THERMAL STABILITY OF CURVED SUPERELEVATED RAILWAYS

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Abstract

Railways are a fundamental infrastructure transportation system for the efficient movement of people and goods, especially over long distances. However, these structures are exposed to increasingly extreme environmental conditions, driven by global climate change. Significant temperature variations throughout the day and across climatic seasons generate thermal stresses on the rails, which can cause structural failures such as thermal buckling, especially in sections with greater geometric complexity. This study aims to analyze the thermal instability of continuously welded rails (CWR) on curves with superelevation, considering the structural interactions involved. Modeling was performed using the finite element method in ANSYS, based on railway profiles. The effects of curvature, cant, lateral and longitudinal ballast resistance, fastener stiffness, and initial rail imperfections were evaluated. The results demonstrate that the loss of ballast resistance and the presence of geometric imperfections significantly reduce the critical buckling temperature. In addition, it was observed that curved geometry with superelevation directly influences the track's sensitivity to temperature increases. The research contributes to the understanding of thermal instability mechanisms in modern railways and reinforces the importance of preventive maintenance, especially in curved alignments, to ensure operational safety in the face of increasing climate variations.

Keywords: Railways. Thermal Buckling. Superelevation. Finite Element Analysis. Stability Analysis. Continuously Welded Rails.

Resumo

A ferrovia é sistema de infraestrutura de transporte fundamental para o deslocamento eficiente de pessoas e mercadorias, especialmente em longas distâncias. No entanto, essas estruturas estão expostas a condições ambientais cada vez mais extremas, impulsionadas pelas mudanças climáticas globais. Variações significativas de temperatura ao longo do dia e das estações climáticas do ano geram tensões térmicas nos trilhos, podendo provocar falhas estruturais como a flambagem térmica, principalmente em trechos com maior complexidade geométrica. Este trabalho tem como objetivo analisar a instabilidade térmica de trilhos soldados continuamente (CWR) em curvas com superelevação, considerando as interações estruturais envolvidas. A modelação foi realizada utilizando o método dos elementos finitos no ANSYS, com base em perfis ferroviários. Foram avaliados os efeitos da curvatura, da superelevação, da resistência lateral e longitudinal do lastro, da rigidez dos fixadores e das imperfeições iniciais dos trilhos. Os resultados demonstram que a perda de resistência do lastro e a presença de imperfeições geométricas reduzem significativamente a temperatura crítica de flambagem. Além disso, observou-se que a geometria em curva com superelevação influencia diretamente a sensibilidade da via ao aumento de temperatura. A pesquisa contribui para a compreensão dos mecanismos de instabilidade térmica em ferrovias modernas e reforça a importância da manutenção preventiva, sobretudo em zonas curvas, para garantir a segurança operacional frente às crescentes variações climáticas.

Palavras-chave: Ferrovias. Instabilidade térmica. Superelevação. Análise de elementos finitos. Análise de estabilidade. Trilhos soldados continuamente.

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Chapter 1

Introduction

1.1 Framework

Railways as guiding and supporting structures were first introduced in the sixteenth century. During that period, wooden tracks were used in England mines to minimize friction and facilitate the movement of mining carts. To keep the vehicles aligned, the tracks featured a raised edge along the running surface [1]. With the advancements brought by the Industrial Revolution, improvements were made by incorporating cast iron or wrought iron plates, which helped reduce wear on the wooden foundations, with the improvements made, it was also possible to verify the increase in speed limits on the railways. [2]. Over time, this innovation led to the development of iron edge rails, allowing for the first use of flanged wheels [3].

The use of Continuous Welded Rail (CWR) has become an increasingly adopted solution on railroads, offering several advantages over segmented rails with expansion joint. This system provides less need for repairs, greater durability and reduced noise impact, resulting in a more comfortable environment for passengers [3]. The replacement of joints with a continuous rail joining process was a major breakthrough, enabling significant improvements in railroad operations. However, the resistance of continuous rails to temperature variations requires precise control, as extreme weather changes can cause internal stresses in the material, affecting its structural stability. This system also contributes to energy efficiency, due to its greater stability and lower friction [4].

Railway are fundamental for adapting rail lines to the terrain, allowing the route to adjust to changes in direction and enabling connectivity between different points. They are essential for operating the rail system in mountainous or urban terrain, because without them, the rail network would be limited to straight paths, making it difficult to build an efficient and comprehensive infrastructure. In order to ensure passenger safety and comfort, as well as the durability of the tracks, superelevation (or cant) must be applied precisely to balance lateral forces such as centrifugal and wind effect, preventing dangerous displacements and ensuring stable operation [1].

Railway track buckling occurs due to thermal expansion of the rails. This phenomenon is amplified in CWR, which do not have joints to accommodate thermal expansion. The increase in temperature generates compression axial forces in the rails, and when these efforts exceed a critical limit corresponding to a critical temperature, lateral buckling of the track occurs. This phenomenon is influenced by the rail's neutral temperature, which defines the point at which it is free of thermal stresses. As the temperature rises beyond this point, the stability of the track can be compromised, leading to lateral displacement of the rails when that critical temperature is exceeded. Studies show that dynamic train loads, such as lateral and vertical forces during braking, reduce the critical buckling temperature, increasing the risk of instability [5].

Preventive measures include defining safe temperature limits for railway operation and applying numerical models, such as the finite element method, to predict and mitigate buckling. In countries with extreme climates, such as Australia, speed restrictions are adopted when temperatures exceed certain values, minimizing risks. Although continuously welded rails have advantages such as lower maintenance costs and greater passenger comfort, their susceptibility to thermal failures requires effective strategies for monitoring and controlling track stability [6].

This paper studies the influence of thermal expansion on the stability of curved railroads with superelevation, applying Finite element analysis (FEA) to determine the critical buckling temperature. The study considers various parameters, including the rail profile, the stiffness of the fastening system, the properties of the sleepers, the magnitude of the superelevation and the radius of the curve. The model is validated by comparison with experimental data and analytical studies, making it possible to predict track instability.

In addition, the analysis evaluates different track configurations, including rail profile variations, while examining the impact of initial misalignments on thermal buckling behavior. In this way, the study contributes to a better understanding of the factors that influence the stability of curved railroads subjected to temperature variations.

1.2 Research Objectives

This work aims to analyze the influence of different parameters on the critical buckling temperatures of rails on a curved railroad, considering different rail profiles. The analysis is carried out by means of Finite Element Analysis (FEA), using numerical simulations to investigate the individual impact of factors such as fastening stiffness, lateral and longitudinal ballast strength, superelevation and initial track misalignment. The data obtained is processed to assess the specific influence of each rail profile on the thermal stability of the track, helping to define strategies to mitigate buckling risks.

1.3 Structure of the Document

The work is structured into five chapters: Introduction, Literature Review, Methodology, Results and Conclusion.

The first chapter, Introduction, presents a general overview of railroads, and their historical evolution, with an emphasis on thermal instability and the phenomenon of rail buckling in curved railroads with superelevation.

The second chapter, Literature Review, defines the concept of a railroad and its structural components, explaining the technical aspects of the study of thermal buckling. This chapter discusses theoretical models, the effects of temperature on rails, and the impact of superelevation and rail profile on track stability.

The third chapter, Methodology, describes the development of the numerical model using Finite Element Analysis, including the boundary conditions applied, the parameters analyzed (rail profile, ballast strength, fastening stiffness, initial misalignments, among others) and the model validation process based on experimental and analytical studies.

Chapters four and five, Results and Conclusion, respectively, present the results of the parametric study and the analysis of the data that was found. The thermal effects on the stability of the curved railway track and the variations in the critical buckling temperatures according to the different rail profiles are discussed.

Chapter 2

Literature Review

2.1 History of Railways

Railroads underwent changes to improve them over time, with the introduction of cast iron rails in the 18th century, which further reduced resistance. At the beginning of the 19th century, the invention of the steam locomotive, with the construction of the first one in 1804 by Englishman Trevithick [7], this first locomotive was very robust, as can be seen in figure 2.1, influenced the creation of the first commercial passenger railroads a while later, such as the line between Stockton and Darlington, inaugurated in 1825. The use of railroads grew rapidly around the world, stimulating economic and political growth, especially in countries such as the United States, Canada and China.

Railroads still play a fundamental role in passenger and freight transport, and the improvements that have taken place over the years have led to the modernization of infrastructure and the adoption of advanced technologies, such as high-speed trains and automated control systems. Railroads are essential in large countries and areas of high population density, where they offer efficient and sustainable solutions for transporting large volumes of cargo and passengers. In addition, concerns about sustainability and reducing carbon emissions have driven investment in railroads as an environmentally friendly alternative to road and air transport.

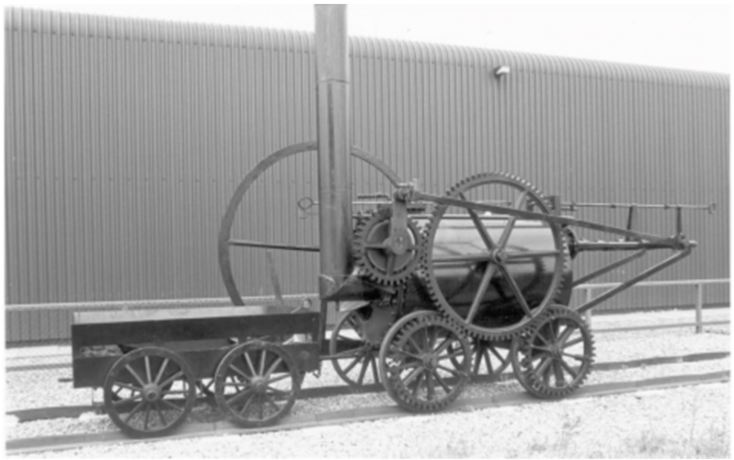

Figure 2.1: First train [7]

2.2 Rails Components

The railway infrastructure is made up of two main subsystems: the superstructure and the substructure. The superstructure includes the rails, sleepers, fasteners and the track bed (ballast and sub-ballast), as can be seen in figure 2.2. This area of the railroad is responsible for supporting and distributing the train loads, ensuring better stability and safety for the system. In addition, it is the outermost section of the system, so it is more exposed to wear and therefore requires more frequent maintenance to preserve its functionality. [8]

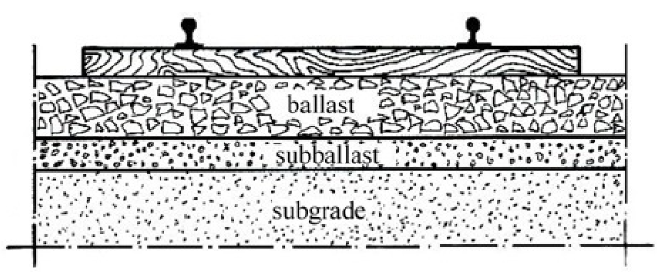

Figure 2.2: Figure of the railway structure adapted from [8]

The rails play a fundamental role in guiding and supporting the train wheels, while the sleepers maintain the spacing between the rails and help distribute the loads. The fasteners connect the rails to the sleepers, as shown in the figure 2.3, ensuring the stability of the track and preventing longitudinal and transverse displacement of the rails due to the forces applied by the trains. The ballast, usually made up of gravel, performs several essential functions, such as absorbing impacts, reducing vibrations and providing an efficient rainwater drainage system. In some cases, a layer of sub-ballast is added to prevent the ballast stones from penetrating the ground and to improve the distribution of loads on the infrastructure [3].

Figure 2.3: Fastener connection to sleeper

The substructure is made up of the subgrade and the foundation layer. The subgrade corresponds to the natural soil or embankment on which the railroad is built, while the foundation layer can be added to reinforce the soil in places where it does not have adequate resistance. The main function of this part of the railroad is to receive the loads distributed by the superstructure and prevent differential settlements that could compromise the stability of the track.

2.2.1 Rails

Railroad tracks play an essential role in railways, guiding and supporting trains safely and efficiently. Since the emergence of railroads, rail profiles have undergone several evolutions, with grooved rail, figure 2.4, being the only rail remaining from the earliest rail models, used where the track is at the same level as the paved surface [8]. Other historical models include the bullhead rail, which despite the expectation of reuse by reversing the position of wear, was eventually replaced by more robust models. The Flat

Bottom profile, also known as the Vignoles rail, figure 2.5, is the current standard for mainline railways. Its head is designed to better distribute loads and reduce localized wear, minimizing problems such as fatigue and shelling in the region of the wheel-rail contact. In addition, its cross-sectional characteristics are the weight w per unit length and the second order moment of area I.

