A new finite element for generalized in-plane pipe loading. Experimental and numerical comparison.

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Abstract

In structural engineering, the geometry of a large number of structural details may involve the combination of straight and curved parts in order to meet requirements of functionality and/or attractive design. Piping systems are structural elements used in the chemical industry, aeronautical and aerospace engineering, conventional and nuclear power plants and fluid transport in general-purpose process equipment. This paper presents a new finite element pipe with 19 degrees of freedom, where shape functions are set-up from the displacement field parallel to a local reference system. A displacement-based formulation was developed with Fourier series for increasing the structural element distortion capabilities. A finite element pipe may be considered as a part of a toroidal shell. The stress field distribution may be calculated for any cylindrical section pipe. Experimental set-up will be presented for in-plane piping system loading case and experimental stress measurement will be compared with the numerical stresses results obtained with this formulation and with other different commercial codes. The main advantage of this formulation is associated with timeless mesh generation with low number of elements and nodes. Considerable computational effort may be reduce with the use of this finite element pipe.

1 Introduction

Pipe bends plays a very important role in the global piping arrangement, as they not only allow flow direction change, but they also absorb thermal expansions and longitudinal deformations from adjacent tangent parts or other straight pipe elements. A computer code has been used extensively to make expedite calculations of deflections, reactions and stresses both, in piping system and pressure vessels. In this paper we will present the results of longitudinal stresses using a formulation for a curved and straight pipe subjected to specific loading case. The numerical results obtained by finite element formulation will be compared with experimental tests and ANSYS® code results, using shell elements.

2 General formulations

2.1. Geometrical definition of a pipe finite element with two nodes

For the development of this element we have included some hypotheses.

a) The thickness is very thin when compared with the transversal section medium radius;

b) The curvature radius is assumed much larger than the section radius;

c) A semi-membrane deformation model is adopted and neglects the bending stiffness along the longitudinal direction of the toroidal shell but considers the meridional bending resulting from ovalization;

d) The circumferential deformation is neglected as expressed by eqn (1), so the medium surfaces are inextensible along the meridional direction.

The geometrical parameters considered for the element definition are: the length of curved arch (*s*), curvature radius (*R*), the thickness (*h*), the circular section radius (*r*) and the centre angle (α), as show in figure 1.

$$\varepsilon_{\theta\theta} = \frac{1}{r}\left(w + \frac{\partial v}{\partial \theta} \right) = 0 \tag{1}$$

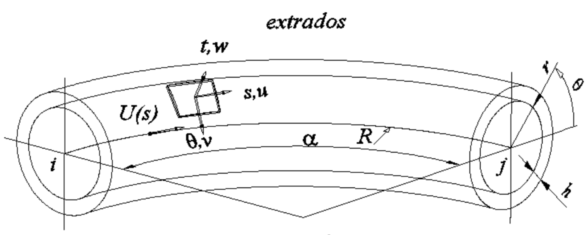


Figure 1: Geometric parameters for the pipe element.

2.2. The displacement field

The displacement field *u*, *v* and *w*, is calculated in the shell structural element surfaces, which is a function of displacement field in the reference medium line of a tubular arch (*U*, *W* and φ). These parameters are obtained by the simple differential equation from the beam bending theory, with the cross section undeformed.

Melo and *all* [1] have considered the following hypotheses:

a) The cross section rotation is related to the transversal displacement *W*, using the following equation:

$$\varphi = \frac{dW}{ds} \tag{2}$$

b) The tangential displacement is related to the transversal displacement *W*, expressed by eqn (3) and the curvature should be calculated according to eqn (4).

$$W = -\frac{dU}{ds} R \tag{3}$$

$$k_s = \frac{d^2 W}{ds^2} \tag{4}$$

The displacement field is calculated in the reference medium arch line, considered as a rigid beam, where: *U* is the tangential displacement, *W* the transversal displacement and φ in-plane rotation, as shown in figure 2.

