



Stem volume ratio equations to variable merchantable limits for sweet chestnut in Portugal

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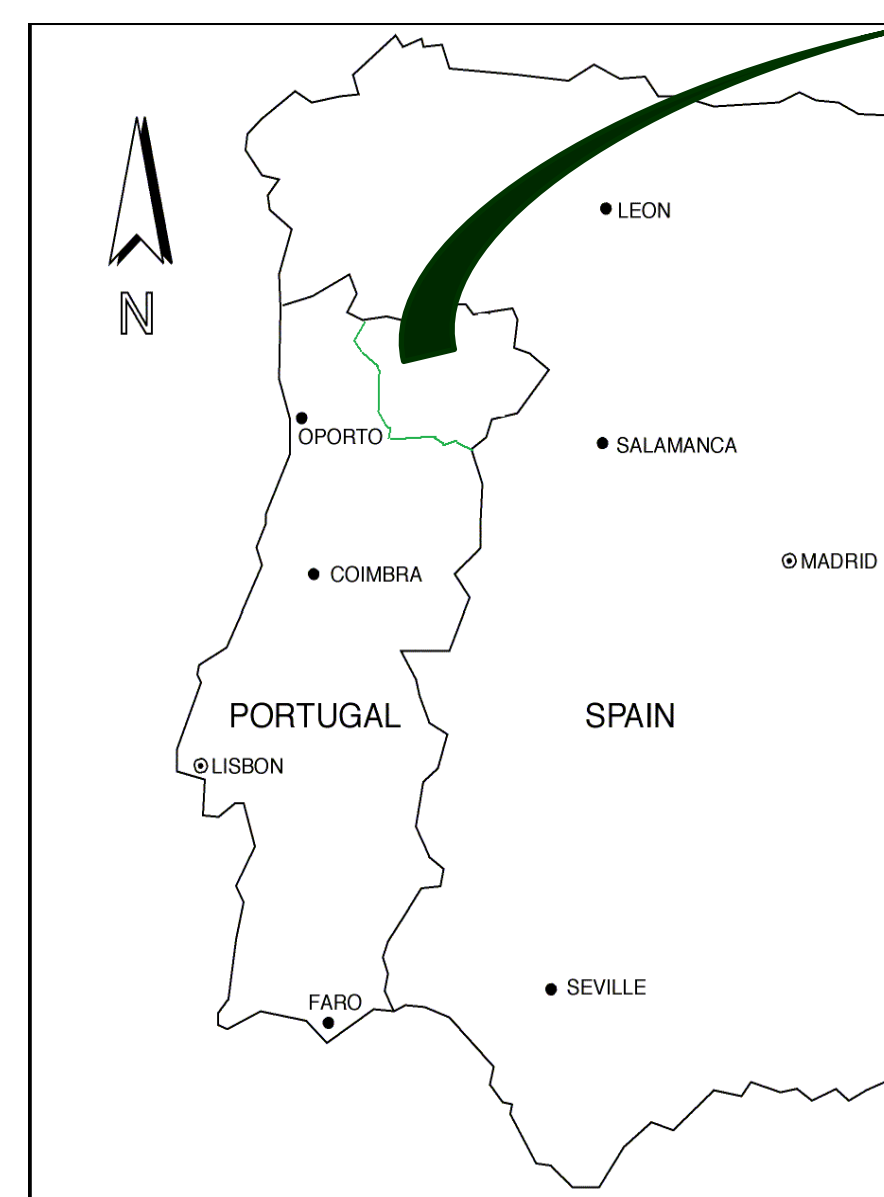


Introduction

Having reliable estimates of wood volume categorized by classes of use is crucial for effective wood utilization. Accurate estimations enable better planning and allocation of resources, ensuring wood is utilized efficiently and appropriately for various purposes. In turn, this leads to informed decisions regarding wood allocation, optimal utilization, and waste minimization, ultimately contributing to maximizing profitability in forest management. However, tools for estimating wood volume are scarce, particularly for sweet chestnut (*Castanea sativa* Mill.) stands in North Portugal.