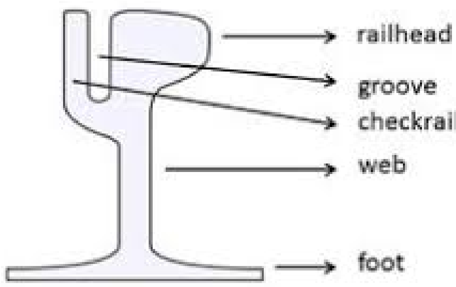

Figure 2.4: Grooved rail

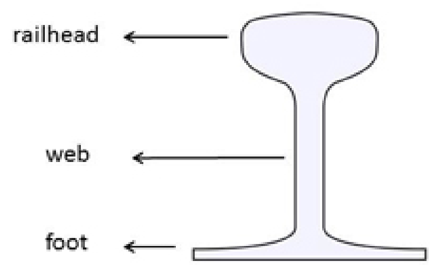

Figure 2.5: Vignoles rail

The material used to manufacture the rails is high-strength steel, produced by processes such as basic oxygen (BOF). The steel in the rails has a balance between tensile strength and toughness, preventing brittle failure. For this reason, different grades of steel are used: R200, which has lower resistance, R260, the standard in Europe, and R260 Mn, which has higher toughness due to its manganese content, as can be seen in Table 2.1 [9], and in table 2.2 contains the chemical information for these materials, the “R” used as a prefix in these steels refers to “Rail”. Quality control in manufacturing is rigorous, including processes such as vacuum degassing to avoid structural failures. [1]

Steel Sample Grade	**Min. Tensile Strength (MPa)**	**Elongation (%)**	**Hardness (HB)**
R200	680	14	200-240
R220	770	12	220-260
R260	880	10	260-300
R260Mn	880	10	260-300

Table 2.1: Mechanical properties according to EN 13674:1 [9].

Grade	C (%)	Si (%)	Mn (%)	P_{max} (%)	S_{max} (%)	Cr (%)
R200	0.38-0.62	0.13-0.60	0.65-1.25	0.040	0.040	≤ 0.15
R220	0.48-0.62	0.18-0.62	0.95-1.30	0.030	0.030	≤ 0.15
R260	0.60-0.82	0.13-0.60	0.65-1.25	0.030	0.030	≤ 0.15
R260Mn	0.53-0.77	0.13-0.62	1.25-1.75	0.030	0.030	≤ 0.15

Table 2.2: Chemical properties according to EN 13674:1 [9].

The renewal and maintenance of rails are complex processes that play a major role in the design and choice of profiles, as they not only influence the performance of the rails but also determine economic costs. The cost of replacing rails involves not only the cost of the material, but also removal, installation and inspection work. Railway managers establish strict acceptance procedures to ensure that the rails delivered meet the required specifications. As rails are subject to wear and structural failure over time, implementing inspection and preventive maintenance programs is essential to avoid unexpected failures and ensure operational safety.

Another relevant factor in the durability of rails is the impact of dynamic and environmental loads. Thermal variation can affect the expansion and contraction of steel, while factors such as humidity and contact with corrosive substances can compromise the integrity of the track. To minimize these problems, techniques such as Continuous Welded Rail (CWR), the application of protective coatings and the monitoring of internal stresses in the steel are adopted.

The development of new metal alloys and heat treatments has been a strategy to extend the useful life of rails and reduce the need for frequent replacement. The use of technologies such as sensors embedded in the rails makes it possible to monitor their structural condition in real time, enabling more precise and efficient interventions.

In this study, four types of profiles were selected, from the S30 profile, which has a lower second order of area and area, to the 136RE, which was the largest of the selected profiles. The UIC50 and 132RE profiles were also selected based on their characteristics, which would be best in their comparisons. Below is table 2.3 with geometrical technical information on some of these profiles and others that are also used around the world.

Country	**Rail section**	**A [m²]**	**I_y [m⁴]**	**I_z [m⁴]**
France	U33	5906 10^{-6}	1588 10^{-8}	338 10^{-8}
	U35-U50	6450 10^{-6}	2019 10^{-8}	405 10^{-8}
Germany	S49	6296 10^{-6}	2819 10^{-8}	320 10^{-8}
	S30	3839 10^{-6}	608 10^{-8}	150 10^{-8}
Great Britain	BS110A	6950 10^{-4}	2345 10^{-8}	417 10^{-8}
	BS113A	7150 10^{-6}	2435 10^{-8}	418 10^{-8}
Netherlands	NP46	5300 10^{-6}	1605 10^{-8}	308 10^{-8}
Switzerland	SBB1	5880 10^{-6}	1631 10^{-8}	298 10^{-8}
UIC	UIC50	6428 10^{-6}	1934 10^{-8}	315 10^{-8}
	UIC60	7693 10^{-6}	3615 10^{-8}	513 10^{-8}
USA	AREA132	8355 10^{-6}	3671 10^{-8}	607 10^{-8}
	AREA136	8612 10^{-6}	3950 10^{-8}	612 10^{-8}

Table 2.3: Rail Section Properties by Country, adapted from [4]

2.2.2 Sleepers

Sleepers are essential components of the railway track, located between the rails and the ballast. Their main function is to properly distribute the loads applied by the rails to the ballast, ensuring the structural stability of the track. In addition, sleepers maintain constant spacing between the rails, according to the track gauge, and allow the rails to be fixed at an appropriate inclination, varying between 1/20 and 1/40. In the case of electrified lines, sleepers also play a role in providing electrical insulation between the rails. [8]

Historically, the first rails were laid directly on blocks in the ground, figure 2.6. However, the need for better load distribution led to the development of sleepers and ballast. Wood was the first material to be widely used due to its availability and ease of handling. However, its natural deterioration and scarcity led to the adoption of other materials, such as steel and concrete. Steel sleepers appeared around 1880 and were widely used for a long time, but since 1950, advances in concrete technology have led to the predominance of concrete sleepers, which can be of the pre-stressed monoblock type or bi-block with reinforced frames.

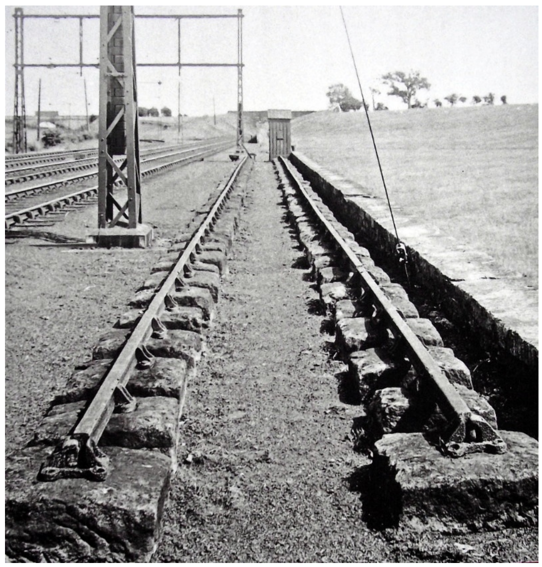

Figure 2.6: Rails supported by blocks

Wooden sleepers are still used in many situations due to their ability to distribute loads more efficiently than other types. This makes them recommended for roads built on subgrade of medium or low quality, where concrete sleepers would require a thicker ballast layer. Despite their high cost and lower durability compared to concrete, wood sleepers continue to be widely used in North America and elsewhere; in Europe they are limited to places where concrete sleepers are not used [8]. Wooden sleepers are subject to chemical and physical degradation due to exposure to variations in humidity and temperature, as well as fungi and insects that deteriorate wooden sleepers.

Concrete sleepers, when introduced, presented challenges such as fragile fractures under dynamic loads and low fatigue resistance. Currently, there are two main types of concrete sleepers: bi-blocks, made up of two trapezoidal concrete sections connected by a metal bar, and monoblocks, which can be pre- or post-tensioned, in figures 2.7 and 2.8 these two forms are represented. The bi-block model was developed in France and is widely used in countries such as Portugal, Brazil, Spain and Tunisia.

Figure 2.7: Monoblock

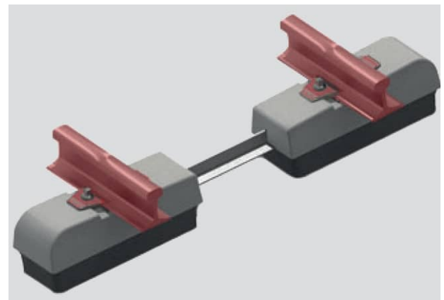

Figure 2.8: Biblock

The pre-tensioned monobloc model, developed in the United Kingdom, is used in countries such as Canada, Japan and the United States, while the post-tensioned model, which originated in Germany, is used in countries such as Italy, Mexico and India [8].

The choice of sleeper type depends on several factors, including the characteristics of the track and the environment in which it will be used. In the case of curved tracks, the use of concrete sleepers is controversial, as their rigidity can influence the track's behavior. For example, in South Africa, concrete sleepers are not used in curves with a radius of less than 300 meters, while in Canada, where there are large temperature variations, these sleepers are used even in tighter curves, with radius of less than 200 meters [8].

Steel sleepers are also used in some specific situations, especially where durability and corrosion resistance are not critical factors. Despite being an industrial product with a simple construction and good transverse stability, steel sleepers are rarely used, especially in Europe, due to their cost and the need for efficient electrical insulation on tracks equipped with train detection circuits. Their lightness makes them easy to handle, but their inverted rail shape can make them difficult to lay in ballast.

The development of sleepers has followed the evolution of railroads, always seeking a balance between strength, durability and cost. While wooden sleepers are still valued for their flexibility and adaptability, concrete sleepers stand out for their longevity and low maintenance. The choice of the most suitable sleeper will depend on the specific requirements of the railroad, the characteristics of the soil and the environmental conditions of each region.

2.2.3 Track Gauges

The gauge is the distance measured between the inner faces of the rail heads, figure 2.9, and is one of the fundamental parameters in the design and maintenance of the railroad. This measurement defines the compatibility between the rail system and the rolling stock, as well as directly influencing train stability, ride comfort, load capacity, and operational safety [10].

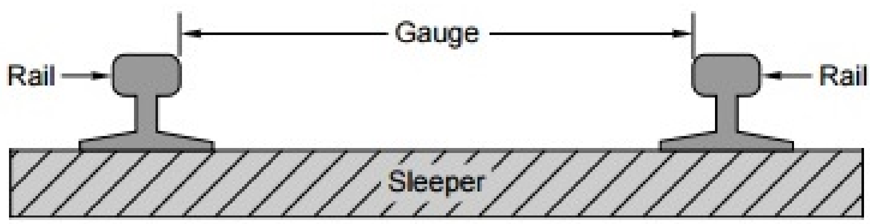

Figure 2.9: Representation of a gauge

Gauges can vary depending on the country or application, the most common being narrow gauge (e.g., 1,000 mm), standard gauge (1,435 mm), and broad gauge (e.g., 1,600 mm). Standardization of gauge is essential to ensure vehicle interoperability and reduce logistics costs in extensive networks. Table 2.4 shows some European gauge standards.

Country	**Gauge**
FR	1000 mm and 1435 mm
PT	1000 mm and 1668 mm
UK	1435 mm and 1600 mm

Table 2.4: Gauge measurements by country [11]

From a structural point of view, gauge significantly affects track behavior under load, on curves, and during temperature variations. On sections with small radius curves and high superelevation, a narrower gauge may present a greater risk of lateral instability, especially in systems with continuously welded rails. In addition, the lateral resistance provided by the ballast and the fastening system must be compatible with the type of gauge adopted in order to minimize the risk of excessive displacement or thermal buckling. Thus, the choice of gauge should consider not only historical and economic aspects, but also technical criteria related to the safety and performance of the railway infrastructure.

2.2.4 Ballast

Railway ballast is one of the fundamental elements of a railway track. It is responsible for guaranteeing the stability of the track, properly distributing the stresses caused by rail traffic and allowing maintenance work to be carried out relatively easily. At the very beginning of the railroads, the importance of ballast was not visualized, making it an incorrectly used component of the railroad. The first railway engineers used unsuitable materials under the sleepers, such as ash, clay and chalk, due to their local availability and low cost [3]. Over time, practical experience revealed the need to use stronger, well-graded materials with suitable properties to support the loads and maintain the track geometry.