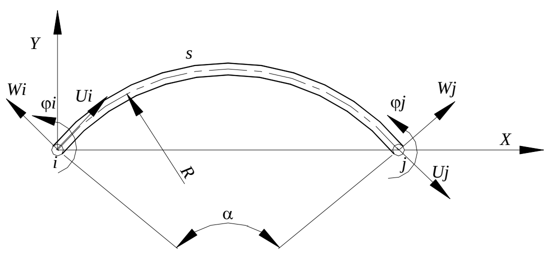


Figure 2: Displacement field for the curved pipe finite element.

A displacement field based on the curved beam theory has been used with a high order formulation. Six different parameters have been used to define the displacement field. The displacement *U* can be approximated by the following 5th order polynomial.

$$U_{(s)} = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 \quad (5)$$

The coefficients a_i are determined as function of the prescribed boundary conditions. The shape functions are considered in the curved referential (*s*) as represented in the figure 2.

For the local reference coordinate system the finite element pipe displacement field is determined by:

$$U_{(s)} = \left(U_i N_{ui} + U_j N_{uj}\right) + \left(W_i N_{wi} + W_j N_{wj}\right) + \left(\varphi_i N_{\varphi i} + \varphi_j N_{\varphi j}\right) \quad (6)$$

$$W_{(s)} = -R\left[\left(U_i N'_{ui} + U_j N'_{uj}\right) + \left(W_i N'_{wi} + W_j N'_{wj}\right) + \left(\varphi_i N'_{\varphi i} + \varphi_j N'_{\varphi j}\right)\right] \quad (7)$$

$$\varphi_{(s)} = -R\left[\left(U_i N''_{ui} + U_j N''_{uj}\right) + \left(W_i N''_{wi} + W_j N''_{wj}\right) + \left(\varphi_i N''_{\varphi i} + \varphi_j N''_{\varphi j}\right)\right] \quad (8)$$

The shape functions are determined according to the following expressions:

$$N_{ui} = \cos\left(\frac{\alpha}{2}\right) + \frac{1}{R}\sin\left(\frac{\alpha}{2}\right)s + \left(-\frac{10}{L^3}\cos\left(\frac{\alpha}{2}\right) - \frac{6}{RL^2}\sin\left(\frac{\alpha}{2}\right)\right)s^3 + \quad (9a)$$
$$+\left(\frac{15}{L^4}\cos\left(\frac{\alpha}{2}\right) + \frac{8}{RL^3}\sin\left(\frac{\alpha}{2}\right)\right)s^4 + \left(-\frac{6}{L^5}\cos\left(\frac{\alpha}{2}\right) - \frac{3}{RL^4}\sin\left(\frac{\alpha}{2}\right)\right)s^5$$

$$N_{uj} = \left(\frac{10}{L^3}\cos\left(\frac{\alpha}{2}\right) + \frac{4}{RL^2}\sin\left(\frac{\alpha}{2}\right)\right)s^3 + \left(-\frac{15}{L^4}\cos\left(\frac{\alpha}{2}\right) - \frac{7}{RL^3}\sin\left(\frac{\alpha}{2}\right)\right)s^4 + \quad (9b)$$
$$+\left(\frac{6}{L^5}\cos\left(\frac{\alpha}{2}\right) + \frac{3}{RL^4}\sin\left(\frac{\alpha}{2}\right)\right)s^5$$

$$N_{wi} = \sin\left(\frac{\alpha}{2}\right) - \frac{1}{R}\cos\left(\frac{\alpha}{2}\right)s + \left(-\frac{10}{L^3}\sin\left(\frac{\alpha}{2}\right) + \frac{6}{RL^2}\cos\left(\frac{\alpha}{2}\right)\right)s^3 + \quad (9c)$$
$$+\left(\frac{15}{L^4}\sin\left(\frac{\alpha}{2}\right) - \frac{8}{RL^3}\cos\left(\frac{\alpha}{2}\right)\right)s^4 + \left(-\frac{6}{L^5}\sin\left(\frac{\alpha}{2}\right) + \frac{3}{RL^4}\cos\left(\frac{\alpha}{2}\right)\right)s^5$$