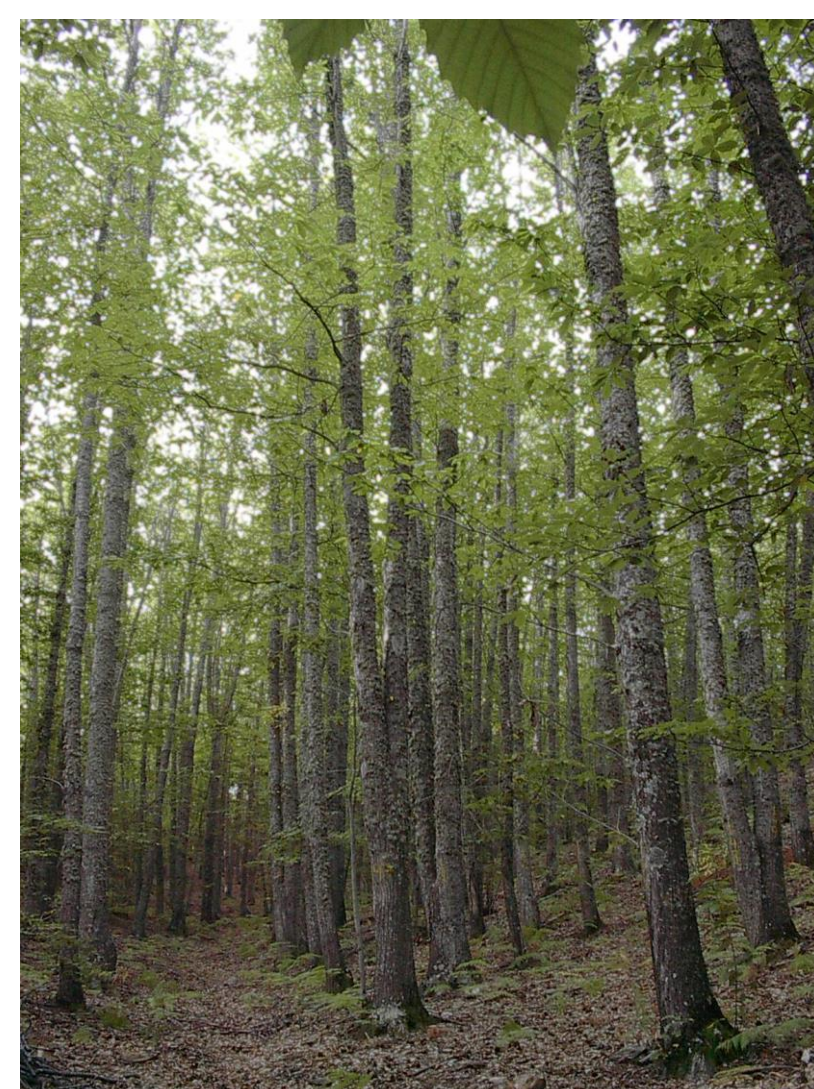
Sweet chestnut is a species with great potential for producing high-quality timber that serves diverse purposes. The wood obtained from sweet chestnut is known for its durability, strength, and appealing aesthetics, making it suitable for construction, furniture making, flooring, and joinery. Furthermore, chestnut can play a significant and renewed role in engineered wood products like Glulam beams and pillars used in sustainable construction practices.

Objective: By utilizing data from high-forest sweet chestnut stands in North Portugal as a baseline, this research aims to fill the gap in tools for estimating wood volume in sweet chestnut and contribute to the sustainable management and utilization of this valuable timber resource.



