

Numerical Modelling of Steel I-beams with Restrained Thermal Elongation

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ASBTRACT: This paper presents a numerical study of the behaviour of steel I-beams with restrained thermal elongation. A geometrical and material non-linear finite element program, specially established for the analysis of structures submitted to fire, has been used to determine the resistance of a beam-column at elevated temperature, using the material properties of Eurocode 3, part 1-2. The numerical results have been compared with those obtained with the Eurocode 3, part 1-2 (1995) and the new version of the same Eurocode (2002).

1. INTRODUCTION

The problem of axially and eccentrically loaded columns was studied by Franssen et al (1995, 1996, 1998) and the problem of lateral torsional buckling of beams was studied by Vila Real et al (1999, 2001, 2002). In this paper the behaviour of beams submitted to combined moment and axial loads due to the restrained thermal elongation will be studied. These are the so called beam-column elements.

The design of beam-columns has to take in to account the axial buckling and the lateral-torsional buckling. This problem at high temperatures has never been studied.

In fact there is a significant difference between the case of a beam acting as a single element free to exhibit axial thermal elongation or being part of a frame where the stiffness of the remaining structure contributes with some restraint to the free elongation of the beam introducing an axial force.

Uniform temperature in the cross-section has been used so that comparison between the numerical results and the Eurocodes could be made.

The program used was the program Safir (Nwosu et al, 1999) which is a finite element code for geometrical and material non-linear analysis, specially developed for studying structures in case of fire.

This paper presents the first results of the study.

2. CASE STUDY

A simply supported beam with fork supports has been analysed, loaded with uniform moment in the major axis and axial compression (Fig.1). It was used an IPE220 of steel grade S235. The results in this paper were obtained with uniform temperature of 400° in the cross section.

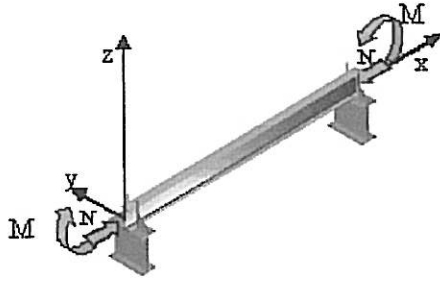


Fig. 1 – Simply supported beam with bending and axial compression.

It has been considered a longitudinal geometric imperfection given by the following expression:

$$y(x) = \frac{l}{1000} \sin\left(\frac{\pi x}{l}\right) \quad (1)$$

The residual stresses adopted are constant across the thickness of the web and of the flanges. Triangular distribution as in figure 2, with a maximum value of 0.3×235 MPa, for the S235 steel has been used (ECCS, 1984).

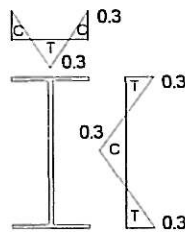


Fig. 2 – Residual stresses: C – compression; T – tension

3. EFFECT OF THE AXIAL RESTRAINT

To evaluate the structure stiffness (K) any current structural analysis program can be used, proceeding according to figure 3, using a similar procedure suggested by Franssen (2000). The beam is replaced by two axial forces. The stiffness considered on a spring that simulate the structure is given by $K=1/d$, where d is the relative displacement between the points of the unit forces application.

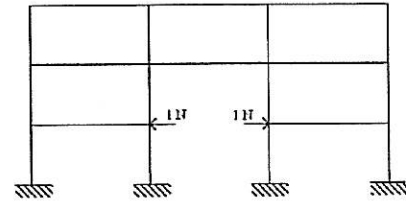


Fig.3 – Calculation of the stiffness of the structure K

The beam in a frame submitted to fire can be simulated by the model shown in figure 4.

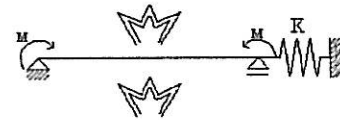


Fig. 4 – Beam with axial restraint in a fire.

The restraint introduced by the stiffness K (Fig. 4) in case of fire produces an axial compression given by the formula (2).

$$N = -\frac{R}{\frac{K_v}{K_{v0}} + R} K_v L_v \alpha \Delta T \quad (2)$$

Where:

L_v - Length of the beam.

