SLIDING MODE CONTROLLERS FOR THE REGULATION OF DC/DC POWER CONVERTERS

Rui Esteves Araújo*, Américo Vicente Leite, Diamantino Silva Freitas***

** Faculty of Engineering of the University of Porto*
Dr. Roberto Frias, 4200-465 Porto , Portugal
** Polytechnic Institute of Bragança
Campus de Sta. Apolónia,
Apartado 134, 5301-587 Bragança, Portugal

Abstract: Sliding mode controllers are derived for the control of the average output voltage in DC/DC power converters. The controller design is carried out on the basis of well-known bilinear models of such circuits. A cascaded control structure is chosen for ease of control realization and to exploit the motion separation property of this power converter. The performance of the proposed sliding mode controllers is tested for the buck and boost converter type. The numerical simulations will demonstrate the efficiency of sliding mode techniques in this field as a powerful alternative to other existing methods.

Keywords: Robust control, Sliding mode control, Nonlinear control, Output feedback stabilization.

1. INTRODUCTION

Switched mode power converters lie at the heart of DC power supplies, bringing the advantages of high efficiency and low mass. They can be modeled as variable structure systems because of the abrupt topological changes suffered by the circuit controlled by a discontinuous control action. They constitute a natural field of application of sliding mode controllers. Sliding mode control is more than a promising technique in the field of automatic control. Moreover, with time it is gaining increasing importance as a universal tool for the robust control of linear and nonlinear systems. It permits the realization of very robust and simple regulators of low cost. Several application of this technique have been developed, mainly in the field of electrical motors control (Utkin, 1993, Araújo, *et al.*, 2000). In fact, since that the power converter is a nonlinear system with variable structure behaviour, the sliding mode control has became the natural solution for direct implementation of its control.

Traditionally, the control problems of DC/DC power converters are solved by using linear controllers that control the average of the output voltage by adjusting the duty cycle of the converter through PWM techniques. Frequently, the state average models of DC/DC power converters are used to derive all the necessary transfer functions to design a controller by using linear control techniques. This is a simple and useful method but it has some very distinct drawbacks like that only the equivalent behavior is represented (in sense that the ripple is excluded), and the control is designed around the steady state solution (equilibrium point), which leaves most of the state-space out of consideration. The regulation is often achieved by a linear controller, showing good performances when disturbances are very small. On the contrary, when disturbances are large, the results of regulation become unsatisfactory. On other hand, since stability cannot be guaranteed more than in a small region around the steady-state operating point, several add-on circuits like: up-ramp,

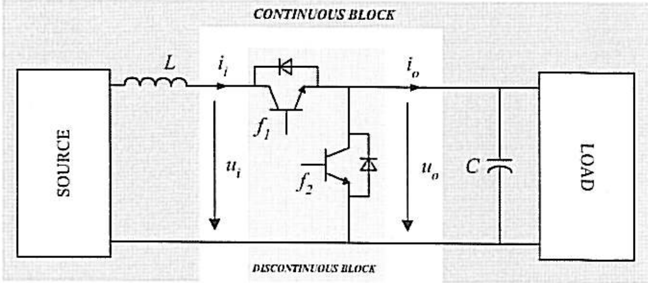

FIG. 1. Functional decomposition of DC/DC converters.

duty cycle limitation, clock, etc., must be added to support the controller.

The sliding mode technique offers an alternative way to implement a control action that exploits the inherent variable structure nature of the power converter. In particular, this technique offers several advantages in DC/DC power converters: - Robustness even for large load and supply variations, - Good dynamic response and - Simple implementation. Although initial ideas of sliding mode control have already been proposal in the 1970 decade, industrial applications are still very rare. Almost all implementations have seen made only for research investigations. Sliding mode control of switched power supplies was first treated by Venkataramanan, (1985) after the survey paper of Utkin (1977), and more recently by Sira-Ramirez and co-authors from a passivity viewpoint (Sira-Ramirez H., *et al.*, 1997). The topic was also treated in the context of the control of the Cúk converter in Oppenheimer M., *et al.*, (1996), where the authors demonstrated the possibility of applying a switching surface that is a linear combination of several state variables. Connections of sliding mode controllers with DC/DC power converters involving the natural time-scale separation properties was first treated by Sira-Ramirez (1988) from a geometric viewpoint.