The structural function of the ballast involves reducing vibrations, distributing the stresses transmitted by the sleepers and resisting the longitudinal and transverse displacement of the rails. It must also ensure good drainage and make geometric corrections to the track possible through processes such as tamping. However, these functions can conflict: a very compacted ballast offers greater structural resistance, but reduces drainage capacity. Therefore, the selection and sizing of ballast must strike a balance between stability, durability and ease of maintenance [8].

As far as the materials used are concerned, preference should be given to hard rock crushed stone, with angular particles between 28 mm and 50 m in size. The presence of very small particles (“fines”) can compromise drainage and cause the foundation to soften, leading to track subsidence problems [3]. For this reason, the granulometric composition of the ballast must be well distributed, free of dust and composed of stones with sharp edges. The elastic properties and acoustic damping capacity of ballast make it a consolidated and economically viable technological choice, despite growing competition with slab track systems [1].

The mechanical behavior of ballast is typically elastoplastic. The first load cycles cause significant plastic deformations, resulting from the rearrangement of the particles until a more stable configuration is reached. After this initial stage, the increase in plastic deformations per cycle tends to decrease. The elastic modulus of the ballast also evolves in the first thousand cycles, doubling compared to the initial value. This characteristic explains the need for a period of "softening" of the new track before its mechanical response can be considered stabilized [8].

In addition to the ballast itself, there is also the sub-ballast layer, which is responsible for protecting the surface of the subgrade, improving drainage and additionally distributing stresses, as shown in figure 2.10. In some cases, this layer is replaced by a thicker formation, thus improving the structure of the railroad. The mechanical behavior of sub-ballast is similar to that of ballast, but it tends to be more elastic, with less variation in the modulus of elasticity over load cycles.

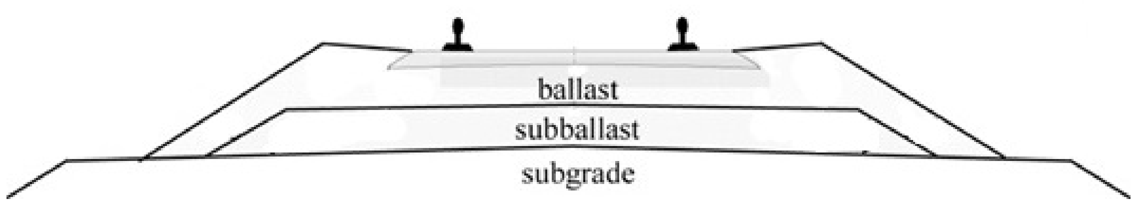

Figure 2.10: Ballast structure adapted from [8]

The durability of ballast is limited by fatigue and mechanical and environmental aggressions. Ballast renewal on high-speed lines usually takes place every 15 to 20 years, while concrete sleepers and rails have a lifespan of around 50 years [1]. Materials such as limestone have low resistance to abrasion in the presence of humidity, with crushed granite being a superior, albeit more expensive, option [3].

There are also cases where ballast can be reused after maintenance, with the ballast being removed to be washed and reused as sub-ballast or foundation layer in secondary lines, as long as it maintains adequate mechanical properties [8]. This practice, as well as being technical, involves economic and environmental considerations. The cost of new ballast and the high cost of disposal make reuse an attractive alternative.

Despite the advantages of traditional ballast, such as low initial cost, ease of maintenance, good drainage and elasticity, modern operational requirements are causing some limitations of the ballast system. This has driven the development of slab tracks, especially in high-speed contexts and high traffic volumes. Even so, the ballasted structure remains a robust and widely used technology.

Finally, rail ballast remains an effective technical solution, provided it is well designed, executed and maintained. The knowledge accumulated over decades makes it possible to optimize its composition, sizing and mechanical behaviour, prolonging its useful life and ensuring safety, comfort and economy in rail operations.

2.2.5 Fasteners

The fastening system between the rail and the sleeper is an essential component of the railway structure, responsible for maintaining the stability of the track and ensuring the safe transfer of loads between the rail and the track bed. Although they may look like simple elements, like the system shown in figure 2.11, fastening systems perform several functions that are simultaneous and critical to the structural performance and safety of the railroad [12].

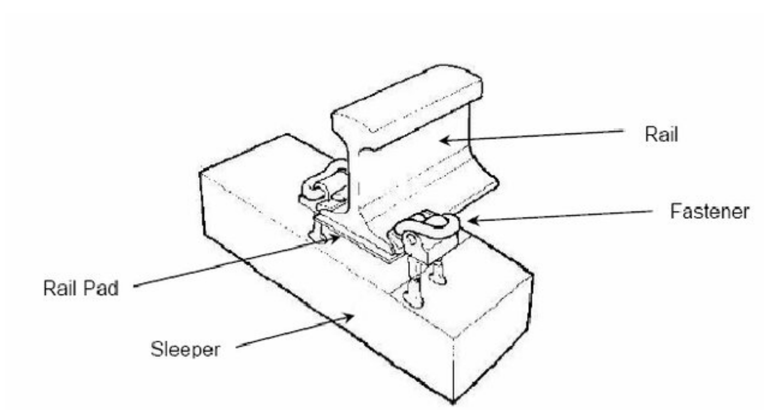

Figure 2.11: Fastening system [13]

The fastening system must elastically absorb the forces acting on the rail and transfer them to the sleeper, keeping the gauge and inclination of the rail within the established limits. In addition, the fastening system must offer sufficient longitudinal strength to prevent unwanted movement of the rail, especially where there are sections of continuous welded rail (CWR) [13]. The system must also provide electrical insulation, which is essential for signaling and control systems [1].

The functional properties that are expected of the fastening system are resistance to corrosion, ease of installation and maintenance, mechanical resilience, controlled deflection, resistance to vandalism and compatible cost. The fixings must maintain their clamping force over time and withstand at least 3 million load cycles at maximum axle stress, without significant degradation of vertical rigidity or longitudinal restraint [8].

Historically, the first fastening systems used wooden wedges inserted into cast iron chairs on which the rail rested. With the introduction of apartment bottom rail in the late 1940s, the need arose for more modern fastenings, capable of accommodating thermal and dynamic variations in rail traffic. It was observed that very rigid fastenings loosened over time, while fastenings with a certain degree of elasticity were more efficient at resisting rail movement and buckling, prolonging the durability of the fastening.

Today, fastening systems can be divided into two main groups: rigid and elastic.

- Rigid fastenings: Mainly used with wooden or steel sleepers, connect the rail directly to the sleeper using screws or nails, figure 2.12. However, they have disadvantages such as plastic deformation of the sleeper under repeated loads, which results in progressive loosening and can compromise track safety [8].

- Elastic fastenings: These fastenings, figure 2.13, have become standard on concrete sleepers and are optional on wooden sleepers. These fixings generally include elements such as threaded bolts, spring steel clips [14], baseplates, electrical insulators and rail pads. The elastic clips exert a clamping force (toe load) sufficient to guarantee the rail's slip resistance, which is greater than the longitudinal shear resistance between sleeper and ballast [8]. The Pandrol, Vossloh and Nabla systems are widely used examples, with variations in the way they are mounted.

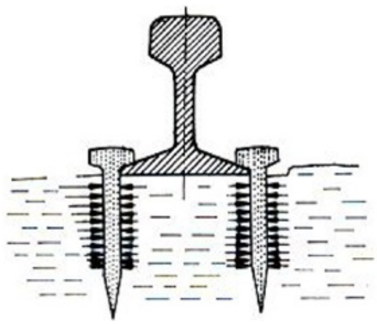

Figure 2.12: Rigid Fastening [8]

Figure 2.13: Elastic Fastening [14]

The longitudinal resistance between rail and sleeper must be at least 15 kN - significantly higher than the shear resistance of the sleeper on the ballasted bed (approximately 5 kN per half sleeper). This ensures that, in the event of high axial loads, the rail remains stable in the fastening and movement only occurs in the sleeper in relation to the ballast [1]. Maintaining the clamping force on the clips is fundamental to the system's good performance over time, and various clamping designs have been tested since the 1940s, with varying levels of success [3].

In addition to mechanical strength, elastomeric rail pads play an important role in vibration damping and electrical insulation, as well as protecting the sleeper from excessive wear. The dynamic stiffness of these pads, especially in concrete sleepers, must be limited (e.g. up to 600 MN/m) to avoid excessive transmission of vertical stresses to the sleeper and ballast [8].

In short, the rail-tie fastening system has evolved from rudimentary solutions to highly sophisticated configurations, with carefully designed mechanical performance and durability. The choice of fastening type depends on the type of sleeper, the local conditions, the need for electrical insulation, the expected loads and the desired level of maintenance. Although elastic systems represent the majority of modern solutions, each project must balance performance, cost and durability according to the specific requirements of the track.

2.3 Buckling

Buckling is a structural phenomenon characterized by a sudden instability that occurs when a structural component subjected to compression exceeds load buckling resistance. Instead of continuing to compress linearly, the structure undergoes significant lateral or torsional deflection, even under a small increase in load, figure 2.14 demonstrates how this buckling can occur. This behavior is not caused by the failure of the material itself, but by a loss of stability in the geometric shape of the structure. Buckling is influenced by the boundary conditions and by initial imperfections, such as geometric deviations or eccentricities in the application of the load [15]. Buckling can manifest itself in two main ways: explosive buckling and progressive buckling.

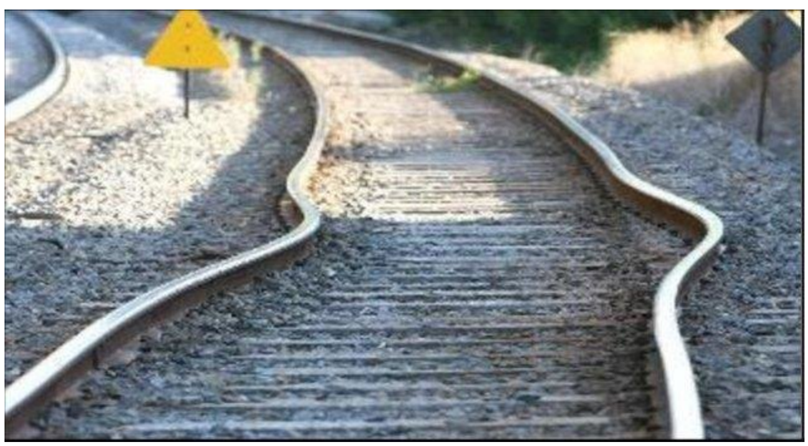

Figure 2.14: Railway buckling [16]

In explosive buckling, the rail deforms abruptly after reaching this maximum temperature, resulting in immediate structural instability. In this scenario, the minimum temperature represents the lower limit below which buckling does not occur, serving as a thermal safety reference, as any temperature below this value would not induce track collapse, while in progressive buckling, there is no sudden drop in axial force after buckling. This behavior makes it impossible to define an exact minimum critical temperature, making it difficult to define the precise thermal safety conditions for this mode of collapse, these two types of buckling were represented in the figure 2.15

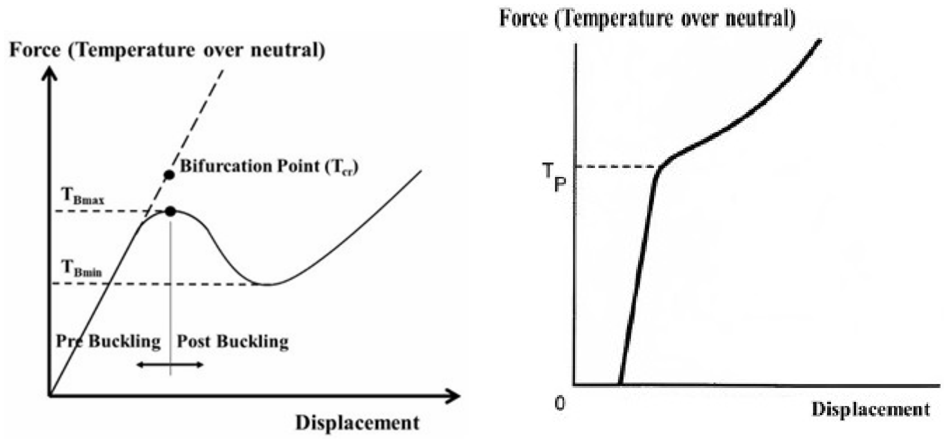

Figure 2.15: Explosive buckling and Progressive buckling

In railroads, thermal buckling occurs especially in continuous welded rails [16], when they are subjected to temperature variations that cause compressive stresses along the longitudinal axis. As this type of rail has no gap joints, the increase in temperature generates confined thermal expansion, which can lead to lateral instability of the rail if the resistance to movement provided by the ballast and fasteners is not sufficient. Rail buckling is a structural instability that is sensitive to the lateral stiffness of the track, longitudinal confinement, the presence of imperfections and the geometry of the curve. As such, it is a global track stability problem that can occur even without previous visible damage, and therefore requires special attention in the design and maintenance of railways exposed to significant thermal variations.