$\Delta T = (\theta - 20)^\circ C$

α - Coefficient of linear thermal expansion

K_v - Axial stiffness of the beam

K_{v0} - Axial stiffness of the beam at room temperature

K - Stiffness of the spring

$$R = \frac{K}{K_{v0}}$$

4. ANALYSIS ACCORDING TO THE EUROCODE 3 (1995)

According to the part 1-2 of the Eurocode 3 the elements with classes 3 and 2 (case studied) submitted to bending and axial compression, in case of fire, must satisfy the condition:

$$\frac{N_{fi,Ed}}{1.2 \frac{\chi_{min,fi}}{Ak_{y,\theta}} \frac{f_y}{\gamma_{M,fi}}} + \frac{K_y M_{y,fi,Ed}}{W_{pl,y} k_{y,\theta} \frac{f_y}{\gamma_{M,fi}}} \leq 1 \quad (3)$$

where:

$$K_y = 1 - \frac{\mu_y N_{fi,Ed}}{\frac{\chi_{y,fi}}{1.2} Ak_{y,\theta} f_y} \quad \text{but} \quad K_y \leq 1.5 \quad (4)$$

and

$$\mu_y = \bar{\lambda}_{y,\theta} (2\beta_{M,y} - 4) \left[\frac{W_{pl,y} - W_{el,y}}{W_{el,y}} \right] \quad (5)$$

but $\mu \leq 0.9$

where:

$\chi_{min,fi}$ is the minimum reduction factor of the axis yy and zz ;

$W_{pl,y}$ is the plastic modulus in axis yy ;

$k_{y,\theta}$ is the reduction factor of the yield strength at temperature θ ;

$\gamma_{M,fi}$ is the partial safety coefficient in case of fire (usually $\gamma_{M,fi} = 1$);

$\beta_{M,y}$ is the equivalent uniform moment factor, in this case ($\beta_{M,y} = 1.1$);

The reduction factor is calculated with the expressions from the Eurocode 3. The reduction factor in case of fire, $\chi_{y,fi}$ and $\chi_{z,fi}$, are determined like at room temperature using the slenderness $\bar{\lambda}_{y,\theta}$ e $\bar{\lambda}_{z,\theta}$ given by equation (6). The constant 1.2 is an empirical correction factor. In the calculation of the reduction factor in case of fire the buckling curve used is the curve c ($\alpha=0.49$).

$$\bar{\lambda}_{y,\theta} = \bar{\lambda}_y \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} \quad \bar{\lambda}_{z,\theta} = \bar{\lambda}_z \sqrt{\frac{k_{z,\theta}}{k_{E,\theta}}} \quad (6)$$

where:

$\bar{\lambda}_y$ e $\bar{\lambda}_z$ are the slenderness of the axis yy and zz at room temperature;

$k_{E,\theta}$ is the reduction factor of the elastic modulus at temperature θ ;

The following values were taken in consideration:

$$N_{fi,\theta,Rd} = Ak_{y,\theta} \frac{f_y}{\gamma_{M,fi}} \quad (7)$$

$$M_{y,fi,\theta,Rd} = W_{pl,y} k_{y,\theta} \frac{f_y}{\gamma_{M,fi}}$$

To compare the results the maximum value of the design moment has been divided by the resistant plastic moment at temperature θ . Taking from equation (3) $M_{y,fi,Ed}$ and divided it by $M_{y,fi,\theta,Rd}$ from equation (7).

$$\frac{M_{y,fi,Ed}}{M_{y,fi,\theta,Rd}} \leq \frac{1}{\left(1 - \frac{\mu_y N_{fi,Ed}}{\frac{\chi_{y,fi}}{1.2} N_{fi,\theta,Rd} \gamma_{M,fi}} \right)} \left(1 - \frac{N_{fi,Ed}}{\frac{\chi_{min,fi}}{1.2} N_{fi,\theta,Rd}} \right) \quad (8)$$