In this paper, we will propose controllers based on the sliding mode technique, which don't employ PWM. The objective is to present a systematic approach of sliding mode controllers for the control of DC/DC power converter. In addition, we intend to put in evidence the most distinctive feature of a sliding mode controller, that is its robustness to parameter uncertainty and external disturbances. These features make this control technique a valid alternative in industrial applications. Finally, we wish to show how the hardware implementation is much easier for sliding mode control techniques when compared with classical control approaches.

This paper is organized as follows. Section 2 presents the basic concepts of this design methodology for DC/DC converters. The basic idea of sliding mode control is also suitable for other power converters. Section 3 develops the sliding mode controllers and demonstrates, for Buck and Boost converter cases, the simplicity of sliding mode control. The simulations' results are presented in section 4. Section 5 contains the main conclusions.

2. DESIGN CONCEPTS OF DC/DC POWER CONVERTERS

The results of this section, regarding the hybrid nature of DC/DC power converters, extend the work found in Araújo (2001), where only the Buck-type DC/DC power converter is treated. It must be noted that the hybrid nature of the power converter is used to structure the controller. The term "hybrid" has many meanings, one of which is: the dynamic evaluation of the converter depends on coupling between variables that take values in a continuum and variables that take values in a finite or countable set.

2.1 Decomposition in Sub-systems and Control Circuit Design

The controlled system of a power converter is constituted of three subsystems: the DC power supply, the static converter and the load as described in Figure 1. Having in account its hybrid nature and independently of the power converter type (buck, boost, Cúk, etc.), it is possible to make a functional decomposition into two blocks: a discontinuous block and continuous one. The first block comprises the switching elements. Its structure largely depends on the power converter type. On the other hand, the continuous block is independent of the static converter and comprises the power supply, load and reactive components. The reactive components are an inductor and a capacitor that work for intermediate storage of energy. It is clear from Figure1 that the system involves inter-action between discrete and continuous dynamics.

The decomposition of the controlled system allows defining a certain systematic approach about control systems design (see Figure 2). The key idea of the design concept for the power converters presented in this work is the cascaded control structure. According to this idea, any controller should consist of two loops: an inner current loop lies with the discontinuous block and an outer voltage control loop lies with the continuous one. This control structure is chosen for ease of control realization and to exploit the time-scale and functional separation properties of power converters. Such characteristic results from the desirable time-scale separation properties of the input filter and the RC output circuit time constant and the hybrid nature of power converters. In fact, for all switched power supplies used in practice, the motion rate of the current is much faster than the motion rate of the output voltage. In this context, we use the sliding mode approach for the control of the inner current loop and the voltage control loop is realized with standard linear control techniques.

The sliding mode control techniques are well suited for controlling static converters because of their natural switching structure. The design consists of two major phases. The first is the selection of a switching surface, so that the system, when restricted to this surface responds in the desired manner. The second is the design of a control law, which satisfies a set of sufficient conditions for the existence and reachability of a sliding mode. The aim of the next sub-section is to formulate the control problem.

2.2 Application of sliding mode control to the power electronics

In general terms the control problem can be formulated as follows:

Given a switching period T>0, determine a feasible control input u^v, so that a controlled variable follows as closely as possible a reference signal, despite influence of disturbances, measurement errors and parameter variations in the system.

The control problem as formulated leads to a number of issues. One is to select a switching surface so that the system produces a desired behaviour when restricted to it. Another is to choose the control law u^v that can induce a sliding motion close to the sliding surface of the system.