$$N_{wj} = \left(-\frac{10}{L^3}\sin\left(\frac{\alpha}{2}\right) + \frac{4}{RL^2}\cos\left(\frac{\alpha}{2}\right)\right)s^3 + \left(\frac{15}{L^4}\sin\left(\frac{\alpha}{2}\right) - \frac{7}{RL^3}\cos\left(\frac{\alpha}{2}\right)\right)s^4 + \quad (9d)$$
$$+\left(-\frac{6}{L^5}\sin\left(\frac{\alpha}{2}\right) + \frac{3}{RL^4}\cos\left(\frac{\alpha}{2}\right)\right)s^5$$

$$N_{\varphi i} = -\frac{1}{2R}s^2 + \frac{3}{2LR}s^3 - \frac{3}{2L^2R}s^4 + \frac{1}{2L^3R}s^5 \quad (9e)$$

$$N_{\varphi j} = -\frac{1}{2LR}s^3 + \frac{1}{L^2 R}s^4 - \frac{1}{2L^3 R}s^5 \tag{9f}$$

The surface shell displacements are obtained by the displacement field in the reference medium line of a tubular arch adding the distortional section effects. The solution presented for the ovalization and warping displacements combine linear functions with trigonometric developed series [2].

The superficial displacement in the radial direction results by the ovalization effect. Thomson [2] proposed the following expression:

$$w_{(s,\theta)} = \sum_{\geq 2}^{9} a_i \cos i\theta \tag{10}$$

In the same form, the meridional displacement is obtained from the ovalization effect:

$$v_{(s,\theta)} = \sum_{\geq 2}^{9} -\frac{a_i}{i}\sin i\theta \tag{11}$$

Finally the longitudinal displacement due to warping section is function of:

$$\Omega_{(s,\theta)} = \sum_{\geq 2}^{9} b_i \cos i\theta \tag{12}$$

The a_i and b_i are constants to be determined as function of developed trigonometric series for ovalization and warping terms, respectively.

Adding this displacement fields to a rigid beam formulation, eqns 6 to 8, the superficial shell displacement field may be obtained:

$$u = U_{(s)} - r\cos\theta\varphi_{(s,\theta)} + \Omega_{(s,\theta)} \tag{13a}$$

$$v = -W_{(s)}\sin\theta + v_{(s,\theta)} \tag{13b}$$

$$w = W_{(s)}\cos\theta + w_{(s,\theta)} \tag{13c}$$

2.3. The deformation and stress fields

As previously referred, the deformation model considers that the pipe undergoes a semi-membrane strain field. The strain field is given by the following equation [3-5]:

$$\underset{\sim}{\varepsilon} = \begin{Bmatrix} \varepsilon_s \\ \gamma_{s\theta} \\ K_\theta \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial s} & -\dfrac{\sin\theta}{R} & \dfrac{\cos\theta}{R} \\ \dfrac{1}{r}\dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial s} & 0 \\ 0 & -\dfrac{1}{r^2}\dfrac{\partial}{\partial\theta} & \dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{14}$$

Where: ε_s is the longitudinal membrane strain, $\gamma_{s\theta}$ is the shear strain and K_θ is the meridional curvature ovalization.

For straight element pipe the Hermite shape functions will be used and in the eqn (14) the term $1/R$ will vanish.

The element stiffness matrix K is calculated from the matrix equation:

$$[K] = \int_{s=0}^{s=L} \int_{\theta=0}^{\theta=2\pi} [B]^T [D][B] r ds d\theta \tag{15}$$

The integration of this equation is extended to the pipe surface, where matrix B results from:

$$[B] = [L] \times [N] \tag{16}$$

Since the pipe is inextensible along the meridional direction, no contribution for elastic strain energy arises from such strains. The elasticity matrix D appears with a simpler algebraic definition, having deleted the contribution of off-diagonal terms with Poisson factor:

$$D = \begin{bmatrix} \dfrac{Eh}{1-\nu^2} & 0 & 0 \\ 0 & \dfrac{Eh}{2(1+\nu)} & 0 \\ 0 & 0 & \dfrac{Eh^3}{12(1-\nu^2)} \end{bmatrix} \tag{17}$$

Where: E is the elasticity modulus, h is the pipe thickness and ν is Poisson ratio.