As the study is about beam-columns, it is necessary to consider the lateral-torsional buckling, and the following formula must also be verified:

$$\frac{N_{fi,Ed}}{1.2 \frac{\chi_{z,fi}}{Ak_{y,\theta}} \frac{f_y}{\gamma_{M,fi}}} + \frac{K_{LT} M_{y,fi,Ed}}{1.2 \frac{\chi_{LT}}{W_{pl,y}} k_{y,\theta} \frac{f_y}{\gamma_{M,fi}}} \leq 1 \quad (9)$$

with:

$$K_{LT} = 1 - \frac{\mu_{LT} N_{fi,Ed}}{\frac{\chi_{z,fi}}{1.2} Ak_{y,\theta} f_y} \quad \text{but} \quad K_{LT} \leq 1.0 \quad (10)$$

and

$$\mu_{LT} = 0.15 \bar{\lambda}_{z,\theta} \beta_{M,LT} - 0.15 \quad \text{but} \quad \mu \leq 0.9 \quad (11)$$

where:

$\beta_{M,LT}$ is the equivalent uniform moment factor corresponding to lateral-torsional buckling, in this case ($\beta_{M,LT} = \beta_{M,y} = 1.1$);

The reduction factor for lateral-torsional buckling is calculated according to the expressions of Eurocode 3, if the slenderness $\bar{\lambda}_{LT,\theta}$ at the temperature θ , exceeds 0.4. The reduction factor in case of fire, $\chi_{LT,fi}$, is determine like at room temperature using the slenderness $\bar{\lambda}_{LT,\theta}$ given by:

$$\bar{\lambda}_{LT,\theta} = \bar{\lambda}_{LT} \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} \quad (12)$$

To compare the results the maximum value of the design moment has been divided by the resistant plastic moment at temperature θ . Taking from equation (9) $M_{y,\beta,Ed}$ and divided it by $M_{y,\beta,\theta,Rd}$ from equation (7).

$$\frac{M_{y,\beta,Ed}}{M_{y,\beta,\theta,Rd}} \leq \frac{\chi_{LT}}{1.2 \left(1 - \frac{\mu_{LT} N_{\beta,Ed}}{\chi_{z,\beta} N_{\beta,\theta,Rd} \gamma_{M,\beta}} \right)} \left(1 - \frac{N_{\beta,Ed}}{\chi_{z,\beta} N_{\beta,\theta,Rd}} \right) \quad (13)$$

5. ANALYSIS ACCORDING TO THE NEW VERSION OF EUROCODE 3

According to the new version of Eurocode 3 the elements with classes sections 1 and 2 (case studied) subjected to bending and axial compression, in case of fire, must satisfy the condition:

$$\frac{N_{\beta,Ed}}{\chi_{\min,\beta} A k_{y,\theta} \frac{f_y}{\gamma_{M,\beta}}} + \frac{K_y M_{y,\beta,Ed}}{W_{pl,y} k_{y,\theta} \frac{f_y}{\gamma_{M,\beta}}} \leq 1 \quad (14)$$

where:

$$K_y = 1 - \frac{\mu_y N_{\beta,Ed}}{\chi_{y,\beta} A k_{y,\theta} \frac{f_y}{\gamma_{M,\beta}}} \quad \text{but} \quad K_y \leq 3 \quad (15)$$

and

$$\mu_y = (1.2\beta_{M,y} - 3)\bar{\lambda}_{y,\theta} + 0.44\beta_{M,y} - 0.29 \quad (16)$$

but $\mu \leq 0.8$

with:

$$\chi_{\beta} = \frac{1}{\phi_{\theta} + \sqrt{[\phi_{\theta}]^2 - [\bar{\lambda}_{\theta}]^2}} \quad (17)$$

where:

$$\phi_{\theta} = \frac{1}{2} \left[1 + \alpha \bar{\lambda}_{\theta} + (\bar{\lambda}_{\theta})^2 \right] \quad (18)$$

$$\alpha = 0.65 \sqrt{235/f_y} \quad (19)$$

χ_{β} is the reduction factor to the axis yy and zz in case of fire;

$$\bar{\lambda}_{y,\theta} = \bar{\lambda}_y \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} \quad \bar{\lambda}_{z,\theta} = \bar{\lambda}_z \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} \quad (20)$$

with:

$\bar{\lambda}_y$ e $\bar{\lambda}_z$ are the slenderness of the axis yy and zz at room temperature;

$k_{E,\theta}$ is the reduction factor of the elastic modulus at temperature θ ;

