with sides, closed. (S6) Flat figure bounded by a line or line segment. (S8) A polygon is a figure bounded by lines [adding the drawing of an octagon]. (S17)	9
Association with three-dimensional objects	A polygon is the union of two geometric figures forming a 3D object. (S4) A polygon can be considered in 3 dimensions and in contrast to the figure in 2D. (S5) Polygons are solids that have 3 dimensions: length, width and height. (S14) A polygon is three dimensional. (S25)	7
Association only with geometric figures	It is a flat figure. (S9) It's a geometric figure. (S10)	3

In the first subcategory, Association with figures bounded by line segments, we considered the answers of all students who explicitly or implicitly refer to the boundary line as a polygonal line and express some idea of delimitation of the figure: "It is a figure bounded by lines (S2) or "It is a figure with sides, closed" (S6).

Similar to what occurred with the concept of geometric figure, the confusion of polygon with the concept of geometric solid is also present in the students' answers. This situation is visible in the second subcategory, Association with three-dimensional objects, in answers such as "...are solids that have 3 dimensions: length, width and height" (S14) or "A polygon is three-dimensional" (S25).

In the third subcategory, Association only with geometric figures, there is a failure to explicitly clarify the concept of polygon, classifying it only as a geometric figure. This answer, for what it leaves open, may mean that students do not have a more concrete idea of the concept of polygon or cannot express it.

3.3 Concept of angle

As to what concerns the concept of angle, it was possible to verify that the students' answers (two of them assumed they did not know) focused on four subcategories, with the association to: (i) a region of the plane; (ii) constituent elements of angles; (iii) amplitude, measure; and (iv) simultaneously to constituent elements and amplitude, measure, as shown in Table 3.

Table 3. Concept of angle

Subcategories	*Evidence (examples)*	*No.*
Association with a region of the plane	It is the region of a plane detremined by the joining of two semi-straight lines presenting a common origin. (S6)	1
Association with constituent elements of angles	An angle is the opening of a point which connects two sides [adding the drawing of a straight angle]. (S2) An angle is formed by two rays that have the same origin (S7) The junction of two rays (S11) Angle is the joining of two straight lines. For instances: I have two straight lines exemplifying I make an angle [adding the drawing of an acute angle]. (S14) An angle in the meeting of two straight line with a common origin named vertex. There ares several kinds of angles. (S20)	10
Association the amplitude, measured	I would initially explain that there are several types of angles such as 90º, 30º, 360º. The angle is the amplitude that can exist between one line and the other as in a right triangle, an angle of 90º. (S1) Angle is the measure in º that there is between two straight lines. (S4) It is a distance/measurement between one side and another of a figure with the vertex as the reference point. (S9) It is the amplitude between two lines [exemplifying with an obtuse angle]. (S13)	8
Association simultaneously to constituent elements and amplitude, measured	This is the junction of two rays, measured in degrees. (S10) When two lines intersect each other, they form an angle, for example when a vertical line intersects with a horizontal line, they form together a right angle of 90 degrees. (S12) I used a drawing. I would draw two perpendicular lines and explain that we can identify the amplitude of the angle through the intersection of the two lines. (S25)	5

The first subcategory, Association with a region of the plane, refers to a possibility of correctly defining an angle, and only one student explained it using appropriate geometric terms: "It's the region of a plane determined by the joining of two semi-straight lines presenting a common origin." (S6).

The second subcategory, Association with elements that constitute angles, includes answers that refer to important aspects related to the notion of angle, such as rays, vertex or opening, although incompletely or with recourse to imprecise geometric terms: "An angle is formed by two rays that have the same origin" (S7) or "The junction of two rays" (S11).

As for the third subcategory, Association the amplitude, measure, the answers expressed focus exclusively on the amplitude or "measure" of angles and not on the geometric concept, confusing a characteristic of an angle with the angle itself: "I would initially explain that there are several types of angles such as 90º, 30º, 360º. The angle is the amplitude that can exist between one line and the other as in a right triangle, an angle of 90º" (S1) or "The angle is the measure in º that is between two lines" (S4).

But there were answers that associate, at the same time, ideas from the two previous subcategories, which were grouped in the fourth subcategory, Association simultaneously to constituent elements and to constituent elements and the amplitude, measure. For example, when a student says: "When two lines intersect, they form an angle, for example when a vertical line intersects with a horizontal line, they form together a right angle of 90 degrees" (S12), it is visible the mixture between the concepts of angle and amplitude of the angle.

3.4 Concept of rectangle

Regarding the concept of rectangle, the students' answers (all answered) originated four subcategories, with the association: (i) only to the congruence of internal angles; (ii) to the congruence of internal angles and other characteristics; (iii) to the sides; and (iv) to the concept of square, as presented in Table 4.