Over the last two decades, a large part of the railroads with jointed track in the Netherlands and other western countries have been replaced by continuously welded track. This system uses factory-welded rails, with sections of up to 360 meters, which are then joined together on site, forming long continuous stretches without expansion joints. This configuration, while advantageous for reducing maintenance and providing greater operational comfort, prevents the rails from naturally expanding thermally. For this reason, at critical locations such as bridges and switches and crossings, expansion joints are still often installed to mitigate the effects of excessive stresses and deformations. However, with accurate prediction of these behaviors, it would be possible to avoid the use of these joints, which are expensive and uncomfortable for railway operation.

In CWR rails, thermal stresses are zero when the rail temperature is around the so-called neutral temperature, usually around 25 °C. Below this value, tensile stresses arise which can, at very low temperatures (such as -20 °C), lead to brittle fracture of the rails, resulting in the formation of gaps that can cause derailments. On the other hand, when the temperature of the rail exceeds 70 °C, strong compressive stresses develop, which are even more dangerous as they can lead to lateral buckling of the track, with displacements of up to 1 meter over a length of around 20 meters, usually in a sinusoidal shape. In hot summers, this type of instability can occur on hundreds of stretches around the world.

In order to understand and prevent such phenomena, the European Railway Research Institute (ERRI) began a four-year research program in 1993 (committee D202), the aim of which was to provide guidelines for the stability of continuously welded rails. Delft University of Technology was commissioned to develop simulation software, CWERRI, based on the discrete element program TILLY. CWERRI makes it possible to model and calculate the stability of CWR in three directions, both on flat track and over bridges, and is available in a user-friendly graphical environment for use by railway companies. In addition to supporting the development of safety concepts, the results also support the revision of the European standard Leaflet 720R, which governs the installation and maintenance of CWR tracks [4].

The rail buckling model presented in the study uses CWERRI to evaluate the behavior of the track under different conditions. Experimental tests and sensitivity analyses are carried out to understand the impact of various parameters on curve buckling, such as curve radius, ballast lateral resistance (peak and limit values), corresponding deformation, ballast longitudinal resistance, fastener torsional stiffness and imperfection geometry, amplitude and wavelength. Based on these analyses, recommendations for the design and maintenance of CWR tracks with greater thermal and structural stability are discussed.

2.3.1 Experimental Investigations

Thermal buckling tests on continuously welded rails were carried out on real stretches of railroad, with the aim of understanding and predicting the structural behavior of the track in the face of rising temperatures. In one of the main experimental campaigns, 200-meter sections were prepared with sinusoidal imperfections introduced in the center, creating an initial condition favorable to buckling. The rails were de-stressed to define the neutral temperature, approximately 22 °C, and heated with electric current supplied by locomotives. During the tests, lateral displacements and temperature variations were monitored, allowing the complete characterization of the thermal buckling phenomenon under real operating conditions [17].

Analysis of the data obtained experimentally showed that buckling occurs in a localized manner, and that the response of the track is strongly influenced by parameters such as the lateral resistance of the track, the stiffness at the ends and the presence of previous longitudinal displacements. The buckling manifested itself with greater intensity in the straight section, where the critical temperature measured was 59 °C above neutral, compared to 44 °C in the curve. The axial force at the moment of buckling exceeded 130 tons per rail. These values were compatible with the expected behavior for well-anchored tracks, and reinforced the importance of considering the effects of test length, ballast strength and imperfection mode introduced [17].

After the track was stabilized, detailed inspections were carried out along the section to identify possible geometric deviations resulting from the settling process and operating conditions. During these measurements, longitudinal imperfections were detected along the railway line, characterized by smooth but continuous undulations distributed along a section approximately 11 meters long. These irregularities manifest themselves in the form of variations in the vertical profile of the rail, with undulating behavior, which can influence both passenger comfort and the dynamic response of the rolling stock. The geometry of these imperfections suggests the existence of symmetry around the center point of the deformation, indicating a relatively uniform distribution of stresses or systematic settlement of the track in that specific segment. Table 2.5 shows the values of these imperfections.

Distance from the center (m)	**Deflection (mm)**
0.0	38
0.5	36
1.0	32
1.5	28
2.0	24
2.5	20
3.0	15
3.5	10
4.0	6
4.5	3
5.0	0

Table 2.5: Misalignment from previous tests, adapted from [17]

The good agreement between the experimental and theoretical results validates the application of analytical models to predict both the onset and post-buckling behavior of the track. The integration of theory and practice has shown that it is possible to safely dimension the installation temperature of the rails, establish preventive criteria against buckling and guide maintenance interventions. Thus, these tests provided not only essential quantitative data, but also a robust methodological framework for the study and control of lateral stability in modern railroads [18][17].

2.3.2 Numerical Models

The growing demand for rail transportation in recent decades has required greater capacity and performance from rail infrastructure. Continuously welded rails have become standard on many modern rail networks due to their operational advantages, such as noise reduction, greater passenger comfort, reduced component wear and less need for maintenance. However, one of the main challenges associated with the use of CWR is their susceptibility to lateral thermal buckling, especially in sections subjected to high temperatures, with curved geometry and the presence of superelevation [5].

The thermal response of the permanent way can be characterized based on three critical temperatures: the neutral temperature (T), corresponding to the condition free of thermal stresses; the buckling temperature (T_{max}), at which structural instability occurs; and the safe temperature (T_{min}), which represents the new equilibrium after buckling has occurred. Accurate prediction of these temperatures depends heavily on the numerical model adopted and the simplifying assumptions involved. Classical analytical models, such as the one proposed by Kerr, segment the track into zones, a central region subject to large lateral displacements, where an imperfection is applied, and adjacent areas with purely axial deformations, enabling the theoretical description of buckling based on structural mechanics [18].

With the advance of computer technology, two-dimensional numerical models have been implemented with different levels of complexity. These models take into account periodic and non-periodic imperfections and use beam elements to simulate the behavior of the rails. Elements were introduced to simulate the presence of sleepers, fasteners and ballast. Despite the advances, these 2D models proved to be limited for applications on curved sections with cant, since they do not adequately capture the three-dimensional interactions that occur on the track [5].

The three-dimensional modeling of the problem was improved, where a complete model was proposed consisting of thin-walled beam elements for the rails, beams on elastic foundations for the sleepers, elastic springs to represent the fasteners and lateral and longitudinal non-linear resistances for the ballast, figure 2.16. A 200-meter section was modeled with a 132RE profile and 1435 mm gauge, under different conditions of ties at the ends of the rails. The results obtained showed that three-dimensional models offer a more accurate representation of thermal buckling in real operating conditions [19].

Continuing this type of study, models were developed using BEAM elements to represent rails and sleepers, and COMBIN39 elements to simulate the interaction with the ballast, incorporating non-linear elasto-plastic behavior. Buckling was induced with central imperfections of up to 20 mm in amplitude. The load arc method, modified RIKS,

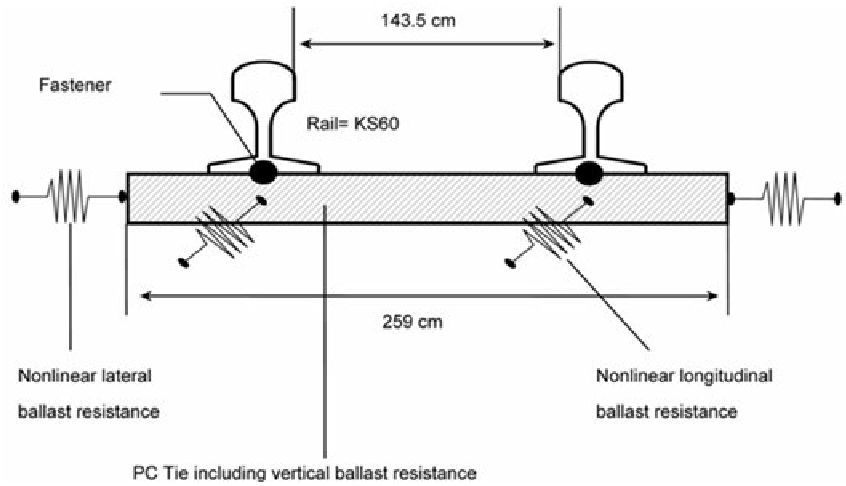

Figure 2.16: Railway model representation [19]

was used to capture structural collapse, dealing with the snap-through effect accurately. The results demonstrated the sensitivity of buckling to the lateral stiffness of the ballast and the quality of the rail-tie connections, especially under the effect of superelevation [20].

In a more recent study, advances were made in modeling by integrating dynamic effects using a multi-body model for the train coupled with a three-dimensional finite element model of the track. The study analyzed different scenarios: constant movement, braking with perfect wheels and with defects, applying quasi-static loads progressively via the Modified RIKS method. The results revealed that dynamic loads significantly alter both the critical buckling temperature and the safe temperature, an effect that is even more pronounced on sections with complex geometry, such as curves with cant, where the stress distribution becomes asymmetrical [5].

The lateral stability of the track in curves with superelevation requires special attention due to the intensification of centrifugal forces on the outer rails, especially at high speeds or in scenarios with superelevation deficiency, as shown in the figure 2.17. The inclination of the track plane alters the distribution of internal forces in the rails and directly affects their behavior in the face of thermal buckling. When not loaded, for example, when at rest under high temperatures, sleepers become more prone to lateral displacement [5].

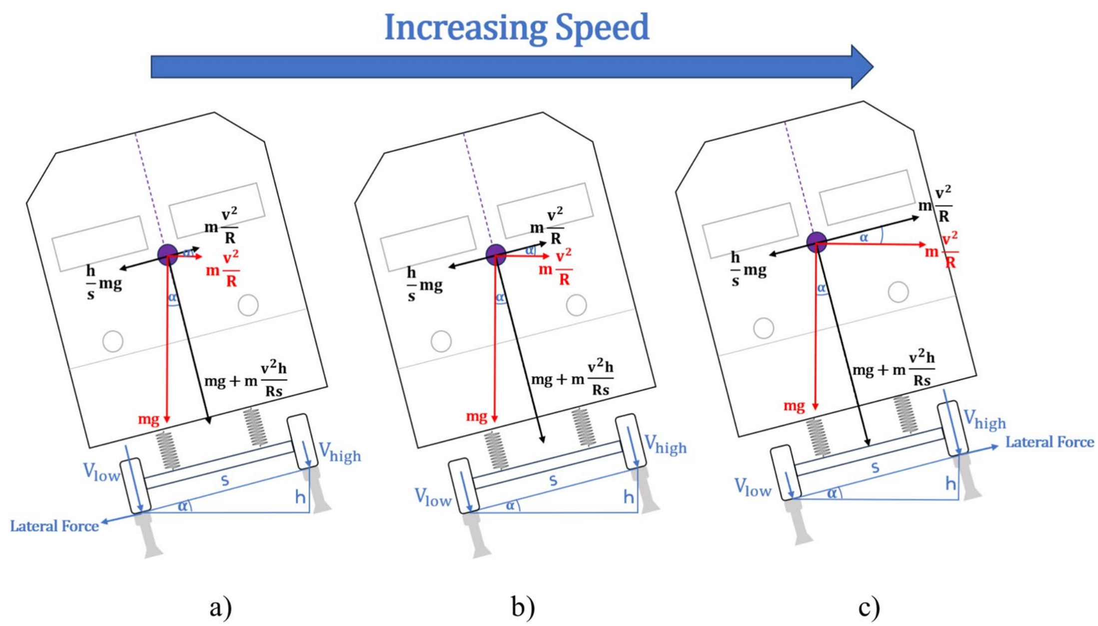

Figure 2.17: Forces at (a) underbalanced speed (b) balanced speed (c) overbalanced speed [21].

