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Mathematical Model for the Assessment of Fracture Risk Associated with Osteoporosis

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Abstract. Osteoporosis is a skeletal disease characterized by low bone mass. It is considered a worldwide public health problem that affects a large number of people, in particularly for women with more than 50 years old. The occurrence pattern of osteoporosis in a population may be related to several factors, including socio-economic factors such as income, educational attainment, and factors related to lifestyle such as diet and physical activity. These and other aspects have increasingly been identified as determining the occurrence of various diseases, including osteoporosis. This work proposes a mathematical model that provides the level of osteoporosis in the patient. Preliminary numerical results are presented.

Keywords: Nonlinear optimization. Risk of fracture. Osteoporosis.
PACS: 02.60.Pn

INTRODUCTION

Osteoporosis is a skeletal disease characterized by low bone mass, predominant in older adults, particularly in postmenopausal women [1, 2]. This pathology is characterized by the degeneration of bone microstructure, leading to increasing bone brittleness and susceptibility to fracture, [3, 4]. Osteoporotic fractures are a major cause of morbidity and mortality in the population [3, 4, 5, 6]. Particularly, hip fracture, has a deep impact on quality of life, increasing difficulties regarding the activities of daily life [7, 8]. In this work we analyze the influence of some specific risk factors related with osteoporosis to identify the level of osteoporosis in the patient.

STUDY METHODOLOGY

For the proposed mathematical model we considered 97 women questionnaires aged over than 60 years old. This is only valid for densitometry examinations marks obtained in the DEXA equipment and software EnCORE 2004 GE Medical Systems.

We indicate that the patient belong to the Zone I if the patient has a normal bone. Zone II and III, if the patient has osteopenia and osteoporosis, respectively. We used the reference curve, for the femur anatomical region, to define the Zone I, II and III (see Fig. 1, where the dashed lines are the boundaries of each zone). This reference curve is indicated through the DEXA equipment.

Table 1 presents the average of input variables in each Zone.

TABLE 1. Average of the input variables for the Zone I, II and III.

Input variables	Zona I	Zona II	Zona III
Age - years (id)	66.3	70.1	73.0
Age of menopause - years (idm)	49.5	46.8	46.3
Body mass index - kg/m^2 (imc)	29.9	28.0	23.4
Coffee consumption - n (cof)	1.5	1.5	1.5
Bone mineral density at the femoral neck -g/cm^2 (BMD_f)	0.9	0.8	0.7
Bone mineral density of the ward -g/cm^2 (BMD_w)	0.8	0.6	0.5

Analyzing the Table 1 we can conclude that in older patients the risk of osteoporosis increases and observe that for the early menopause, the risk for developing osteoporosis increases also. We also observe that the body mass index is

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smaller for the women that are in the Zone III (women with osteoporosis). As we expected, the bone mineral density (at the femoral neck and at the wards) is lower in the women group that has osteoporosis.

In the neighborhood of the reference curves, the proposed mathematical model is not valid. So, the mathematical model is not valid in the following region:

$$BMD_L = [BMD - \delta, BMD + \delta], \tag{1}$$

where the δ is the region tolerance and BMD is the value in the reference curve for the current patient age.

Figure 1 represents the Zone I, II and III and the region BMD_L.

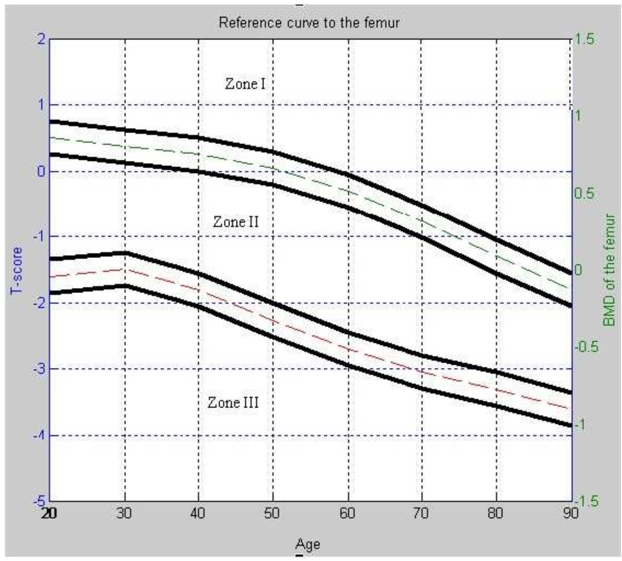

FIGURE 1. Femur region curve and the tolerance limits.