Table 4. Concept of rectangle

Subcategories	*Evidence (examples)*	*No.*
Association only with the congruence of internal angles	It is a quadrilateral that has all congruent internal angles, right angles. (S6) It is a quadrilateral with 4 right angles. (S9) Geometric figure which has four congruent internal angles. (S10) It has congruent angles. (S11)	4
Association with congruence of internal angles and other characteristics	A rectangle is a quadrilateral with 4 right angles, with edges parallel to each other. (S7) A rectangle has 2 equal sides and 2 more sides equal to each other, forming 4 90-degree angles. (S12) A rectangle has 4 right angles and has greater length than width. (S21) Rectangle consists of four sides being parallel two by two and constructing four right angles. (S25)	7
Association to the sides	A rectangle is a 4-sided geometric figure in which 2 parallel sides have the same length and the other 2 sides also have the same length, but 2 sides have to be longer. [drawing a rectangle] (S2) Rectangle is a geometric figure with four unequal sides, that is, equal to two. (S14) Rectangle is a geometric figure with 4 sides, which can be equal or not. (S15) A rectangle is a four-sided geometric figure where we have length and width. (S23) A rectangle has four sides which are parallel 2 by 2. (S26)	11
Association with the square concept	A rectangle is a square, both have four sides, but the rectangle has two equal sides and another two equal sides, that is, it has two sides bigger than the other two. (S5) A rectangle is a four-sided geometric figure, similar to a square, but with two sides with smaller measurements and two sides with larger measurements. (S22)	4

The first subcategory, Association only with the congruence of internal angles, includes answers that only refer to the main characteristic of rectangles - quadrilaterals with the four congruent internal angles - regardless of other characteristics also verified by rectangles. The answer "It is a quadrilateral that has all congruent internal angles, right angles" (S6) is an example of this situation using appropriate geometric terms.

The consideration of the congruence of angles, simultaneously with other properties, is reflected in the second subcategory, Association with the congruence of internal angles and other characteristics. For example, when it is stated that the "Rectangle consists of four sides being parallel two by two and building four right angles" (S25), the reference to the congruence of angles seems not to be assumed as the main characteristic of this quadrilateral.

The third subcategory, Association with sides, corresponds to the very frequent situation of associating the essential characteristic of rectangles with properties of the sides, whether it is their length: "Rectangle is a geometric figure with four unequal sides, i.e. equal to two" (S14), or its parallelism: "A rectangle has 4 parallel sides 2 to 2" (S26), or, again, both properties: "A rectangle is a geometric figure with 4 sides in which 2 parallel sides have the same length and the other 2 sides also, but 2 sides must be longer" (S2). In this study, many students express this view of the concept of rectangle, not focusing on the essential characteristic of a rectangle.

There are also links between rectangle and square, grouped in the fourth subcategory, Association to the concept of square. One student considers a rectangle "...similar to a square, but with two sides with smaller measures and the other two wider" (S22), and two students state that "...it is a square", but point out differences regarding the congruence of the sides: "...both have four sides, but the rectangle has 2 equal sides and the other 2 equal sides, that is, it has 2 sides bigger than the other 2" (S5).

4 CONCLUSIONS

This final section presents the main results regarding the students' knowledge about the four concepts addressed - geometric figure, polygon, angle, and rectangle - supported by the answers given in the questionnaire. Overall, the results of the study follow evidence already revealed in other works, namely in the studies of Couto and Vale [6] and Menezes *et al*. [7]. The results indicate that the students, although they feel more confident in certain concepts (geometric figure, rectangle), continue to reveal some difficulties in geometric concepts that they worked on throughout their non-higher education and use a not very rigorous language when communicating their ideas.

When referring to the concept of geometric figure, students preferably use vocabulary associated to the most explored polygonal figures, such as triangles and quadrilaterals, with some connections to shapes in the surrounding environment (even confusing them with geometric solids). From the answers they gave, students often showed a lot of difficulty in using an appropriate mathematical language to explain or clarify the concept of geometric figure, also showing that, for some students, this concept is still not very clear or precise.

The concept of polygon is associated by students with geometric figures bounded by line segments but is also identified as a geometric solid. It can be seen that some students have a valid intuitive idea of the concept of polygon, although they may not always use the most correct language in geometric terms. But it is clear that students still reveal some difficulties with the concept, either considering the significant number of students who gave no answer or mentioned not knowing how to answer, or the most prominent misconception of associating the concept of polygon to three-dimensional objects.

Only one student links the concept of angle to a region of the plane. The rest associate it, often inaccurately, with terms related to an angle, such as semi-straights or aperture, as well as with its amplitude or 'measure'. Many students express insecure and unconfirmed ideas, revealing some confusion and mixing of the concept of angle with its amplitude, or between angle and an attribute of it.

All students reveal some knowledge about the concept of rectangle. But few of them associate it only with the congruence of the internal angles, the consideration of this congruence accompanied by other its properties being more frequent. In addition to links to the concept of quadrangle, there is also a large association with characteristics of the sides of the rectangle. It is clear that many of them have not stabilized the main attribute associated with the concept of rectangle (congruence of internal angles), dispersing themselves in other important, but not essential, features, such as properties related to the sides. The idea that, in a rectangle, sides can only be congruent two by two is very strong for many students, revealing some difficulty in understanding that the square is a particular case of the rectangle.

We know that teachers with insufficient conceptual knowledge tend to treat concepts and procedures incompletely and incorrectly [2], [7], not responding in the best way to the difficulties and interests of their students. Therefore, we reinforce the importance that initial training can and should have in identifying possible gaps in future teachers' previous (geometric) knowledge in order to better help them to overcome them and to restructure their knowledge.

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