INTERNAL WAVES AND ANGULAR MOMENTUM TRANSPORT IN THE SUN

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ABSTRACT

The low frequency gravity waves emitted at the base of the solar convection zone are able to extract angular momentum from the radiative interior. As a first step, we evaluate the efficiency of this transport with the following simplifying assumptions: we ignore the Coriolis force, approximate the spectrum of turbulent convection by the Kolmogorov law and couple this turbulence to the internal waves through their pressure fluctuations, following Press (1981) and García López & Spruit (1991).

The local frequency of an internal wave varies with depth in a differentially rotating star, and it may even vanish at some location, thus leading to complete dissipation (Goldreich & Nicholson 1989). It is that mechanism only that we take into account in the exchange of momentum between waves and stellar rotation. We find that it operates on a timescale of 10^7 years, and is probably responsible for the flat rotation profile detected through helioseismology.

1. Properties of internal waves

Most of the properties of internal waves propagating in stellar interiors have been described by Press (1981) and by Goldreich & Nicholson (1989). Like them, we shall also treat the waves as if they were pure gravity modes which are not modified by the Coriolis acceleration. Their horizontal variation is then described by the spherical functions $Y_\ell^m(\theta,\phi)$, and their local frequency is Doppler shifted by the differential rotation:

$$\sigma(r) = \omega - m\,[\Omega(r) - \Omega_c] \equiv \omega - m\,\Delta\Omega(r). \qquad (1)$$

Here m is the azimuthal order of this monochromatic wave and ω its frequency when it is emitted at the base of the convection zone, which rotates at the angular velocity Ω_c.

An important property of the internal waves is that they conserve their angular momentum in the adiabatic limit. When damping is taken into account, the angular momentum luminosity associated with a monochromatic wave may be written

$$\mathcal{L}_J(r) = \mathcal{L}_J(r_c)\,\exp[-\tau(\omega,\ell,m,r)], \qquad (2)$$

where τ (similar to an optical depth) accounts for radiative damping (as above, the subscript $_c$ refers to the base of the convection zone).

The flux of kinetic energy associated with such a wave is given by

$$\mathcal{F}_K(r_c) = \frac{1}{2}\,\rho\,\frac{(N_c^2-\omega^2)^{\frac{1}{2}}}{k_h}\,\frac{\omega^2}{N_c^2}\,\overline{u}^2, \qquad (3)$$

with the usual notations for the density ρ, the Brunt-Väisälä frequency N, the horizontal wavenumber $k_h = [\ell(\ell+1)]^{1/2}/r$, and $\frac{1}{2}\rho\overline{u}^2$ being the kinetic energy density of the wave. We shall assume that the waves are excited by the pressure fluctuations of the turbulent convection, that this turbulent spectrum obeys Kolmogorov's law, and that the spectral energy of the internal waves is distributed uniformly over their azimuthal order m, for given ℓ. Since the angular momentum flux is related to the kinetic energy flux by $\mathcal{F}_J = 2\,\frac{m}{\sigma}\,\mathcal{F}_K$, we reach the following expression for the luminosity of angular momentum integrated over the whole wave spectrum:

$$\mathcal{L}_J(r) = 4\pi r^2\,\frac{\rho_c v_c^3}{N_c \ell_c} \qquad (4)$$

$$\times \int_{\omega_c}^{N_c} \frac{d\omega}{\omega}\left(1-\frac{\omega^2}{N_c^2}\right)^{\frac{1}{2}}\left[\frac{\omega}{\omega_c}\right]^{-3}\int_0^{\ell_u}\frac{d\ell}{\ell}\int_{-\ell}^{\ell} e^{-\tau}\,m\,dm.$$

2. The angular momentum flux

In this first approach, we neglect the deposit of angular momentum related to the different damping experienced by waves of opposite azimuthal order m. Instead, we concentrate on those waves which are completely damped because their local frequency vanishes at some critical depth. In doing so, we shall approximate the damping exponential by the step function $\mathcal{H}(r-r^*)$; we thus take $\exp[-\tau(r)] = 1$ for $\omega > m\,\Delta\Omega$ and $\exp[-\tau(r)] = 0$ for $\omega < m\,\Delta\Omega$, simplifying the integral (4). At a certain depth r^*, we will only consider the waves which have experienced little damping before; their frequency is given by

$$\omega^4 \geq I(r^*)\,\ell^3 \quad \text{with} \quad I(r) = \int_r^{r_c} K\,N\,N_t^2\,\frac{dr}{r^3}, \qquad (5)$$

where K is the radiative diffusivity and N_t that portion of the Brunt-Väisälä frequency which is due to the stratification in temperature.

We now perform the integration (4) in the interval of I and $\Delta\Omega$ relevant for the solar interior, namely $\ell_c^{-3} < (\frac{I}{\omega_c^4}) < 1$ and $\frac{\omega_c}{N_c}(\frac{I}{\omega_c^4}) < (\frac{\Delta\Omega}{\omega_c})^3 < (\frac{I}{\omega_c^4})$.