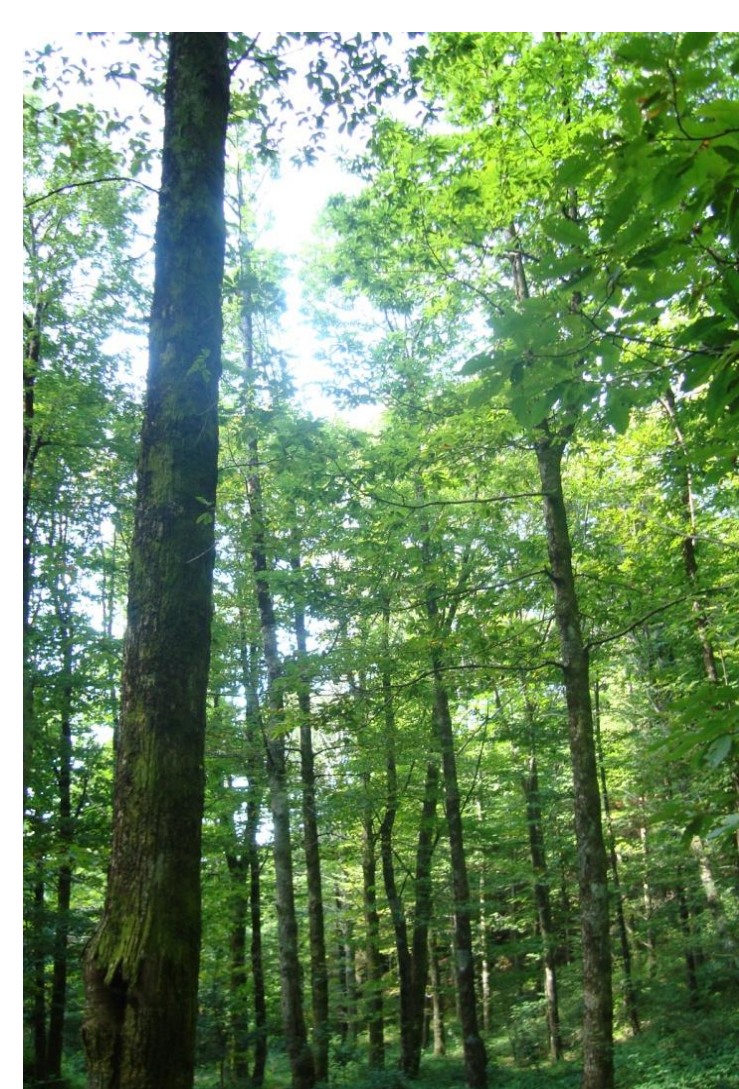
North Portugal

Bornes



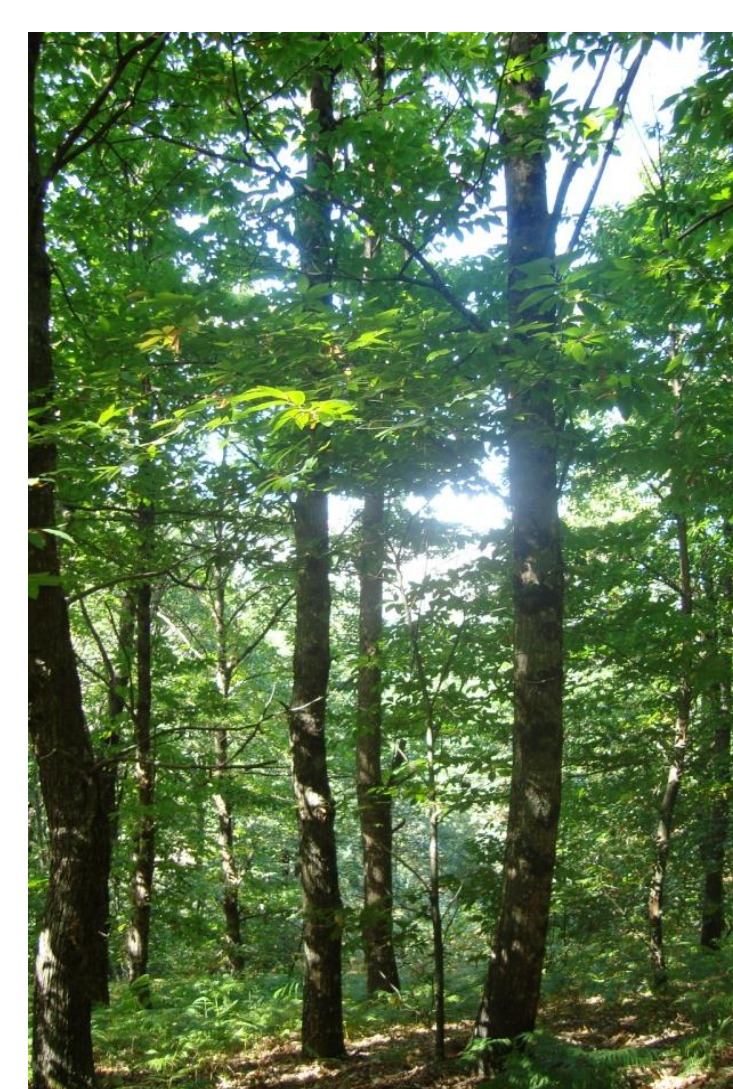
(41° 29' 42" N, 6° 55' 12" W)