Usually the DC/DC power converters can be described with a unified state-space formulation in the form of a hybrid system defined on $\Re^n$.

$$\dot{x} = Ax + Bu^v x \qquad (1)$$

where $x \in \Re^n$ is the state vector, $A \in \Re^{n \times n}$ and $B \in \Re^{n \times n}$ are matrices with constant values and u^v is an m-dimensional vector that lives in a finite set

$$u^v \in \mathcal{U} = \{u^1, u^2, \cdots, u^N\} \subset \Re^m, \quad N \geq 2 \qquad (2)$$

where each element u^v can be one between the 2^m different vectors, such as:

$$u^v = \left\{ \underset{1}{\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}} \underset{2}{\begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}} \cdots \underset{2^m}{\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}} \right\}, \quad i = 1, \cdots, 2^m. \qquad (3)$$

The switching instants are determined by appropriate surfaces (that will be determined later), which are chosen to achieve a desired dynamic response. Between two commutations, the control signal u^v is a constant vector.

Note that the differential equation (1) describing the generic converter corresponds to a bilinear system due to the multiplicative character of the control variable u^v with respect to the state vector. Bilinear systems have been found in diverse processes and fields and many control strategies have been developed in the past, such as the quadratic feedback control proposed by Gutman, (1981). Here, we will focus on a class of the controllers designed by sliding mode techniques (Ping-Chen, Y., *et al.*, 2000). It must be noticed that for the DC/DC converters the control signal u^v only takes values from the discrete set $u^v \in \mathcal{U} = \{0, 1\}$.

Now, for the system (1), we advocate a discontinuous control as

$$u^v = \frac{1}{2}\left(1 - sign\ (s)\right) \qquad (4)$$

where s is a switching function in the sense of sliding mode theory, see Utkin (1992). This control action is a direct large signal control approach and DC/DC converter is self-oscillating. Explicitly, this means that the switching frequency depends on the rate of change of function s and on the amplitude of the hysteresis band. This problem can be unacceptable if the range of variations becomes too high.

3. SLIDING MODE CONTROLLERS FOR DC/DC POWER CONVERTERS

3.1 The model and controller for the Buck converter

Consider the buck-type converter circuit shown in Figure 3. The differential equations that describe the circuit are given in (5).

where x_1 and x_2 represent the input inductor current and the output capacitor voltage variables, respectively. The positive quantity V_{cc} represents the constant voltage value of the power supply.

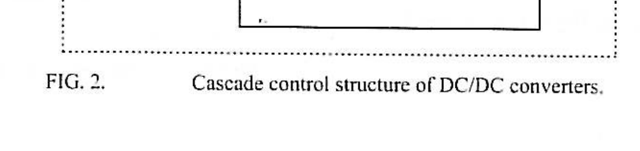

FIG. 2. Cascade control structure of DC/DC converters.

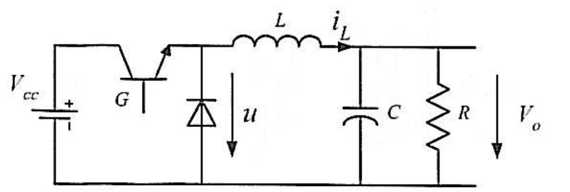

FIG. 3. Buck converter circuit.

$$\begin{aligned} \dot{x}_1 &= -\frac{1}{L}x_2 + \frac{V_{cc}}{L}u^v \\ \dot{x}_2 &= \frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{aligned} \quad . \tag{5}$$

The variable u^v denotes the switch state, acting as a control input.

We now consider the problem of constructing an appropriate control law that may ensure an asymptotic voltage tracking. Suppose it is desired to regulate the output capacitor voltage to a constant value. This is expressed mathematically by

$$\lim_{t\to\infty} (V_0 - V_{ref}) = 0, \qquad \dot{V}_0 = 0 . \tag{6}$$

Now, corresponding to this objective for the output voltage, the required input current may be represented by a function $I_{ref}(t)$, to be determined in thefollowing.