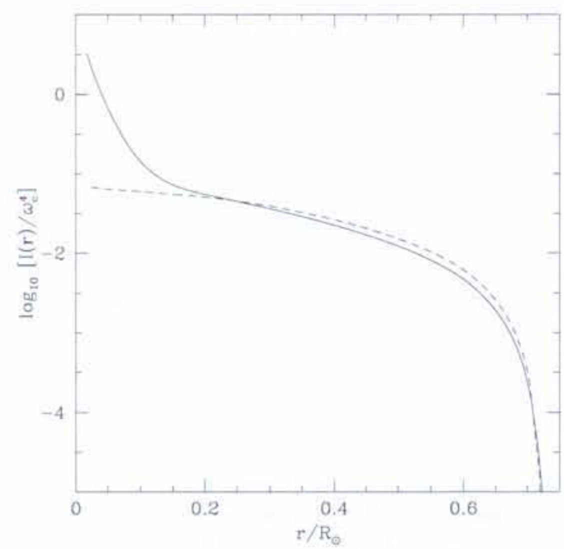

Figure 1: Variation with depth of the damping integral defined in (7), scaled by the fourth power of the convective turnover frequency: I/ω_c^4. The dashed line refers to the Sun at 200 Myr, the continuous line to the present Sun.

The angular momentum luminosity is then a linear function of $\Delta\Omega$:

$$\mathcal{L}_J(r) = \mathcal{L}_J(r_c) - \frac{4\pi r^2}{3} \frac{\rho_c v_c^3}{N_c \ell_c} \left(\frac{\omega_c{}^4}{I}\right) \frac{\Delta\Omega}{\omega_c}. \qquad (6)$$

3. Angular momentum transport in the Sun

If momentum is transported only by the internal waves we have considered here, the angular velocity evolves with time according to

$$\frac{\partial}{\partial t}(\rho r^4 \Omega) = \frac{1}{2} \frac{\rho_c v_c^3}{N_c \ell_c} \frac{\partial}{\partial r} \left[r^2 \left(\frac{\omega_c{}^4}{I}\right) \frac{\Delta\Omega}{\omega_c} \right]. \qquad (7)$$

We may estimate the magnitude of this transport by neglecting all variations except those of Ω, hence $\frac{\partial\Omega}{\partial t} \approx V_w \frac{\partial\Omega}{\partial r}$. Within this approximation the rotation profile, whose slope depends on the rate at which angular momentum is lost through the wind, propagates inwards with the velocity V_w, which is directly related to the damping integral $I(r)$:

$$V_w = \frac{1}{2} \frac{\rho_c v_c^3}{\rho r^2} \frac{1}{N_c \omega_c \ell_c} \left(\frac{\omega_c{}^4}{I}\right). \qquad (8)$$

Approximating $\rho_c v_c^3$ by 1/10 of the convective flux (see Cox & Giuli 1968), and this flux by $L_\odot / 4\pi r_c^2$, we obtain a crude estimate for the synchronization time $t_{sync} = r/V_w$:

$$t_{sync} \approx 60 \frac{M_\odot R_\odot^2}{L_\odot} \frac{\rho}{\overline{\rho}} \left(\frac{r}{R_\odot}\right)^3 \left(\frac{r_c}{R_\odot}\right)^2 N_c \omega_c \ell_c \left(\frac{I}{\omega_c{}^4}\right), \qquad (9)$$

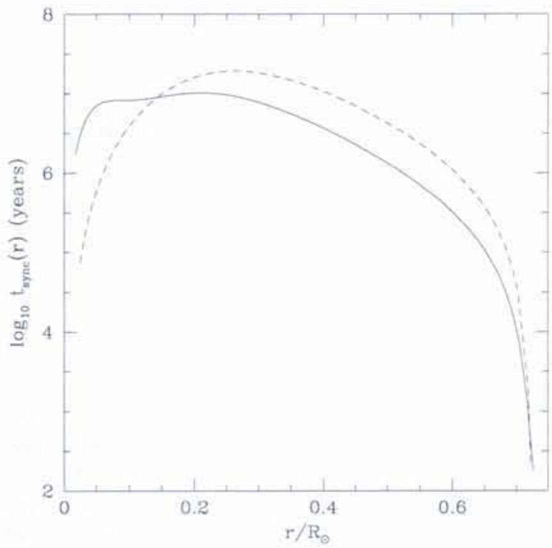

Figure 2: Variation of the synchronization time $t_{sync} = r/V_w$ (see eq. 9).

$\overline{\rho}$ being the mean density. This time is displayed in Fig. 2; it is of the order of 10^7 years, much shorter than the present spin-down time of the Sun.

4. Conclusion

We conclude that the internal gravity waves are quite efficient in extracting angular momentum from the radiative interior of the Sun, a result anticipated by Schatzman (1993) and confirmed independently by Kumar & Quataert (1996). During the later stages of the solar spin-down, this process prevails over the other mechanisms which have been proposed so far, except perhaps magnetic torquing. It is probably responsible for the flat rotation profile observed in Sun. We are currently implementing this transport in our rotational evolution codes, where it will compete with other mechanisms, such as meridian circulation and shear turbulence.

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