If the patient's BMD not belong to BMD_L the mathematical model is applied. This mathematical model has as main objective to predict what is the situation of the patient (patient in the Zone I, II or III).

In this study we considered two different mathematical models to approximate the value of T-score for femoral neck region

$$f_1(x) = \quad x_1(bmdf + bmdw)^2 + x_2\frac{imc}{30} + x_3\frac{60}{id} + x_4\frac{1}{cof} + x_5\frac{idm}{50} \tag{2}$$

and

$$f_2(x) = \quad x_1(bmdf + bmdw)^2 + x_2\frac{imc}{30} + x_3\frac{60}{id} + x_4\frac{1}{cof} + x_5\frac{idm}{50} + x_6. \tag{3}$$

To identify the optimal solution x, it was solved the following constrained nonlinear optimization problem:

$$\begin{array}{ll} \min & g(x) = \sum_{i=1}^{n} (f_j^i(x) - ts_i)^2 \\ s.a & x_1 \geq x_2 \\ & x_2 \geq x_3 \\ & x_3 \geq x_4 \\ & x_5 \geq x_4 \end{array} \tag{4}$$

where ts_i represents the T-score of the patient i, $f_j^i(x)$, for $j = 1,2$, represents the T-score approximation of the patient i.

The constraints are used to assign different weights to each input variable, risk factors. In this propose, we considered more weight assigned to the BMD of the femoral neck (*bmdf*) and bone mineral density of the wards (*bmdw*), then to the body mass index (*imc*), followed by patient age (*id*), age at menopause (*idm*) and finally the coffee consumer (*cof*).

To identify the optimal parameters, we used three optimization methods from Matlab [9]: Genetic Algorithms (GA), Pattern Search method (PS) and Sequential Quadratic Programming method (SQP).

NUMERICAL RESULT AND DISCUSSION

To choose the best mathematical model, we use twenty questionnaires. Table 2 shows the objective function values for the three optimization methods.

TABLE 2. Objective function values.

Algorithms	f_1	f_2
SQP	8.10	1.23
PS	8.09	1.48
GA	8.93	3.68

According to the results presented in Table 2 the best function is f_2.

The Table 3 presents the results obtained with twenty questionnaires considering the function f_2 and different values of δ. The table presents the number of tests that do not belong to the region BMD_L (NTN), the number of questionnaires with a proper identification (NQP), the number of questionnaires with a wrong identification (NQW) and the correct identification rate (CIR).

TABLE 3. Numerical results for the function f_2.

δ	Algorithms	NTN	NQP	NQW	CIR
0.050	SQP	11	9	2	81.2%
0.050	PS	11	9	2	81.2%
0.050	GA	11	8	3	72.7%
0.025	SQP	13	11	2	84.6%
0.025	PS	13	11	2	84.6%
0.025	GA	13	11	2	84.6%
0.015	SQP	17	14	3	82.23%
0.015	PS	17	13	4	76.47%
0.015	GA	17	14	3	82.23%

Table 3 shows that the best result is obtained for $\delta = 0.025$.

To validate the mathematical model twenty new questionnaires of patients aged over 60 years old were used. These data were not used to define the optimal parameters of the mathematical model.

The motivation for using these new questionnaires is to test the reliability of the model. The Table 4 presents the validation results.

TABLE 4. Numerical results for function f_2 with new questionnaires.

Tolerance	Algorithms	NTN	NQP	NQW	CIR
0.025	SQP	16	16	0	100%
0.025	PS	16	16	0	100%
0.025	GA	16	16	0	100%

We can verified in Table 4, that the proposed mathematical model (f_2) has 100% of correct identification rate.

CONCLUSIONS AND FUTURE WORK

Several risk factors for osteoporosis and osteopenia were identified, some of them are considered modifiable, such as, age, coffee consumption and body mass index (imc). The numerical results of the propose mathematical model were satisfactory, since the correct identification rate was 100%. However, the model should be tested with more and new data.

Some ideas for future work are to extend this study to patients under the age of 60 years old and different gender. And propose different mathematical models to approximate the T-score in different anatomical parts of the human body.

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