To properly model these effects, it is essential to consider the lateral resistance of the track, which is made up of three main components: the base of the sleeper, the crib ballast and the ballast shoulders. Studies by Le Pen show that the geometry of the sleeper, the type of ballast, the degree of compaction and the friction at the interfaces all determine the magnitude of the resistance. Realistic modeling of these interactions can be done using spring-type elements (such as COMBIN39), with multilinear behavior adjusted from experimental data. Lateral resistance at the shoulders, for example, has a limit beyond which increases in width or height do not result in further improvements in stability [22].

Complementing the modeling, the prediction of the rail temperature itself is essential to assess the risk of thermal deformation. Frigeri, in his study [23], validated a rail temperature prediction model based on an improved version of the Chungnam National University (CNU) model, comparing it with experimental measurements and finite element simulations. His approach takes into account solar radiation, convection, radiation losses, and includes the calculation of the position of the sun and the shadow of the rail using a Python package called railtemp. The results showed a high correlation with experimental data ($R^2 = 0.947$ and RMSE = 2.6 °C), providing a reliable tool for estimating temperature loads as boundary conditions in structural deformation models.

In addition, a comparative finite element thermal model was developed using the PLANE55 and COMBIN39 elements in ANSYS to validate the Python tool. The FEM model showed nonlinear heat transfer behavior and realistic boundary conditions. The model results confirmed that concentrated thermal models assuming a uniform temperature distribution along the rail section are valid for predicting maximum temperatures, thus justifying their integration with structural buckling models [23].

Therefore, the numerical analysis of thermal buckling in CWRs must incorporate not only the geometric and thermal parameters of the track, but also structural and operational factors, as well as factors such as superelevation. Complete three-dimensional models, combining static and dynamic analysis with non-linear representation of resistances, currently represent the most robust approach to predicting and mitigating the risks of thermal buckling on curved sections with cant. The integration of theory, simulation and experimental data is essential for the safe and efficient design of permanent railway track [17].

In previous studies by Nava [24] and Frigeri [6], parametric analyses were developed to investigate the static buckling behavior of continuously welded rails. To do this, they used ANSYS software, adopting the Newton-Raphson method, which is widely used in non-linear simulations. The track geometry was modeled with BEAM188 elements, both for the rails and the sleepers, allowing for an adequate representation of the track's structural flexibility. The model mesh can be seen in Figure 2.18.

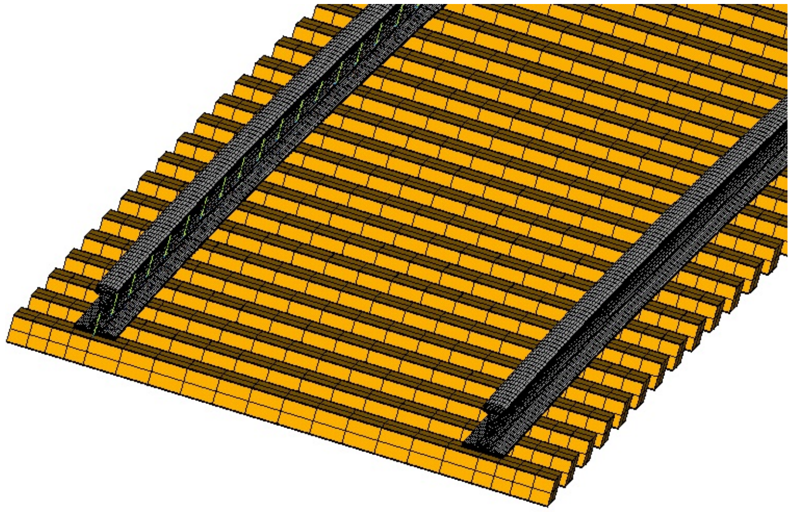

Figure 2.18: BEAM188 elements

In order to include the effects of the torsional stiffness of the fasteners and the resistance offered by the ballast, both in the lateral and longitudinal directions, COMBIN39 elements were incorporated into the model, in figure 2.19 one can see an example of how the connection between the sleeper and the rail may be conceived using COMBIN39. These elements, which represent connectors with specific mechanical behavior, were configured based on data from technical literature and adjusted to follow multilinear behavior throughout the simulation.

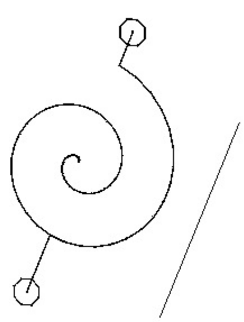

Figure 2.19: COMBIN39 elements

To simulate the effect of thermal loading, the rails were subjected to a controlled heating process. The initial temperature of 20 °C was gradually raised until it reached 300 °C. This procedure made it possible to observe the progressive development of axial and lateral forces and the structural response of the system to thermal expansion, providing important data for assessing the stability of the track under high temperatures.

The numerical model considered the presence of an initial imperfection located at the central longitudinal node of both rails, acting in the lateral direction. This imperfection served as the starting point for the parametric analysis and was later varied in amplitude in order to assess its influence on the buckling behavior.

2.3.3 Analytical Models

Analytical models play an essential role in the theoretical understanding of thermal buckling in continuously welded rails, providing a solid mathematical basis for initial estimates of the critical buckling load, the influence of structural parameters and the behavior of the track in unstable equilibrium. Since Kerr's work [18], various approaches have been developed with the aim of more accurately representing the effects of ballast resistance, geometric imperfections and sleepers.

Kerr's classic formulation [18] used the principle of minimum potential energy to determine the critical buckling load on thermally compressed rails. His model, based on a continuous beam on an elastic foundation with constant lateral resistance, provided exact solutions for four different buckling configurations. However, the torsional stiffness of the fasteners was disregarded as it was considered insignificant in the case of cut spikes. Subsequently, Grissom and Kerr incorporated an additional stiffness coefficient into the model, representing the contribution of the fasteners to the overall behavior of the track [18].

More recent models, such as those proposed by Navarro et al. and Van, have refined the representation of track-mast interaction. In Navarro's study, the rail-tie structure was replaced by an equivalent beam, adopting an energy approach based on minimizing the system's total potential. His analytical model also made it possible to evaluate the effects of maintenance operations (such as ballast denting), the type of sleeper (wood, monoblock concrete and biblock), the geometry of the initial imperfection (half-wave or full wave) and the height of the ballast over the sleepers. The results showed that loaded rails present buckling loads up to 200% higher, due to the increase in base resistance provided by the vertical loads [25].

Van's model [4] applied the analytical methodology to a curved line with a radius of 400 meters, evaluating central imperfections with amplitudes between 8 and 50 mm. The lateral and longitudinal resistance of the ballast was represented with multilinear behavior. The model also considered the effect of a test wagon with a vertical load of 293 kN per wheel. The sensitivity analysis indicated that the curvature of the track, the horizontal resistance of the ballast and the amplitude of the imperfections are the most critical factors for the stability of the track. Although the stiffness of the fasteners has a limited impact, their inclusion significantly increased the robustness of the model in regions with dynamic loading and irregular geometry [4].

In more recent studies, Navarro has developed a more comprehensive analytical model that includes parameters often ignored by previous approaches, such as the variation in ballast cover on sleepers, the consolidation of ballast over time and the loss of resistance associated with maintenance operations, such as denting. The formulation of the total potential energy takes into account the contributions of rail bending, the resistance offered by the ballast (divided into base, crib and shoulders), and the work of external forces. The resolution of the system makes it possible not only to calculate the critical buckling load, but also the amplitude of the deformation and the length of the most critical imperfection. The results indicated, for example, that the type of sleeper and the amplitude of the imperfection are more relevant factors in unloaded rails, while the application of a vertical load substantially increases the overall stiffness of the system, reducing the risk of buckling. The model also highlighted the importance of dynamic stabilization as a way of recovering up to 80%of the lateral resistance lost through maintenance [25].

These models reinforce the usefulness of analytical approaches as tools for preliminary assessment of thermal buckling. Despite simplifications, such as constant lateral resistance and the disregard of lifting waves on loaded rails, they make it possible to understand how variations in materials, track geometry, maintenance and loading affect track safety. When used in conjunction with empirical data or in support of three-dimensional numerical models, analytical models provide a reliable basis for the design and maintenance of CWR tracks, especially in curved regions with superelevation, where thermal stresses and centrifugal forces are most critical.

2.4 Buckling Parameters

The thermal buckling of continuously welded rails is influenced by various structural, geometric and operational factors, whose combined action determines the stability of the track. On sections with curvature and superelevation, these factors become even more relevant, since the geometry of the track alters the distribution of internal forces and enhances the acting lateral forces, especially on the outer rails. Elements such as the type of sleeper, the stiffness of the fasteners, the lateral resistance of the ballast and the presence of initial geometric imperfections have a direct impact on the critical buckling load.

Studies such as those by Navarro [25], Van [4], Frigeri [6] and Nava [24] have shown that the amplitude of imperfections, the type of sleeper and track maintenance operations are determining factors for the behavior of the track under thermal variations. In addition, the application of a vertical load, such as the weight of the train, significantly increases the base lateral resistance of the system, increasing the overall stiffness and safety against buckling, especially on inclined and curved sections.

2.4.1 Sleeper-Ballast Interaction and Resistance

The analysis of rail systems is essential to ensure safety and efficiency in rail transportation, especially when it comes to the longitudinal strength of rails and the failures associated with the fastening system. A study by the Federal Railroad Administration (FRA), referring to accident data from US railroads between 1999 and 2018, revealed an average of 12.5 derailments per year caused by "defective or missing pins or rail fasteners" [18]. These derailments have often been associated with flaws in the design of fastening systems, which makes it urgent to improve the design methodology of these systems. Fastener failure usually occurs due to a combination of vertical, lateral and longitudinal loads [17], which makes it essential to quantify all the loads applied to the fasteners.

Longitudinal loads are responsible for many of the most common fastener system failures, such as broken pins, deterioration of rail seats and direct fastening rod failures [26]. However, longitudinal loads are the least quantified, and the subject has not yet received the attention it deserves in research. To fill this research gap, analytical models have been developed and used to quantify the loading demands on fasteners.

The RailTEC (Rail Transportation and Engineering Center) at the University of Illinois Urbana-Champaign conducted research on the longitudinal strength of rails using track panel pull tests (TPPT) for both wood and concrete sleepers. This data helped provide crucial information to improve the accuracy of analytical models and contribute to the development of more robust fastening systems. Track Panel Pull Tests (TPPTs) are performed to quantify the longitudinal strength of rails. During the test, a load is applied to the rail panel until a maximum value or constant resistance is reached. The applied force and displacement of the panel are recorded to calculate the longitudinal strength of the rail panel, figure 2.20. Longitudinal resistance is independent of panel length when comparing results from different panel lengths. Field tests have the advantage of reflecting actual service conditions, while laboratory tests offer the flexibility to investigate more rail configurations and allow more time to replicate results [26].

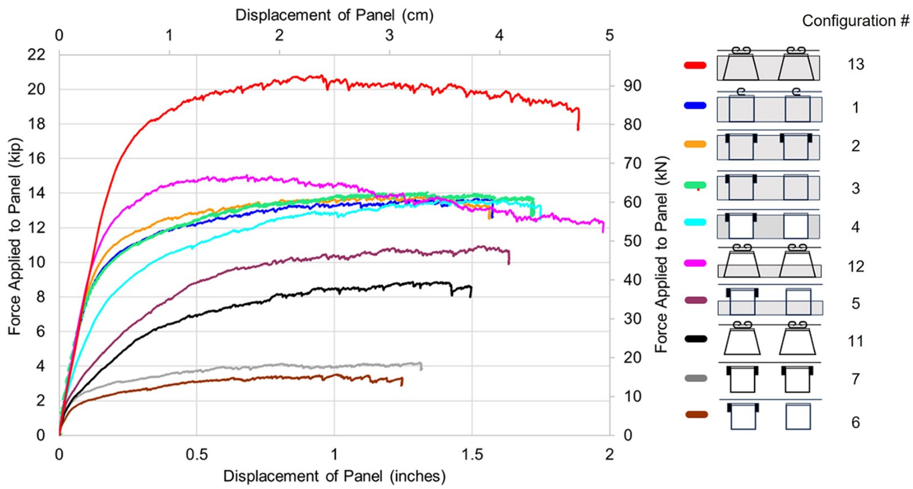

Figure 2.20: Longitudinal resistance [26]

The use of TPPT results to perform a load propagation analysis. The longitudinal resistance of rails is measured as the force per unit length provided by ballast, sleepers, fasteners, and rails, which implies that these variables should be considered in future studies. The results of these laboratory tests provide a basis for improving the understanding of fastening system behavior, especially with regard to longitudinal resistance [26].