The stress field is determined by:

$$\underset{\sim}{\sigma} = \begin{Bmatrix} N_{ss} \\ N_{s\theta} \\ M_{\theta\theta} \end{Bmatrix} = [D] \begin{Bmatrix} \varepsilon_s \\ \gamma_{s\theta} \\ K_{\theta\theta} \end{Bmatrix} \tag{18}$$

The matrix force-displacement equation for the curved pipe element is:

$$[K]\{\delta\}_e = \{F\} \tag{19}$$

Where: K is the stiffness matrix and δ_e is a nodal unknown displacement vector.

4 The case study

The studied case is shown in figure 3, representing a tubular steel structure submitted to an axial force in extremities. The elasticity modulus is 2.1GPa and the Poisson coefficient is equal to 0.3. The experimental setup used for this test is represented in figure 4.

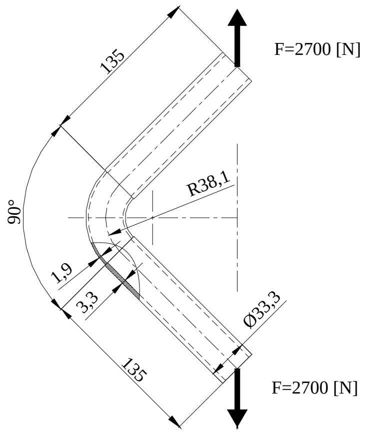


Figure 3: Geometric parameters for a tubular structure.

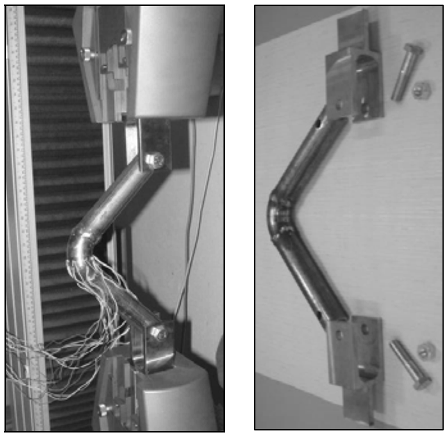
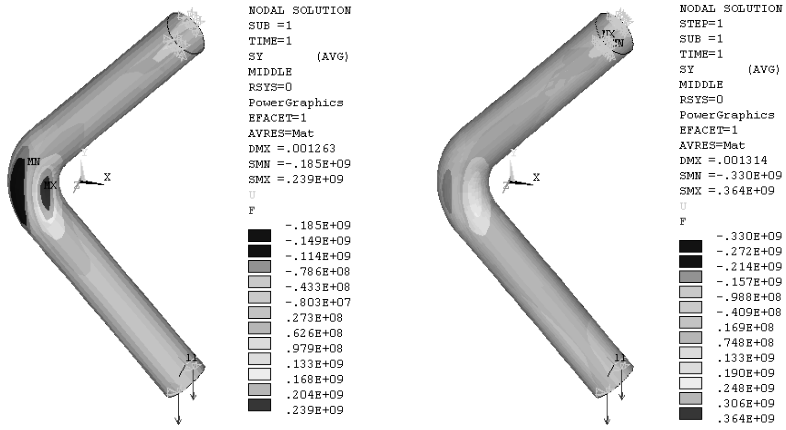


Figure 4: Experimental assembly for tubular steel structure.

The longitudinal stresses will be analysed at the middle of the structure over the radial cross section, using the experimental strain gauge method and numerical results obtain with ANSYS® code and with the new developed finite element. The loading system is implemented by the universal machine as represented in the figure 4.

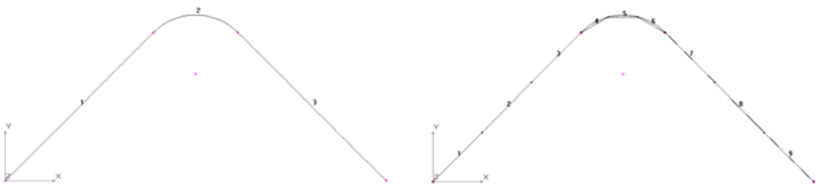
In the ANSYS® code, two different meshes using a shell63 element were used. Figure 5 show the stress field in the tubular structure.