This methodology is justified by the inspection of the model of the system in the following way: first, it is verified that the inductor current (state variable x_1) is manipulated by means of the control variable u^v (first equation of 5). Second, taking care of the purpose of control the following can be derived from equation (6):

$$\dot{V}_0 = 0 \Leftrightarrow \dot{x}_2 = 0 \Rightarrow I_{ref} = \frac{V_{ref}}{R} . \tag{7}$$

Since the objective of the controller is to drive V_0 to some constant value, it is clear that the inductor current will play a central role. Consequently, the first objective of the controller is to guarantee that the state variable x_1 follows the trajectory established in equation (7). Then, the first loop (of greater dynamics) is controlled through the establishment of a function of commutation defined as:

$$s = I_L - I_{ref} \tag{8}$$

where s is the surface variable and the superscript *ref* indicate the reference or set-point. The set point value is determined by the upper level controller. On s=0, the inductor current will converge to its reference. In order to enforce sliding mode in the surface s in (8), a control action based on (4) is used. The condition for sliding mode to exist is derived from $s\dot{s} < 0$. Thus, we have

$$s\dot{s} < 0 \Leftrightarrow s(\dot{I}_L - \dot{I}_{ref}) < 0 \Leftrightarrow s\left(-\frac{x_2}{L} + \frac{V_{cc}}{L}u^v\right) < 0 \tag{9}$$

taking care of equation (9) that defines the control action, results in two situations:

if $s > 0 \Rightarrow u^v = 0$, as it is necessary to guarantee $\dot{s} < 0$ for the existence condition to be true, it is enough then to guarantee that $x_2 > 0$;

if $s < 0 \Rightarrow u^v = 1$, as it is necessary to guarantee $\dot{s} > 0$, for the existence condition to be true it is enough that $x_2 < V_{cc}$. In conclusion, the domain of attraction of the specified surface is verified if

$$0 < x_2 < V_{cc} . \tag{10}$$

This mathematical result has immediate physical interpretation: the output voltage of this static converter is always lesser than that of the source.

We now follow on to verify if the control objective is fulfilled. The dynamics of the output voltage of the converter in sliding mode is given by:

$$\dot{V}_o = -\frac{1}{RC}V_o + \frac{1}{RC}V_{ref} \tag{11}$$

and the solution of this first order differential equation (11) is:

$$V_o(t) = V_{ref} + (V_o(t_a) - V_{ref})\, e^{-(t-t_a)/RC} . \tag{12}$$

The second term of the solution will decay rapidly to zero. Under this condition $\lim_{t\to\infty} (V_0 - V_{ref}) = 0$ is true, see (6).

3.2 *Model and Controller Design for the Boost type converter*

Consider now the boost converter circuit shown in Figure 4. The converter model is described by the following set of differential equations, with variables defined as before:

$$\begin{aligned} \dot{x}_1 &= -(1-u^v)\frac{1}{L}x_2 + \frac{V_{cc}}{L} \\ \dot{x}_2 &= (1-u^v)\frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{aligned} \tag{13}$$

Following the same procedure as in the previous case, a desired current is obtained from the outer voltage loop as

$$I_{ref} = \frac{V_{ref}^2}{RV_{cc}} \tag{14}$$

where V_{ref} is the desired output voltage. The switching function for the inner current control is defined , like in (8):

$$s = I_L - I_{ref} . \tag{15}$$

This choice is, as usual, motivated by the motion

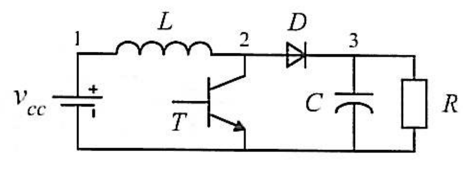

FIG. 4. Boost converter circuit.

separation property of power converters. This means that the dynamics of the inductor current is much faster than the dynamics of the output voltage. In order to enforce the inductor current to track the desired value, the control action can be similar to (4). Now, the condition that guarantee the existence of sliding mode is found by applying the convergence condition $s\,\dot{s} < 0$ to the system (13), given

$$x_2 > V_{cc}\,. \qquad (16)$$

Intuitively, (16) implies that the output voltage will always be greater than the input voltage.