In addition, TPPTs can also be used to test different ballast conditions, such as the effect of compacted ballast versus disturbed ballast. The behavior of the fastening system is significantly influenced by the state of the ballast, and tests should be performed taking this into account.

Analysis of the TPPT results reveals that rail panels with concrete sleepers offer greater longitudinal resistance than those with wooden sleepers, especially when the ballast is completely compacted. However, when the ballast is disturbed or the amount of ballast is reduced, the stiffness of the panel decreases considerably. This indicates that the load transfer between the rails and the ground is strongly dependent on the ballast before fastener slippage occurs.

The data also indicate that the fastening system plays an important role in longitudinal resistance. In tests with wooden sleepers, the longitudinal resistance was similar regardless of the type of fastener or the presence of ballast at the sleeper edges. This suggests that the fasteners were well engaged in the sleepers and were stronger than the ballast, since slippage occurred at the sleeper-ballast interface and not at the rail-fastener interface.

2.4.2 Fastener Stiffness and Torsional Resistance

The rigidity of fasteners and their resistance to torsion are essential structural parameters in the performance of permanent railway tracks, especially in continuously welded rails, where thermal variations and dynamic loads impose severe stability conditions. These elements play a decisive role in containing lateral movements and rail rotations, acting directly to prevent thermal buckling, especially in sections with curvature and superelevation. The torsional resistance presented by the fastener generally presents a linear graph, as shown in figure 2.21.

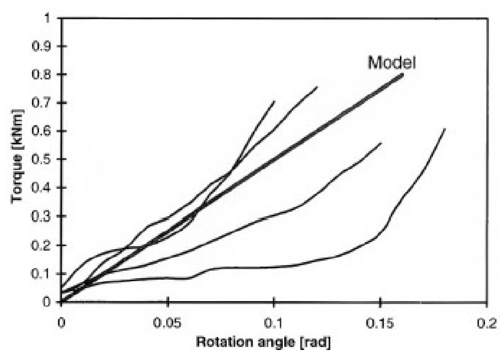

Figure 2.21: Graph of a linear resistance

The rigidity of fasteners can be defined as the ability to resist deformation under external loads, and is commonly broken down into longitudinal, lateral, and vertical components. In thermal stability analyses, lateral stiffness is of central importance, as it contributes to the overall resistance of the track to transverse displacements induced by thermal compressions. Longitudinal stiffness, on the other hand, acts to restrict the axial movement of the rails, influencing the accumulation of thermal stresses and the response of the track to braking and acceleration. Insufficient values for these parameters can lead to premature thermal collapse of the track [17].

The torsional resistance of fasteners is equally important, especially on curved sections with cant, where the forces resulting from the weight of the train, combined with the track gradient, produce torsional moments around the longitudinal axis of the rail. Fastening devices with low torsional resistance allow the rail to rotate, which can compromise track alignment, reduce the effectiveness of fastening systems, and aggravate lateral instability. Analytical models such as those developed by Kerr [18] and later improved by Navarro [25] often incorporate this resistance indirectly, through effective stiffness coefficients applied to the equivalent rail beam.

The study conducted by the Volpe National Transportation Systems Center (VNTSC) evaluated the torsional stiffness associated with different types of fasteners and tie materials, as shown in Table 2.6.

	Fastener System	**Average τ_0 (kips-in./rad)**	**Standard Deviation σ**	**No. Tests**
Wood ties	Pandrol	5026	1957	3
	8 spikes/plate	2165	589	6
	4 spikes/plate	1203	384	22
	2 spikes/plate	386	145	4

Table 2.6: Torsional Stiffness Values [17]

The numerical representation of these behaviors is performed using spring-type elements with linear or elastoplastic stiffness, such as the COMBIN39 element, which allows rotational, lateral, and longitudinal constraints to be included in the model. In thermal buckling simulations, the correct adjustment of these parameters directly influences the buckling mode, the amplitude of the deformation, and the critical temperature (ΔT_{max}). The use of small variations in lateral stiffness can cause significant differences in the simulated results [20].

From an experimental point of view, it was also possible to observe that increasing the stiffness of the fasteners is associated with a substantial increase in the critical buckling temperature, increasing the track's ability to withstand thermal variations without collapsing [17]. However, excessive stiffness values can cause concentrated stresses in the fasteners and compromise durability due to fatigue, in addition to limiting the flexibility of the system in the face of cyclic rail movements.

Therefore, calculating the stiffness of fasteners and their resistance to torsion is essential to ensure track stability, especially in critical areas such as cant curves, where the interaction between thermal and dynamic loads is more intense. The integration of experimental data, numerical modeling, and design criteria must be constantly improved to adapt fastening systems to the operational requirements of each railway line.

2.4.3 Track Misalignment and Initial Imperfections

Track misalignments and initial geometric imperfections are very important factors in the analysis of thermal buckling. These deviations from the ideal track geometry act as disturbances which, under the action of compressive thermal stresses, can be amplified, triggering lateral instabilities. The importance of these imperfections is even greater in sections with curvature and cant, where centrifugal forces and track inclination increase the structure's sensitivity to transverse displacements [27].

Some analytical models treat buckling as a phenomenon that arises from the amplification of a pre-existing imperfection in the rail [18]. The use of an energy model with initial deformations assumed as sinusoidal functions, evaluating different amplitudes and wavelengths, can be used, thus showing that the presence of half-wave or full-wave imperfections significantly reduces the critical buckling load, especially in unloaded rails [25]. Sensitivity to the shape and amplitude of the imperfection was also observed when testing central imperfections of up to 50 mm on a 400 m radius curve, confirming that curvature and misalignment act together in anticipating instability [4].

In numerical simulations, imperfections were introduced with amplitudes between 5 mm and 20 mm, showing that thermal buckling is strongly dependent on the initial geometry of the track [20]. The point where the imperfection is located also influences the buckling mode, which can generate symmetrical or antisymmetrical forms of lateral deformation. Experimental studies reinforce this conclusion, showing that deformations located in the center of the track tend to amplify in a predictable manner, figure 2.22, while asymmetrical or irregular imperfections can cause sudden and unstable buckling, especially under conditions of low lateral resistance [17].

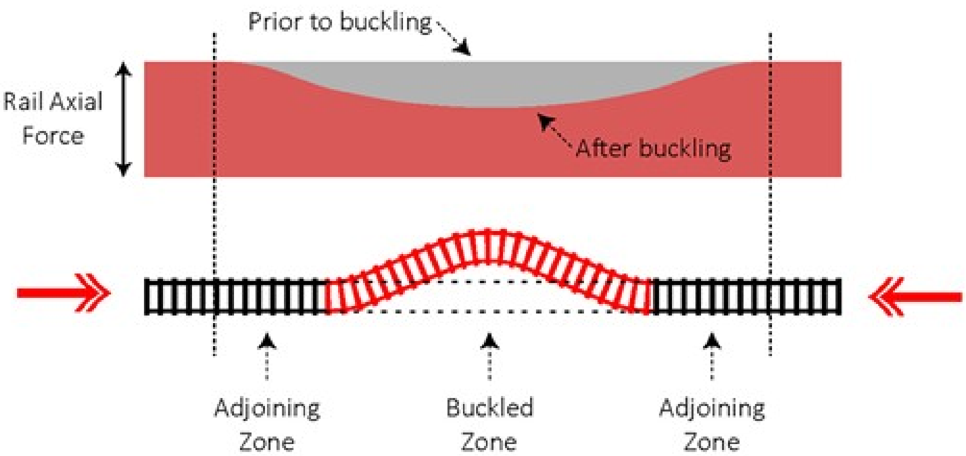

Figure 2.22: Deformation located in the center of the track

Thus, the presence and control of initial track imperfections play a decisive role in predicting the structural behavior of CWR rails. The adoption of strict geometric inspection criteria, combined with the correction of misalignments through preventive maintenance, is essential to prevent the worsening of these imperfections and reduce the risk of thermal buckling, especially in curved sections with cant, where the combination of unfavorable factors makes the structural system more vulnerable to instability.

2.4.4 Rail Profile and Geometrical Properties

Although thermal buckling of continuously welded rails occurs predominantly in the lateral plane, the stifness of the rail cross section in the vertical direction, represented by the vertical second order of area (I_y), has an indirect but relevant influence on track stability. This occurs because the rail, when displaced laterally, rotates around its longitudinal axis, and this torsional-lateral coupling causes the cross section to bend. In this case, a higher I_y increases the overall flexural stiffness of the rail, which resists this lateral deformation.

Thus, although deformation occurs in the horizontal plane, vertical bending stiffness contributes to the system's ability to resist lateral displacement. This parameter is directly related to the rail's ability to resist lateral bending, since buckling occurs as a deformation in the horizontal plane but is controlled by the sectional stiffness of the rail in that same plane.

The following figures show the geometries of the S30, UIC50, 132RE, and 136RE profiles, respectively, demonstrating how important it is to follow their characteristics.

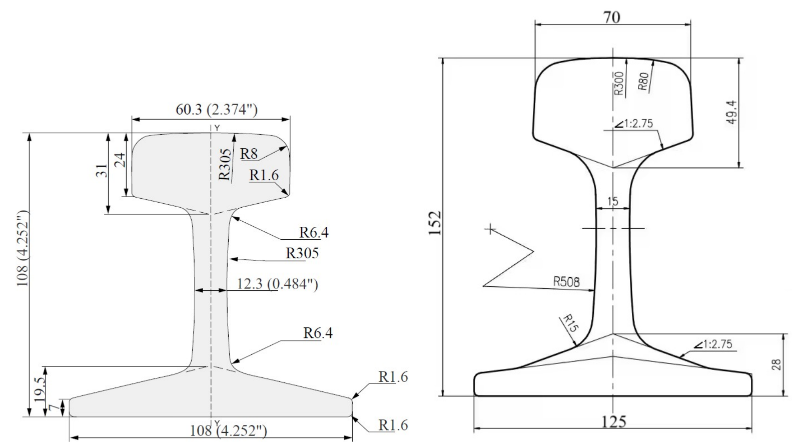

Figure 2.23: S30 and UIC50 profiles.

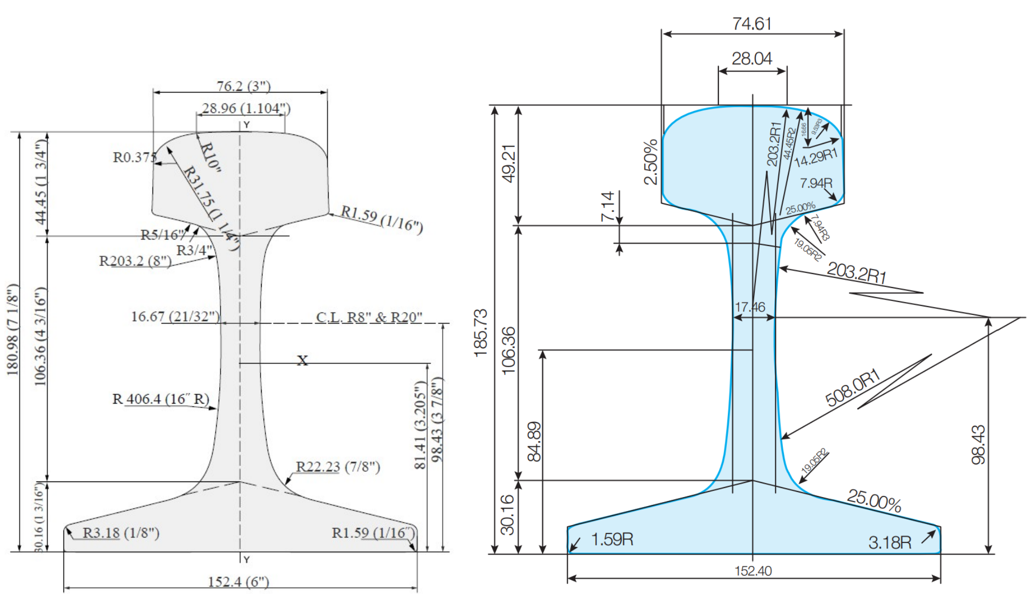

Figure 2.24: 132RE and 136RE profiles

The second order moment of area relative to the weak axis of the rail cross-section plays a key role in lateral thermal buckling resistance. Although deformation occurs predominantly in the horizontal plane, buckling resistance is directly related to the lateral stiffness of the rail, which is controlled by the second order moment of area around the vertical axis (I_y). Sections with higher I_y offer greater resistance to lateral bending under thermal compression loads, resulting in higher critical buckling temperatures. More robust profiles, such as 136RE, which have higher I_y values compared to lighter profiles, such as S30, demonstrate greater thermal stability due to their greater lateral bending stiffness. Thus, the proper choice of rail profile, considering its geometric properties and the respective second order moment of area, is essential to reduce the risk of lateral buckling due to thermal variation.