To compare the results the maximum value of the design moment has been divided by the resistant plastic moment at temperature θ . Taking from equation (14) $M_{y,\beta,Ed}$ and divided it by $M_{y,\beta,\theta,Rd}$ from equation (7).

$$\frac{M_{y,\beta,Ed}}{M_{y,\beta,\theta,Rd}} \leq \frac{1}{\left(1 - \frac{\mu_y N_{\beta,Ed}}{\chi_{y,\beta} N_{\beta,\theta,Rd}} \right)} \left(1 - \frac{N_{\beta,Ed}}{\chi_{\min,\beta} N_{\beta,\theta,Rd}} \right) \quad (21)$$

Again as the study is about beam-columns, it is necessary to consider the lateral-torsional buckling, therefore beyond the previous verification it is necessary to make the following verification.

$$\frac{N_{\beta,Ed}}{\chi_{z,\beta} A k_{y,\theta} \frac{f_y}{\gamma_{M,\beta}}} + \frac{K_{LT} M_{y,\beta,Ed}}{\chi_{LT,\beta} W_{pl,y} k_{y,\theta} \frac{f_y}{\gamma_{M,\beta}}} \leq 1 \quad (22)$$

where:

$$K_{LT} = 1 - \frac{\mu_{LT} N_{\beta,Ed}}{\chi_{z,\beta} A k_{y,\theta} \frac{f_y}{\gamma_{M,\beta}}} \quad \text{but} \quad K_{LT} \leq 1.0 \quad (23)$$

and

$$\mu_{LT} = 0.15 \bar{\lambda}_{z,\theta} \beta_{M,LT} - 0.15 \quad \text{but} \quad \mu \leq 0.9 \quad (24)$$

where:

$\beta_{M,LT}$ is the equivalent uniform moment factor corresponding to lateral-torsional buckling, in this case ($\beta_{M,LT} = \beta_{M,y} = 1.1$);

where:

$$\chi_{LT,\beta} = \frac{1}{\phi_{LT,\theta} + \sqrt{[\phi_{LT,\theta}]^2 - [\bar{\lambda}_{LT,\theta}]^2}} \quad (25)$$

with:

$$\phi_{LT,\theta} = \frac{1}{2} \left[1 + \alpha \bar{\lambda}_{LT,\theta} + (\bar{\lambda}_{LT,\theta})^2 \right] \quad (26)$$

$$\alpha = 0.65 \sqrt{235/f_y} \quad (27)$$

and

$$\bar{\lambda}_{LT,\theta} = \bar{\lambda}_{LT} \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} \quad (28)$$

To compare the results the maximum value of the design moment has been divided by the resistant plastic moment at temperature θ . Taking from equation (22) $M_{y,\beta,Ed}$ and divided it by $M_{y,\beta,\theta,Rd}$ from equation (7).

$$\frac{M_{y,\beta,Ed}}{M_{\beta,\theta,Rd}} \leq \frac{\chi_{LT}}{\left(1 - \frac{\mu_{LT} N_{\beta,Ed}}{\chi_{z,\beta} N_{\beta,\theta,Rd}}\right)} \left(1 - \frac{N_{\beta,Ed}}{\chi_{z,\beta} N_{\beta,\theta,Rd}}\right) \quad (29)$$

6. COMPARISON BETWEEN THE TWO VERSIONS OF EUROCODE 3 AND THE SAFIR PROGRAM

6.1 RESULTS FOR BEAMS AND COLUMNS

The comparison between the two versions of the Eurocode 3 and the Safir program are presented for simply supported beams submitted to pure bending and for axially loaded columns in figures 5 and 6 respectively.