In the ideal case (infinite switching frequency) the control action can be replaced by its mean value u_{eq}, the equivalent control, which is a continuous variable, is derived by formally solving $\dot{s} = \dot{x}_1 = 0$. From (13), proceeding like Utkin (1992), we obtain for the control input u^V:

$$u_{eq} = 1 - \frac{V_{cc}}{x_2}\,. \qquad (17)$$

The last expression for the equivalent control, determines the behaviour of the nominal system restricted to the switching surface. Now, we will see the equivalent equation of the output voltage in the sliding mode. Introducing (17) into the dynamic model of the boost converter, we obtain the output voltage:

$$\dot{x}_2 = -\frac{1}{RC}\left(x_2 - \frac{V_{ref}^2}{x_2}\right). \qquad (18)$$

This equation can be solved explicitly as

$$x_2 = \sqrt{\left(V_{ref}^2 + (x_2^2(t_0) - V_{ref}^2)\right)e^{-2(t-t_0)/RC}} \qquad (19)$$

where t_0 is the reaching instant to the switching surface and $x_2(t_0)$ is the output voltage at time t_0. We can see from (19) that x_2 tends to V_{ref} asymptotically as t goes to infinity. Then, there exists a control action u^V [that satisfying (4)] for the converter model (13) such that

$$\lim_{t\to\infty} (V_0 - V_{ref}) = 0\,. \qquad (20)$$

Consequently, there exists a finite time $t > t_0$ such that $V_0(t) = V_{ref}$, $\forall\, t \geq t_0$. Furthermore, regulation can be achieved with a suitable selection of u^V only.

4. SIMULATIONS RESULTS

Simulations were performed for the closed-loop behavior of a buck and boost circuits regulated by means of the sliding mode controllers. The proposed control scheme for the buck converter, illustrated in Figure 5, that also represents the basis of the hardware, was investigated by means of simulations done using the Vector Control Signal Processing Blockset for Simulink environment (Araújo, *et al.*, 1997, Araújo, *et al.*, 1998). The numerical experiment consists of analyzing the performance of the controller in tracking of the trajectory of reference of the voltage intended to the output of the converter. Thus, the initial reference of the voltage is of 10 V and for the time instant t= 0,05 s the reference is subjected to a step variation towards the value of 15 V. For the presented results, we considered the following parameters:

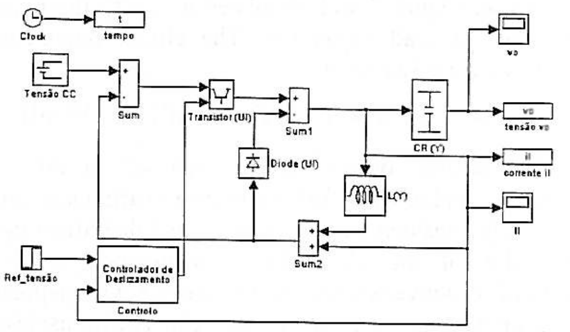

FIG. 5. Simulink model of a sliding mode buck DC/DC converter.

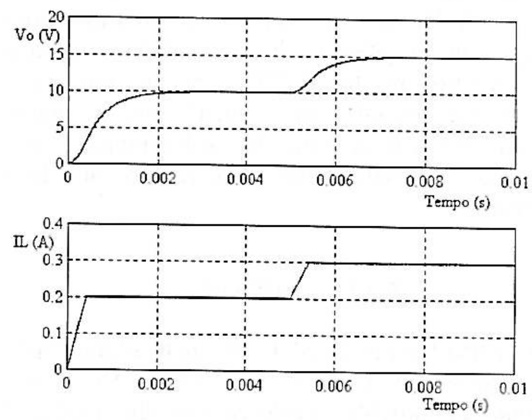

FIG. 6. Output voltage and inductor current waveforms in the buck converter with a sliding control scheme.