Marão



(41° 14' 46" N, 7° 55' 04" W)

Padrela



(41° 30' 41" N, 7° 37' 15" W)

Data analysis:

- ✓ The volume rate equations were fitted using the ordinary least squares method (OLS) with both the PROC REG (linear models) and PROC NLIN (non-linear models) procedures of SAS/STAT. The modified Gauss-Newton iterative method was employed for fitting the non-linear models..
- ✓ The departure from regression assumptions was addressed by applying non-linear iteratively reweighted least squares (IRWLS) using the Huber function (Myers, 1986), with a maximum value of $r=1$

Materials and Methods

In the field data collection process, a total of **466 trees were selected** for volume estimation. Rigorous cubing was carried out using a telerelascope of Bitterlich device, following the existing diameter classes. This resulted in the generation of data set 1, consisting of 3,690 observations. The volume ratio equations with bark were fitted based on this dataset.

To validate the accuracy of the volume estimations, an independent sample of 39 felled trees underwent rigorous cubing. This produced data set 2, comprising 330 observations. Measurements were taken at the base, at breast height (1.30 m), and at regular intervals of 2.25 m from breast height to the 7 cm tip. However, in specific instances, measurements were made every 2 m in length from the diameter to the height of the breast.

For calculating tree volume, Smalian's formula was employed. The details and characteristics of data sets 1 and 2 can be found in Table 1..

Table1. Dendrometric characterization of data sets 1 and 2 for adjusting the volume ratio equations (volumes obtained through rigorous cubing with a telerelascope) and validation (volumes obtained through rigorous cubing in felled trees), respectively

| Site - plot | Tree variable | Nº Trees | Mean | s | Minimum | Maximum |
|--------------------------------------|---------------|----------|-------|-------|---------|---------|
| Data set 1 (model fit) | | | | | | |
| Bornes | d (cm) | 76 | 20.80 | 5.81 | 10.30 | 34.00 |
| | h (m) | | 19.05 | 2.67 | 11.94 | 24.01 |
| | v (m³) | | 0.33 | 0.22 | 0.04 | 1.09 |
| Marão - A | d (cm) | 26 | 28.35 | 5.45 | 19.70 | 41.50 |
| | h (m) | | 14.14 | 2.48 | 11.06 | 21.39 |
| | v (m³) | | 0.38 | 0.15 | 0.16 | 0.70 |
| Marão - B | d (cm) | 29 | 37.28 | 8.98 | 22.50 | 54.10 |
| | h (m) | | 27.77 | 2.84 | 22.91 | 33.90 |
| | v (m³) | | 1.20 | 0.66 | 0.33 | 2.50 |
| Padrela - A | d (cm) | 66 | 32.35 | 10.07 | 9.00 | 56.00 |
| | h (m) | | 23.27 | 2.81 | 12.15 | 26.86 |
| | v (m³) | | 0.93 | 0.56 | 0.04 | 2.33 |
| Padrela - C | d (cm) | 269 | 37.30 | 12.16 | 9.00 | 69.30 |
| | h (m) | | 20.34 | 4.76 | 6.11 | 30.92 |
| | v (m³) | | 1.11 | 0.74 | 0.02 | 4.02 |
| Total (data set 1) | d (cm) | 466 | 33.41 | 12.19 | 9.00 | 69.30 |
| | h (m) | | 20.66 | 4.81 | 6.11 | 33.90 |
| | v (m³) | | 0.92 | 0.70 | 0.02 | 4.02 |
| Data set 2 (model validation) | | | | | | |
| Felled trees | d (cm) | 39 | 34.81 | 13.61 | 10.25 | 64.20 |
| | h (m) | | 22.28 | 4.51 | 11.55 | 30.40 |
| | v (m³) | | 1.20 | 0.91 | 0.05 | 3.38 |