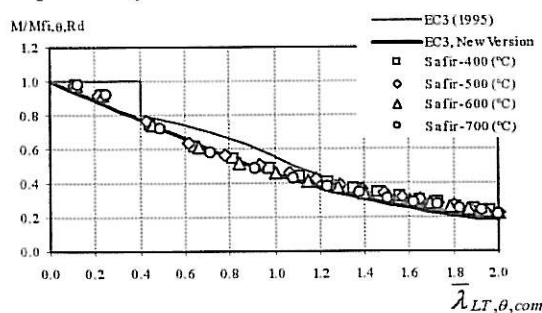


Fig. 5 – Design curves for lateral torsional buckling of beams

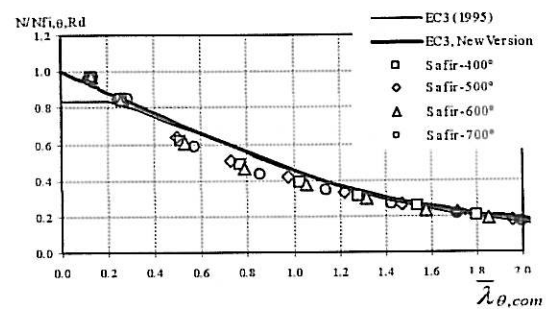


Fig. 6 – Design curves for buckling of columns

6.2 RESULTS FOR BEAM-COLUMNS

The resistant moments given by Safir for the beam in figure 1 under 400°C were divided by $M_{y,\beta,\theta,Rd}$, to compare with Eurocode 3 (1995) and with the new version of Eurocode 3. The charts from figure (7) to figure (23) present $M/M_{y,\beta,\theta,Rd}$ in function of $N/N_{\beta,\theta,Rd}$, where N is the design axial compression load due to the restraint and M the resistant moment of the beam for the three solutions we are considering.

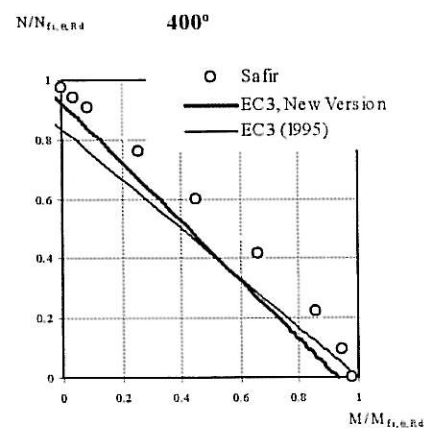


Fig.7- $L=250mm$; $\bar{\lambda}_{LT,\beta} = 0.12$; $\bar{\lambda}_{y,\beta} = 0.03$;

$$\bar{\lambda}_{z,\beta} = 0.13$$

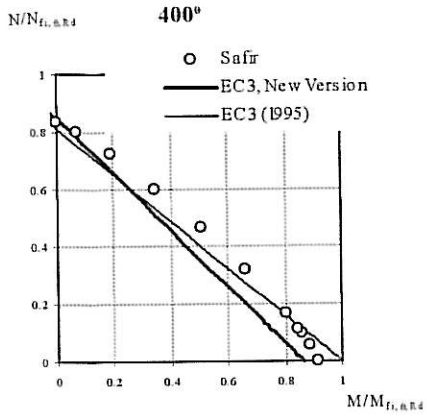


Fig.8- $L=500\text{mm}$; $\bar{\lambda}_{LT,fi} = 0.23$; $\bar{\lambda}_{y,fi} = 0.07$;
 $\bar{\lambda}_{z,fi} = 0.26$

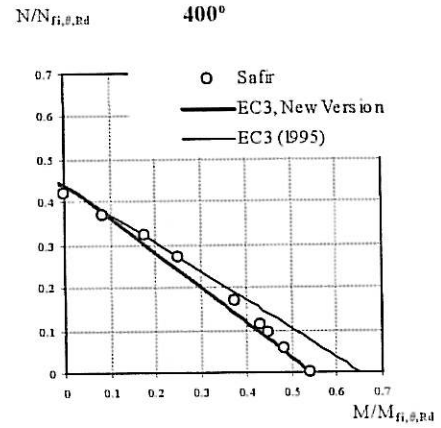


Fig.11- $L=2000\text{mm}$; $\bar{\lambda}_{LT,fi} = 0.82$; $\bar{\lambda}_{y,fi} = 0.28$;
 $\bar{\lambda}_{z,fi} = 1.03$