Changes in the second order moment of area have an influence on lateral buckling, as they are directly linked to the geometry of the profile, such as the base, height, and web. This results in a change in stiffness in the buckling plane, but as the study will be conducted in the lateral direction. As observed in Kerr's models [18] and Navarro's parametric analyses [25], the second order moment of area directly enters the terms of flexural deformation energy used to calculate the total potential energy of the track. An increase in I_y influences the critical buckling load and, therefore, the operating temperature limit of the rail without destabilizing it.

This effect is even more relevant in curved sections with cant, where the outer rail is subject to greater lateral and torsional stresses. The different types of I_y act as an additional "structural barrier" against the accumulation of lateral deformations under thermal compression, contributing to different types of buckling that can occur, which is why the choice of profile is so important when modeling the track [4] [20].

Therefore, the value of the vertical moment of inertia of the rail section not only contributes to the support of vertical operational loads, but also plays an important role in the resistance to lateral thermal buckling.

2.4.5 Track Curvature

The presence of curvature in the railway track significantly alters the structural behavior of the track, especially with regard to thermal buckling in continuously welded rails. The curved geometry introduces additional centrifugal forces during railway traffic, redistributes internal stresses between the rails, and intensifies the action of torsional moments, making the system more sensitive to lateral instability. These changes are even more relevant when associated with cant, which seeks to compensate for part of the centrifugal force but, when insufficient, aggravates the stresses acting on the outer rail.

Classic analytical and numerical models, such as those by Kerr [18] and Navarro [25], already pointed out that track curvature reduces the critical buckling load due to the loss of structural symmetry and the asymmetrical accumulation of thermal stresses. Van [4], in his parametric study of a curved line with a radius of 400 m, demonstrated that the combination of sharp curvature, geometric imperfections, and reduced lateral resistance can lead to collapse even with small temperature variations. Pucillo [28], in simulations in ABAQUS, confirmed that buckling behavior is significantly altered in curves, requiring greater rigidity in fasteners and higher lateral resistance to compensate for the additional stresses.

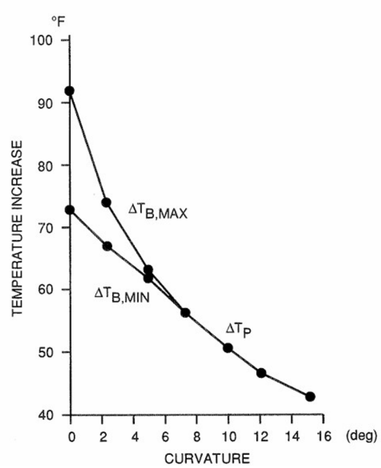

Figure 2.25: Relation between curve radius and buckling [17]

The integration of the influence of curvature with dynamic parameters, such as train speed and superelevation imbalance, shows that curvature not only reduces track stability in terms of static buckling, but also increases risks in dynamic regimes, with amplified deformations and anticipation of thermal instability [21].

Therefore, track curvature can not be treated as a secondary element in the study of thermal buckling, as seen in figure 2.25 it is possible to see how curvature influences buckling. Its influence directly affects stress distribution, buckling modes, and critical temperature. In designs for lines with CWR rails, especially in hot climates and under heavy traffic, it is essential to consider the curve radius and the applied cant.

2.4.6 Superelevation

Superelevation, also known as cant, is the difference in height between the outer rail and the inner rail of a railway curve. Its main purpose is to compensate, either totally or partially, for the centrifugal forces acting when the train travels through a curve, reducing the lateral forces transmitted to the track and improving passenger comfort. In the context of continuously welded rails (CWR), which are sensitive to instabilities caused by temperature variations, cant becomes a critical factor, as it directly affects the balance of lateral forces and the probability of thermal buckling, especially on small-radius curves or at high speeds.

The ideal cant value, called theoretical cant h_{th}, is calculated based on the train speed and the curve radius in order to completely cancel out the centrifugal force [8]. However, on mixed lines, where passenger and freight trains operate at different speeds, it is necessary to adopt an intermediate value called applied cant (h). This has two effects: cant deficiency h_d, figure 2.26, for fast trains, which still suffer centrifugal action, and cant excess h_e, figure 2.26, for slow trains, which are overcompensated, which can hinder traction when starting on curves [29]. The cant value applied must therefore balance safety, performance, and comfort, considering all types of operation.

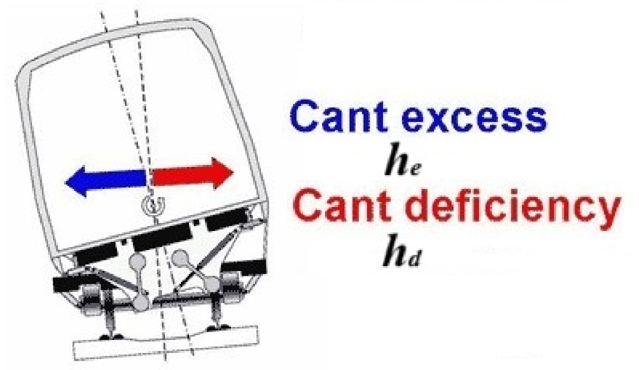

Figure 2.26: Cant deficiency and cant excess [29]

In the context of thermal buckling, the presence of cant significantly influences track behavior. The cross-sectional slope affects the distribution of normal forces and the effective lateral stiffness of the rail-sleeper-ballast system. This creates an asymmetrical condition in which the outer rail is more prone to buckling, especially when there is a cant deficiency and high thermal loads [5] [21]. In numerical modeling, this effect must be carefully incorporated, as it alters the buckling shape and can reduce the critical temperature ΔT_{max} of the system. The interaction between the track inclination and the lateral resistance of the ballast can also cause antisymmetric buckling, which is more unstable and dangerous.

In addition, the presence of cant interacts with the initial geometric imperfections and the stiffness of the fasteners, two parameters that strongly influence thermal buckling [17] [25]. The combined effect of cant with these imperfections can aggravate the loss of stability even under moderate operating conditions, especially on poorly maintained tracks or with low-rigidity fasteners. On the other hand, technologies such as tilting trains can mitigate part of the cant deficiency, allowing higher speeds on curves without increasing the physical cant of the track [8].

According to the study by Aela et al. [30], it was found that the lateral resistance of monoblock sleepers increases with the degree of superelevation. Lateral resistance increased by about 100% for a superelevation of 150 mm compared to rails without superelevation. This extra resistance is mainly attributed to the ballast shoulder, whose contribution intensifies with cant, while the contribution of the base tends to decrease.

The width of the ballast shoulder is also crucial: increasing it from 300 mm to 500 mm resulted in an increase in resistance of approximately 27%, regardless of the degree of cant. This finding reinforces the importance of proper ballast shoulder maintenance, especially on sections with cant [30]. Numerical simulations using the discrete element method (DEM) showed that with increasing cant [30]:

- The total resistance to lateral movement of the sleeper increased.
- The contribution of the ballast shoulder increased both in absolute and proportional terms, from about 40% to over 60%.
- The resistance of the lateral ballast increased slightly.
- The resistance of the base decreased in relative terms, from over 30% to less than 10%.

Therefore, the correct design of cant on curves with CWR is essential not only for operational comfort and safety, but also as a strategy to mitigate thermal buckling. Numerical and analytical models that consider the inclined geometry of the section, the interaction with the ballast, and the dynamic behavior of the train are indispensable for a realistic prediction of track performance under critical conditions.

Chapter 3

Methodology

This chapter describes the steps taken to model and analyze the phenomenon of thermal buckling in continuously welded rails in curved sections with superelevation. It defines the physical and geometric parameters of the track components (rails, sleepers, fastenings, and ballast), as well as the numerical methods used to simulate structural behavior under temperature variations. The modeling was performed using three-dimensional finite elements, with detailed representation of the torsional stiffness of the fasteners, in addition to the nonlinear resistance of the ballast, also considering the presence of initial imperfections in the track geometry.

The numerical model developed was configured to accurately represent the combined effects of curvature and superelevation on the thermal stability of the track. Thermal loads were applied incrementally, allowing the behavior of the system to be observed until stability was lost. The methodology also includes parametric analyses that investigate how different geometric configurations, such as changes in the lateral stiffness of fasteners, ballast resistance, and initial imperfections, influence thermal buckling, with a special focus on track performance in canted curves.

3.1 Material and Structural Properties

The physical and geometric parameters that define the mechanical behavior of the railway are fundamental to this study, as the thermal stability of continuously welded rails (CWR) depends directly on the properties of its components, such as the profile and elasticity modulus of the rail, the type of sleeper, the stiffness of the fasteners, and the lateral and longitudinal resistance provided by the ballast. Each of these elements contributes in a distinct way to the overall stiffness of the track, affecting the response to thermal compression and lateral buckling.

Understanding these properties allows to build a numerical model consistent with the actual conditions of the track and identify which characteristics most strongly influence buckling in curves with superelevation. For example, the influence of a particular rail profile, figure 3.1, on the critical buckling load [31], while fasteners with low lateral stiffness or torsional resistance can drastically reduce stability. Thus, the study of these properties establishes the physical basis necessary for all subsequent numerical and parametric analyses.

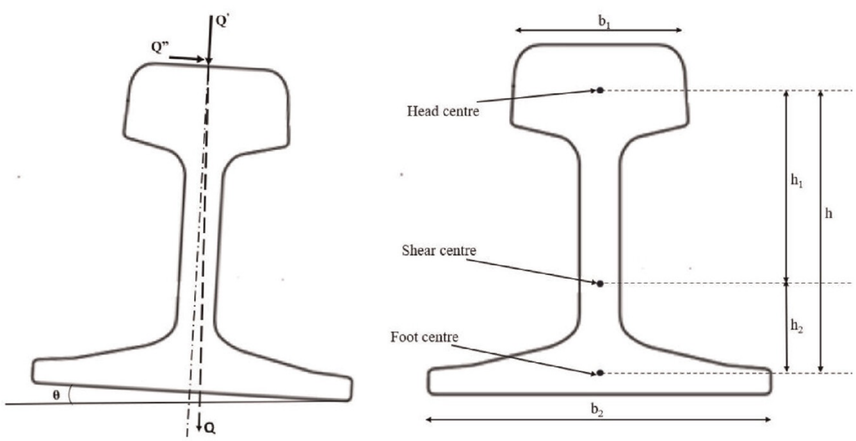

Figure 3.1: Rail with and without cant [31]

3.1.1 Rail Profile and Steel Properties

In this study, four rail profiles were considered: S30, UIC50, 132RE, and 136RE, all widely used in mixed traffic and high-density railways. Each of these profiles has different geometric characteristics, which directly influence the stiffness of the track and, consequently, its stability under thermal variations. The lateral second order moment of area I_y is relevant in the analysis of thermal buckling, since instability usually manifests itself in the lateral plane. [8][21].

All profiles were considered with R260 steel [9]. This pearlitic steel has a yield strength of approximately 260 MPa, making it suitable for applications in continuously welded rails (CWR) due to its good mechanical strength and dimensional stability under thermal loads. To simulate the behavior of the material more realistically, especially in states close to buckling collapse, the Ramberg-Osgood model was adopted [32]. This model allows the gradual transition between the elastic and plastic regimes of the stress-strain curve to be represented, even in the absence of detailed experimental data.