d, diameter at 1.30 m height with bark (cm); h, total height of the tree (m); v, total volume with bark (m³); s, standard deviation.

Model selection:

- ✓ The final equations were selected based on the following criteria:
- ✓ Measures of Fit and Prediction Ability: Various metrics such as modelling efficiency (EM), mean square error (MSE), and prediction R2 (R2pred) were used to assess how well the models fit the data and their predictive performance.
- ✓ Significance of Model Parameters: The significance of the parameters in the models was considered to ensure their meaningful contribution to volume estimation.
- ✓ Collinearity Assessment: The correlation matrix, variance inflation factor (VIF), and matrix condition number (NCOND) were examined to identify and address any issues related to collinearity among predictor variables.
- ✓ Normality of Residuals: The normality of studentized residuals was evaluated using normal QQ plots to ensure that the models did not exhibit significant departures from normality.
- ✓ Heteroscedasticity Check: Studentized residuals were plotted against predicted values to detect any patterns indicating heteroscedasticity, which could affect the accuracy of the models.
- ✓ The final equations were selected based on their overall fit, prediction accuracy, significance of parameters, degree of collinearity, normality of residuals, and absence of heteroscedasticity.

Results and Discussion

Table2. Models tested for the development of volume ratio equations as a function of topping diameter (di) or topping height (hi)

| Ref. | Model | Designation | Obs. |
|--|---|--------------------------|------------|
| Volume ratio equations as a function of topping height, hi | | | |
| EVPH1 | $R=1+\beta_1(h-h_i)^{\beta_2}$ | Cao et al. (1980) | |
| EVPH2 | $R=1+z\beta_1(z^2-z)+\beta_2(z^2-z)+\beta_3(z^2-z)+\beta_4(z^2-z)$ | Cao et al. (1980) | |
| EVPH3 | $R=1-[1-\exp(-\beta_1 \tan(\beta_2 h_i^{\beta_3}))]^{\beta_4}$ | Matney e Sullivan (1980) | Non-linear |
| EVPH4 | $R=1-z^{\beta_1}$ | Reed e Green (1984) | linear |
| EVPH5 | $R=\exp(\beta_1 z^{\beta_2})$ | Parresol et al. (1987) | |
| EVPH6 | $R=\exp(\beta_1 (p^2/h^{\beta_2}))$ | Parresol et al. (1987) | |
| EVPH7 | $R=1+\beta_1(h_i/h-1)+\beta_2(h_i/h^2-1)$ | Honer (1967) | Linear |
| Volume ratio equations as a function of topping diameter, di | | | |
| EVPD1 | $R=1+\beta_1(d_i/d)^{\beta_2}$ | Burkhardt (1977) | |
| EVPD2 | $R=1-\beta_1(d_i/d)^{\beta_2}$ | Clutter (1980) | |
| EVPD3 | $R=1-[1-\exp(-\beta_1 \tan(\beta_2 h_i^{\beta_3}(d_i/d)))]^{\beta_4}$ | Matney e Sullivan (1980) | Non-linear |
| EVPD4 | $R=\exp(\beta_1 (d_i/d)^{\beta_2})$ | Deusen et al. (1981) | linear |
| EVPD5 | $R=1+\beta_1(d_i/d)^{\beta_2}$ | Reed e Green (1984) | |
| EVPD6 | $R=1+\beta_1(d_i/d)^{\beta_2}+\beta_2(h_i/d)^{\beta_3}$ | Reed e Green (1984) | |
| EVPD7 | $R=\exp(\beta_1 (d_i/d)^{\beta_2})$ | Parresol et al. (1987) | |
| EVPD8 | $R=1+\beta_1(d_i/d)+\beta_2(d_i/d)^2-\beta_3(d_i/d)+\beta_4(d_i/d)^2-\beta_5(d_i/d)+\beta_6(d_i/d)^2$ | Cao et al. (1980) | Linear |