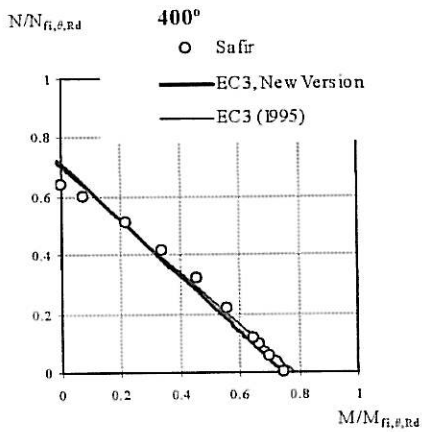


Fig.9- $L=1000\text{mm}$; $\bar{\lambda}_{LT,fi} = 0.45$; $\bar{\lambda}_{y,fi} = 0.14$;
 $\bar{\lambda}_{z,fi} = 0.51$

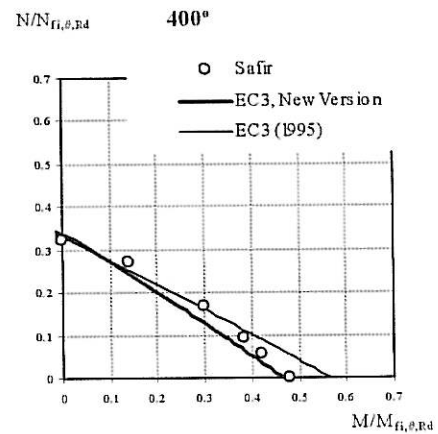


Fig.12- $L=2500\text{mm}$; $\bar{\lambda}_{LT,fi} = 0.98$; $\bar{\lambda}_{y,fi} = 0.35$;
 $\bar{\lambda}_{z,fi} = 1.28$

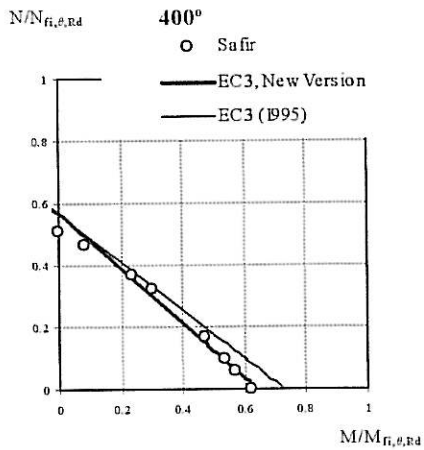


Fig.10- $L=1500\text{mm}$; $\bar{\lambda}_{LT,fi} = 0.64$; $\bar{\lambda}_{y,fi} = 0.21$;
 $\bar{\lambda}_{z,fi} = 0.77$

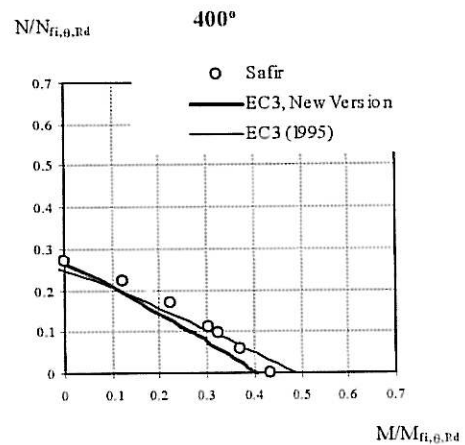


Fig.13- $L=3000\text{mm}$; $\bar{\lambda}_{LT,fi} = 1.12$; $\bar{\lambda}_{y,fi} = 0.42$;
 $\bar{\lambda}_{z,fi} = 1.54$

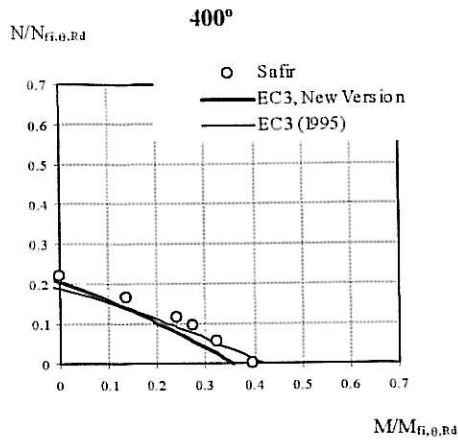


Fig.14- $L=3500\text{mm}$; $\bar{\lambda}_{LT,fi}=1.24$; $\bar{\lambda}_{y,fi}=0.49$;
 $\bar{\lambda}_{z,fi}=1.80$