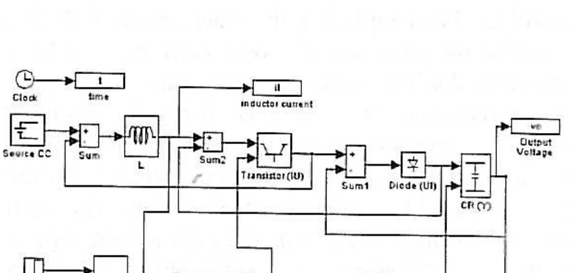

FIG. 7 Simulink model of a sliding mode boost DC/DC converter.

$V_{cc} = 20\text{ V}$, $R = 50\ \Omega$, $C = 10\ \mu\text{F}$, $L = 40\text{ mH}$.

Figure 6 shows the simulation results of the proposed control scheme for the buck DC/DC converter. From its analysis a good functioning may be verified. Notice that the inductor current and the output voltage converge rapidly to their references values.

A boost circuit with a feedback control circuit shown in Figure 7 was simulated to verify the main concept of load regulation. The circuit parameter values were taken to be:

$V_{cc} = 10\text{ V}$, $R = 40\ \Omega$, $C = 10\ \mu\text{F}$, $L = 3.5\text{ mH}$.

The desired output voltage was set to 20 V. Because unknown load resistance variations are normally considered as the main source that affect the behavior of the closed-loop performance of the controlled converter, the load resistance was stepped in a large range during the experiment. The robustness to load resistance uncertainties is verified, when an unmodeled sudden change in the load resistance was set to 80 per cent of its nominal value. As can be seen from Figure 8 the controller manages rapidly to restore the desired steady-state conditions immediately after the load perturbation disappears. Note that the output voltage converges rapidly to their reference value.

In conclusion, as is proven in the two simulations examples, the proposed methodology to design sliding mode controllers for DC/DC power converters achieves the desired direct regulation of the output voltage. This type of controller also exhibits a good degree of robustness with respect to the load disturbance.

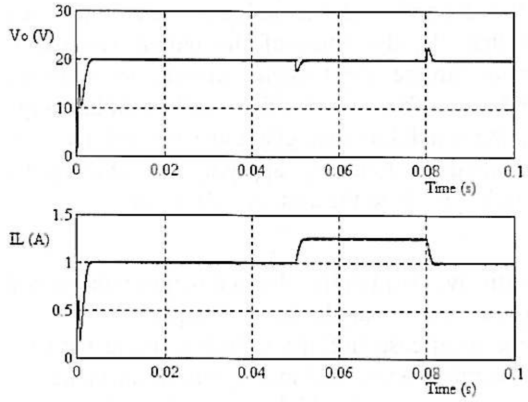

FIG. 8. Robustness test of the sliding mode controller in a boost converter to load disturbance.

5. CONCLUSIONS

This paper has presented the design of sliding mode controllers for DC/DC power converters. The design methodology provides evidence that the proposed sliding mode controller is capable of regulating the output voltage of the buck and boost converters. The advantages of sliding controllers over a classical control alternative lie in the hardware simplicity and closed loop robustness. The main objectives of this work have therefore been satisfied: - sliding mode control has been applied in the inner current loop that is similar for all types of power converters and the controllers don't introduce any additional complexity. Such possibilities were derived from the desirable time-scale separation properties between the L input filter and the RC output circuit time constant. On the other hand, the simplicity makes this approach attractive for implementation since the switch state is determined only from sign information about an affined functional of the converter state. Simulations results confirmed the effectiveness of the proposed control approaches.

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