✓ d represent the diameter breast height (DBH), di the stem diameter at height hi, h the total height of the tree, hi height of the tree to the diameter di, p = h-hi, z = (h-hi)/h relative tree height. The volume rate, denoted as R, is defined as the ratio of the merchantable volume of a tree (vm) to the total volume (v)

Model validation

Table 4. Measures of predictive capacity obtained from the validation process for the volume ratio equations over bark as a function of the height at the top (hi) or the diameter at the top (di).

| Model | MSrp | R²rpred | mrp | marp | Vrp |
|--|--------|---------|---------|--------|--------|
| Number of observations – data set 2' (n=330) | | | | | |
| Volume ratio equations as a function of topping height, hi | | | | | |
| EVPH1 | 0.0012 | 0.9842 | -0.0218 | 0.0269 | 0.0007 |
| EVPH2 | 0.0007 | 0.9905 | -0.0044 | 0.0225 | 0.0007 |
| EVPH3 | 0.0012 | 0.9842 | -0.0226 | 0.0271 | 0.0007 |
| EVPH4 | 0.0013 | 0.9828 | -0.0218 | 0.0272 | 0.0008 |
| EVPH5 | 0.0016 | 0.9785 | -0.0284 | 0.0324 | 0.0008 |
| EVPH6 | 0.0016 | 0.9796 | -0.0273 | 0.0288 | 0.0008 |
| EVPH7 | 0.0013 | 0.9833 | -0.0256 | 0.0313 | 0.0006 |
| Volume ratio equations as a function of topping diameter, di | | | | | |
| EVPD1 | 0.0027 | 0.9646 | -0.0034 | 0.0370 | 0.0027 |
| EVPD2 | 0.0027 | 0.9646 | -0.0034 | 0.0370 | 0.0027 |
| EVPD3 | 0.0024 | 0.9684 | -0.0076 | 0.0346 | 0.0024 |
| EVPD4 | 0.0034 | 0.9564 | -0.0002 | 0.0406 | 0.0034 |
| EVPD5 | 0.0023 | 0.9689 | -0.0068 | 0.0341 | 0.0023 |
| EVPD6 | 0.0024 | 0.9687 | -0.0067 | 0.0342 | 0.0024 |
| EVPD7 | 0.0032 | 0.9584 | 0.0013 | 0.0399 | 0.0032 |
| EVPD8 | 0.0038 | 0.9505 | 0.0024 | 0.0391 | 0.0029 |

Prediction residuals (rp); mean square of prediction residuals (MSrp); mean of prediction residuals (mrp); mean absolute values of prediction residuals (marp); variance of prediction residuals (Vrp).

Conclusions

- ✓ This study has provided valuable volume ratio equations that offer precise estimates of the merchantable stem volume over bark for high forest sweet chestnut, utilizing specific trunk diameter or height as variables.
- ✓ Forest managers can utilize the equations to estimate the volume of merchantable timber available in a given area, facilitating informed decision-making regarding harvesting.
- ✓ By accurately estimating the volume of merchantable timber, landowners can better negotiate fair prices for their products and contribute to timber industry supply chains.