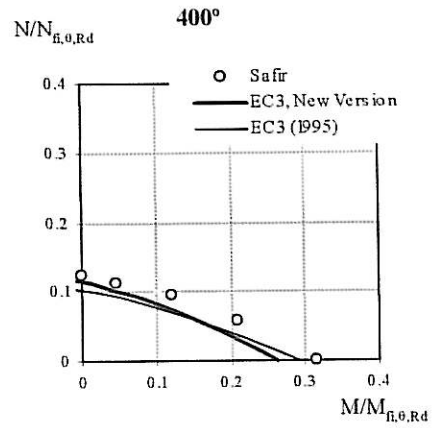


Fig.17- $L=5000\text{mm}$; $\bar{\lambda}_{LT,fi}=1.56$; $\bar{\lambda}_{y,fi}=0.70$;
 $\bar{\lambda}_{z,fi}=2.57$

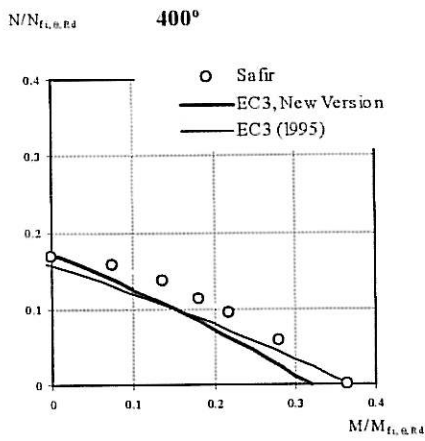


Fig.15- $L=4000\text{mm}$; $\bar{\lambda}_{LT,fi}=1.35$; $\bar{\lambda}_{y,fi}=0.56$;
 $\bar{\lambda}_{z,fi}=2.05$

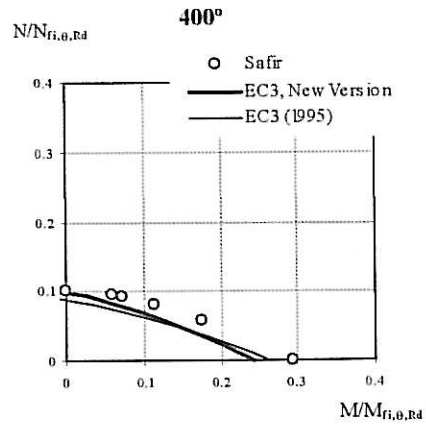


Fig.18- $L=5500\text{mm}$; $\bar{\lambda}_{LT,fi}=1.65$; $\bar{\lambda}_{y,fi}=0.77$;
 $\bar{\lambda}_{z,fi}=2.82$

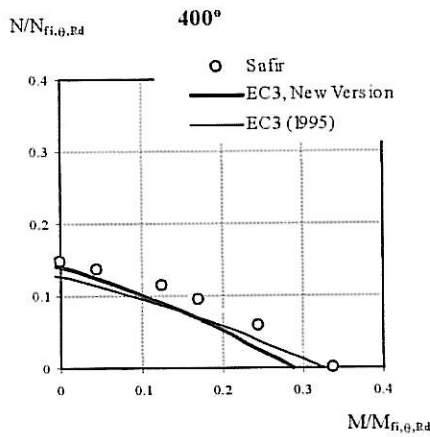


Fig.16- $L=4500\text{mm}$; $\bar{\lambda}_{LT,fi}=1.46$; $\bar{\lambda}_{y,fi}=0.63$;
 $\bar{\lambda}_{z,fi}=2.31$

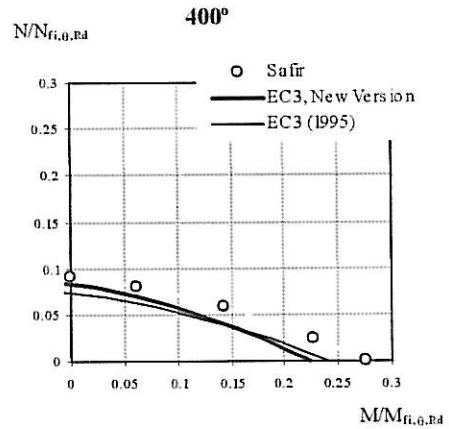


Fig.19- $L=6000\text{mm}$; $\bar{\lambda}_{LT,fi}=1.73$; $\bar{\lambda}_{y,fi}=0.84$;
 $\bar{\lambda}_{z,fi}=3.08$