Mesh 1 (624 elements) Mesh 2 (2688 elements)

Figure 5: Stress field using the shell68 in ANSYS® code.

The numerical results obtained by the new developed finite element will be presented with two different uni-dimensional meshes, as represented in figure 6. The stress results will be obtained for the transverse section at the middle of structure.



3 elements 9 elements

Figure 6: Uni-dimensional meshes used in developed finite element.

The numerical result using 9 elements presents good agreement with the experimental results. Using other higher mesh density, the results will converge to the same final solution, as represented in figure 7.

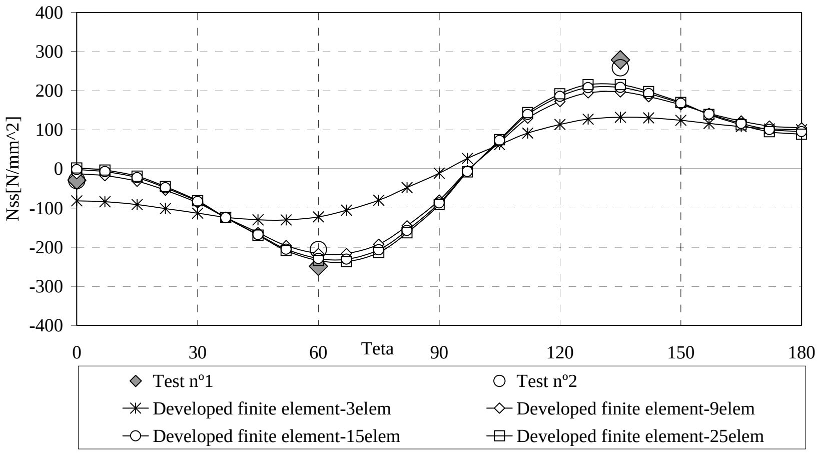


Figure 7: Longitudinal stresses with developed finite element and experimental results.

Figure 8 presents the results obtained with the new developed finite element using 9 and 15 elements, compared with the experimental results and ANSYS® numerical results.

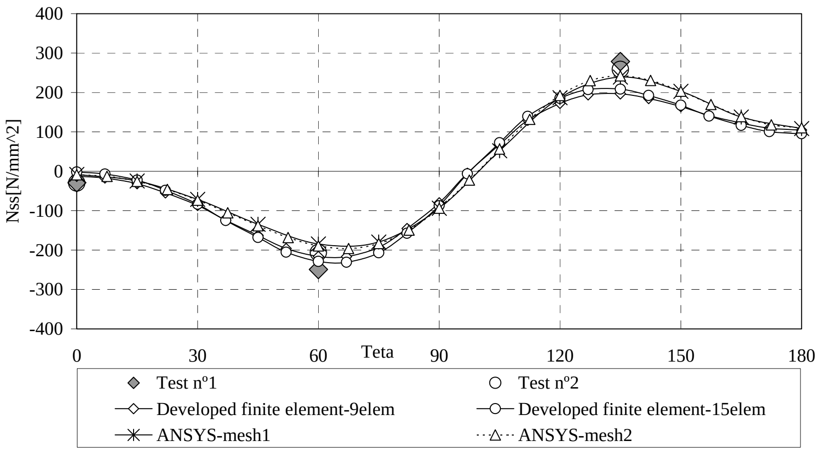


Figure 8: Longitudinal stresses with developed finite element, experimental results and ANSYS® code.

The experimental results are determined over the external tubular structure surface, justifying the position in figures 7 and 8.

7 Conclusions

This paper presents a new formulation for stresses and deformation characterization of thin piping systems, using the finite element method. The finite element pipe developed presents a displacement field for a toroidal shell semi-membrane deformation model. The solution for the pipe cross section distortion depends upon boundary conditions of prescribed displacements or forces. One experimental in-plane loading piping system was developed to validate the linear finite element procedures presented. The new finite element gives excellent results compared with measured stresses and numerical shell model. An easier solution may be obtained with a new uni-dimensional finite element formulation with all shell membrane capabilities, when compared to other possible finite modelling techniques.

References

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