Model fitting

Table 3. Measurements of adjustment and prediction for the volume ratio models over bark as a function of height at the top (hi) and at the top (di), after applying IRWLS.

| Model | MSE | EM | mPRESS | maPRESS | R²pred | P95 | P5 | NCOND |
|--|------------------------|--------|-------------------------|---------|--------|--------|---------|-------------------------|
| Nº of observations (n=3690) | | | | | | | | |
| Volume ratio equations as a function of topping height, hi | | | | | | | | |
| EVPH1 | 0.868 10 ⁻³ | 0.9872 | 0.0025 | 0.0271 | 0.9792 | 0.0683 | -0.0544 | 3090.5862 |
| EVPH2 | 0.866 10 ⁻³ | 0.9872 | 0.0018 | 0.0269 | 0.9788 | 0.0713 | -0.0527 | 1.7879 10 ⁸ |
| EVPH3 | 0.868 10 ⁻³ | 0.9872 | 0.0017 | 0.0270 | 0.9790 | 0.0683 | -0.0544 | 1934.8555 |
| EVPH4 | 0.963 10 ⁻³ | 0.9856 | 0.0034 | 0.0285 | 0.9771 | 0.0721 | -0.0573 | 1.0000 |
| EVPH5 | 0.0011 | 0.9830 | -0.0028 | 0.0306 | 0.9737 | 0.0766 | -0.0631 | 10.1603 |
| EVPH6 | 0.0011 | 0.9834 | -0.0030 | 0.0304 | 0.9742 | 0.0740 | -0.0628 | 2326.2828 |
| EVPH7 | 0.0010 | 0.9843 | 0.0011 | 0.0307 | 0.9752 | 0.0814 | -0.0488 | 180.3401 |
| Volume ratio equations as a function of topping diameter, di | | | | | | | | |
| EVPD1 | 0.0025 | 0.9629 | 0.2896 10 ⁻⁴ | 0.0441 | 0.9376 | 0.1122 | -0.1076 | 4279.3834 |
| EVPD2 | 0.0025 | 0.9629 | 0.2896 10 ⁻⁴ | 0.0441 | 0.9376 | 0.1122 | -0.1076 | 4279.3848 |
| EVPD3 | 0.0022 | 0.9671 | 0.1156 10 ⁻⁴ | 0.0416 | 0.9431 | 0.1087 | -0.1051 | 1841.9179 |
| EVPD4 | 0.0028 | 0.9571 | -0.0037 | 0.0476 | 0.9320 | 0.1233 | -0.1049 | 6.0185 |
| EVPD5 | 0.0022 | 0.9673 | -0.0005 | 0.0415 | 0.9437 | 0.1083 | -0.1068 | 6154.2454 |
| EVPD6 | 0.0022 | 0.9674 | -0.0005 | 0.0414 | 0.9437 | 0.1080 | -0.1069 | 4.1826 10 ¹² |
| EVPD7 | 0.0027 | 0.9585 | -0.0036 | 0.0470 | 0.9333 | 0.1257 | -0.1039 | 3188.4573 |
| EVPD8 | 0.0026 | 0.9615 | 0.0005 | 0.0451 | 0.9362 | 0.1138 | -0.1066 | 3.9845 10 ⁸ |

(EM) modelling efficiency; (MSE) mean square error, (R2pred) prediction R2, (m_PRESS) mean of PRESS residuals, (ma_PRESS) mean absolute values of PRESS residuals, (P95) 95th percentile and (P5) 5th percentile of the PRESS residuals, (NCOND) matrix condition number.

Model selected

Equations that demonstrated the best performance in predicting the volume ratio with bark as a function of top height hi were as follows:

$$\text{EVPH1: } R=1+[-0.8075(h-h_i)^2.7922/h^2.7359]$$

$$\text{EVPH4: } R=1-((h-h_i)/h)^2.9340$$

Equations for predicting the volume ratio over bark as a function of topping diameter di were as follows:

$$\text{EVPD3: } R = 1 - [1 - \exp(-0.8525 * \tan(0.9682 * h^0.0847 * (di/d)))]^3.1504$$

$$\text{EVPD4: } R = \exp[-1.4039 * (di/d)^4.3716]$$

- ✓ To estimate volumes by timber use category for high forest chestnut, the proposed total volume equation from Patrício (2006) and Patrício and Nunes (2017) is recommended: $v = 0.015160 + 0.0000324 d_2h$.

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