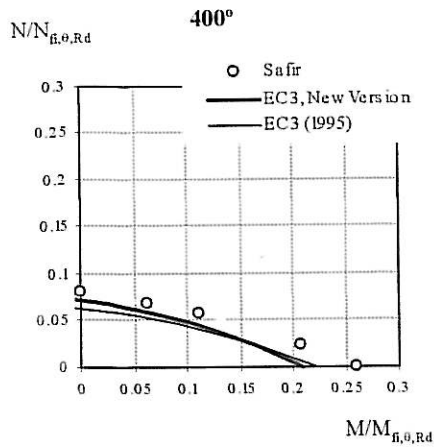


Fig.20- $L=6500mm$; $\bar{\lambda}_{LT,fi}=1.81$; $\bar{\lambda}_{y,fi}=0.91$;
 $\bar{\lambda}_{z,fi}=3.34$

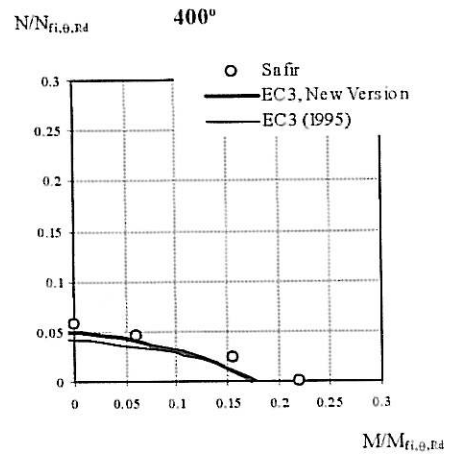


Fig.23- $L=8000mm$; $\bar{\lambda}_{LT,fi}=2.04$; $\bar{\lambda}_{y,fi}=1.12$;
 $\bar{\lambda}_{z,fi}=4.11$

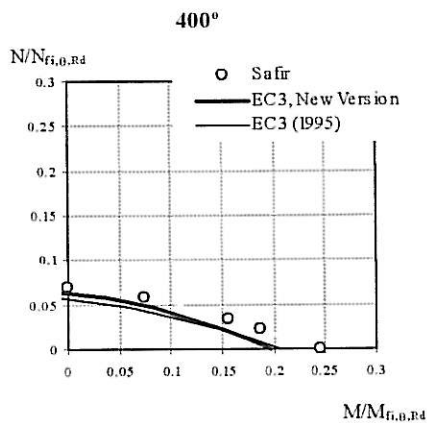


Fig.21- $L=7000mm$; $\bar{\lambda}_{LT,fi}=1.89$; $\bar{\lambda}_{y,fi}=0.98$;
 $\bar{\lambda}_{z,fi}=3.59$

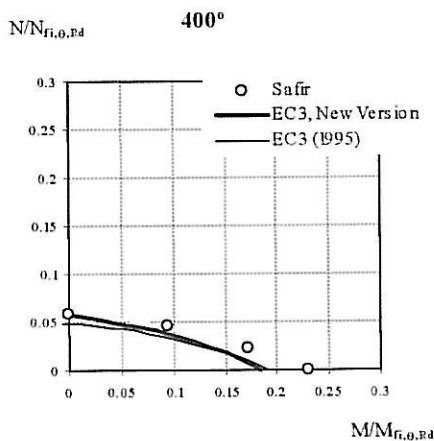


Fig.22- $L=7500mm$; $\bar{\lambda}_{LT,fi}=1.96$; $\bar{\lambda}_{y,fi}=1.05$;
 $\bar{\lambda}_{z,fi}=3.85$

7. CONCLUSIONS

For Beam-columns with length between 500 mm and 3000 mm the new version of Eurocode 3 is safer than the version from 1995. This fact is due to the new proposal presented for the lateral-torsional buckling of beams, (Vila Real et al, 1999, 2001, 2002).

The reason why the numerical results are not in the safe side for beam lengths up to 2500 mm is due to the fact that for axially-loaded columns the two version of the Eurocode 3 are less conservative than the numerical results. It must be said that Franssen et al (1996) has proposed the new version for columns based on experimental tests instead of numerical results.

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