The equation used is given by:

$$\varepsilon = \frac{\sigma}{E} + K\left(\frac{\sigma}{\sigma_y}\right)^n \tag{3.1}$$

where:

- ε is the total deformation,
- σ is the applied stress,
- E is the modulus of elasticity of R260 steel,
- σ_y is the applied stress,
- n is the work hardening exponent,
- K is adjusted based on tensile strength, according to the procedure of Kamaya [32].

The adoption of this stress-strain relationship allows the numerical model to more accurately capture the nonlinear effects associated with thermal buckling, especially in canted curves, where the asymmetric distribution of forces makes the system more susceptible to instability. The use of different profiles also allows the influence of rail geometry on the critical buckling temperature and on the mode of lateral deformation to be evaluated.

The stress-strain curve model is then validated by comparing it with the engineering curve obtained from experimental data [33]. This comparison allows the accuracy of the model in reproducing the actual mechanical behavior of the material to be evaluated, especially in the elastic and plastic regions of deformation, ensuring that the parameters adopted are in accordance with the results observed in laboratory tests. The results obtained using equation 3.1 allowed the data shown in figure 3.2 to be found.

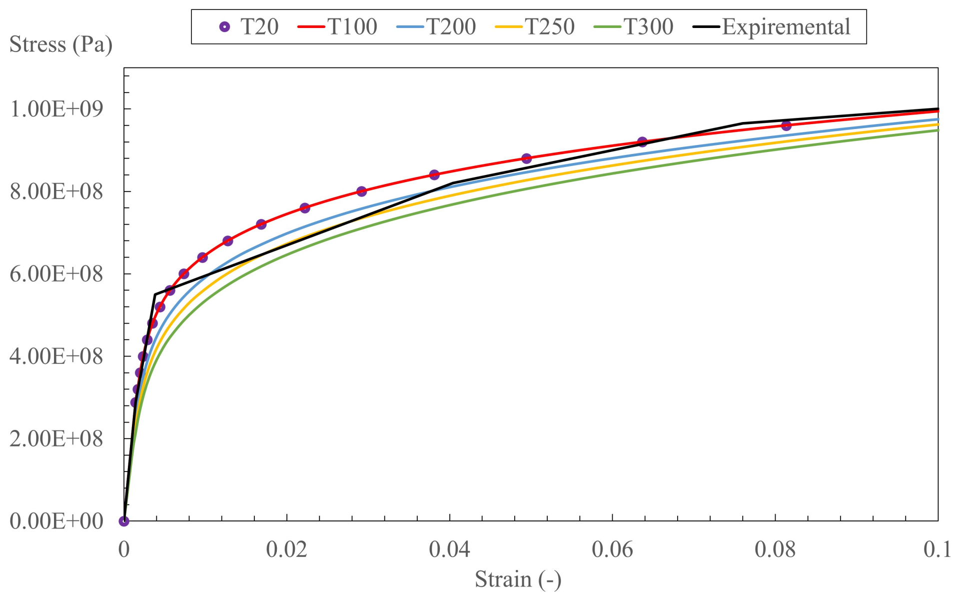

Figure 3.2: Mechanical stress-strain curve of R260 steel.

The mechanical behavior of the material at different temperature ranges was defined using mechanical properties in accordance with the equations recommended by standard EN 1993-1-2 [34]. The thermal expansion of steel was determined based on the equation that expresses thermal elongation as a function of temperature, equation 3.2, which was then derived to obtain the thermal expansion coefficient.

Another important factor is the axial stiffness ($\frac{EA}{L}$) for each profile. This parameter quantifies the axial resistance to thermal expansion and directly influences the accumulation of thermal forces. The $\frac{EA}{L}$ values were calculated using Young's modulus (E) and the cross-sectional area (A) of each profile, divided by L, which corresponds to the length of the element in the numerical model. Table 3.1 shows the values obtained.

Profile	**Area** (m^2)	**EA** (N)
S30	3.839×10^{-6}	0.7716×10^{6}
UIC50	6.428×10^{-6}	1.292×10^{6}
132RE	8.355×10^{-6}	1.679×10^{6}
136RE	8.612×10^{-6}	1.731×10^{6}

Table 3.1: Profiles with Area and EA values

Higher $\frac{EA}{L}$ values indicate greater axial stiffness, resulting in higher thermal compressive forces when expansion is restricted. These forces are one of the determining factors for thermal deformation and interact with the lateral stiffness defined by I_y, making both parameters essential for assessing the stability of the track under temperature variations.

In addition, specific temperature levels were considered, such as 20°C, 100°C, 200°C, and 300°C, to calculate the accumulated thermal elongation of the material, using the formula indicated in the aforementioned standard, see equation 3.2.

$$\frac{\Delta L}{L} = 1.2 \times 10^{-5} T + 0.4 \times 10^{-8} \cdot T^2 + 2.416 \times 10^{-4} \tag{3.2}$$

3.1.2 Sleeper Properties

Wooden sleepers are widely used on railway lines, especially on sections that require greater structural flexibility, ease of maintenance, and good response to dynamic loads. Wood is an orthotropic material, meaning that its mechanical properties vary according to the direction considered, longitudinal (along the fibers), radial, and tangential, which requires careful modeling to correctly represent its structural behavior [35].

The orientation of the wood fibers in the sleepers follows the longitudinal axis of the piece, which ensures high resistance to compression, bending, and shear in that direction, figure 3.3 shows how it works. However, the directions perpendicular and transverse to the fibers have considerably lower stiffness and strength, requiring special attention when representing the sleeper in three-dimensional numerical simulations. Therefore, different properties were assigned to the elasticity modules, Poisson's ratios, and shear modules in each main direction of the material in order to reflect its orthotropic character [36].

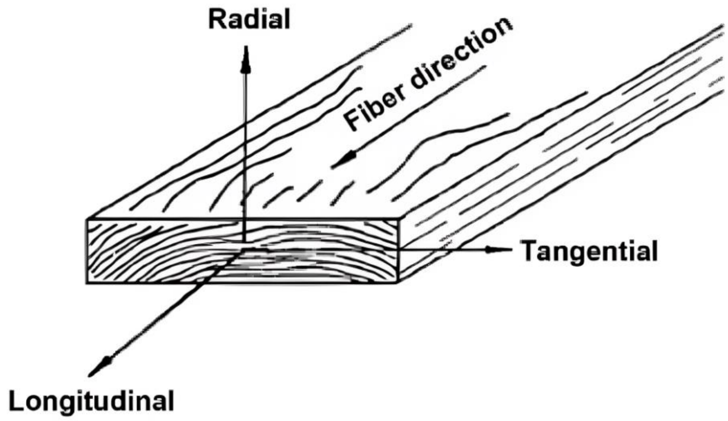

Figure 3.3: Orthotropic wood [36]

The characteristics of wood make it a relatively flexible material when compared to concrete, which directly influences the sleeper-ballast interaction and the distribution of stresses along the track. The rough surface of wood provides greater static friction at the interface with the ballast, resulting in greater base lateral resistance, an important advantage in thermal buckling resistance. This contribution of base friction is strongly dependent on the load condition: loaded sleepers, for example, generate greater normal pressure on the ballast and, consequently, greater resistance to sliding. Modeling wood as an orthotropic material aims to capture this directional variation in performance more accurately, especially in three-dimensional analyses where the combined effects of curvature and cant modify the action of forces along the track.

To properly model this anisotropic behavior in the simulation environment, reference values obtained from the literature for Red Oak were used, considering the typical ratios between the orthotropic properties and the longitudinal modulus of elasticity [37]. These values are presented in Table 3.2. In this table, E_L represents the modulus of elasticity in the direction of the fibers (longitudinal), E_R and E_T the modules in the radial and tangential directions, respectively; G_{LR}, G_{LT} and G_{RT} the shear modules in different planes, and ν_{LR}, ν_{LT} and ν_{RT} the corresponding Poisson's ratios.

Based on these reference parameters and experimental values available in the literature for the longitudinal modulus of elasticity of Red Oak, it was possible to estimate the

Specie	E_T/E_L	E_R/E_L	G_{LR}/E_L	G_{LT}/E_L	G_{RT}/E_L	ν_{LR}	ν_{LT}	ν_{RT}
Red Oak	0.082	0.154	0.089	0.081	0.004	0.35	0.448	0.56

Table 3.2: Orthotropic Red Oak Parameters, adapted from [37]

other orthotropic components proportionally, ensuring a realistic representation of the mechanical behavior of wood under different types of loading. Finally, as the thermal analysis does not consider heat transfer through the sleeper, the thermal properties of wood were disregarded at this stage.

3.1.3 Fastener Stiffness and Torsional Resistance

The stiffness of fasteners and their resistance to torsion are essential factors for the lateral stability of continuously welded rails (CWR), especially in curves with superelevation. These fasteners connect the rail to the sleeper and resist longitudinal and lateral displacements and rotations caused by thermal and dynamic traffic loads. The stiffness of the fasteners determines the restriction of rail movements, directly influencing the critical buckling temperature and the modes of track instability. In the specific case of wooden sleepers, the fastening system generally consists of four or two nails, whose stiffness values were adopted based on previous studies, such as that by Kish [17].

Studies show that thermal buckling is highly sensitive to the lateral stiffness of fasteners. In addition, this high lateral stiffness ends up raising the critical buckling temperature, inhibiting the transverse displacement of the rail [17]. Furthermore, in curves with superelevation, the torsional resistance of the fasteners is particularly important, as the torsional moments generated by the asymmetrical loading of the train can induce rotations in the rail, compromising the overall stability of the track if not adequately restricted [25].

In numerical modeling, the stiffness and torsional resistance of the fasteners are represented by spring elements or connectors, such as COMBIN39 in ANSYS, calibrated with experimental results. The variation of these parameters affects the buckling mode and the critical and safe temperatures ΔT_{max} and ΔT_{min} of the track. Therefore, careful consideration of these aspects is essential to improve the accuracy of numerical models and guide the design and maintenance of railway infrastructure, especially in critical sections with complex geometry and different types of sleepers.

As can be seen in figure 3.4, the graph of the torsional moment versus angular rotation relation, shows linear behavior, typical of modeling with torsional spring elements, such as COMBIN39 in ANSYS. This linear behavior indicates a constant stiffness of the fastener against rotations, which directly contributes to restricting the movement of the rail and increasing the critical buckling temperature.

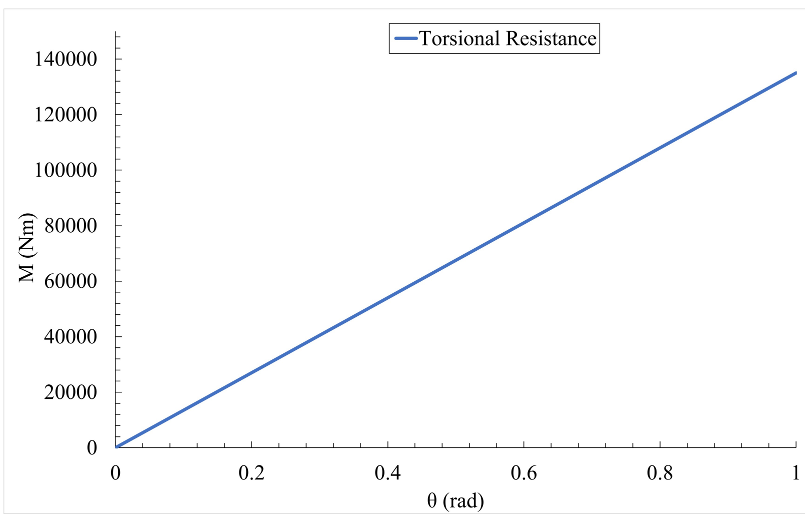

Figure 3.4: Torsional Resistance of the Rail Fastening System

3.1.4 Ballast Properties

Railway ballast is one of the most important layers in the permanent track structure, responsible for distributing the vertical loads from passing trains, allowing water drainage, and maintaining track alignment and leveling. Consisting of granular aggregates, usually crushed stone with a particle size between 40 and 75 mm, ballast also acts to contain the sleepers, providing lateral and longitudinal support. Its efficiency is directly linked to the shape, degree of compaction, angularity of the particles, and state of degradation of the material.

With continuous use of the track, the ballast layer tends to undergo progressive degradation, either through particle fragmentation due to cyclic loading or contamination with fines (such as coal dust, subgrade intrusion, or clayey materials). This degradation, called fouling, compromises the stiffness of the system and affects both the vertical support and lateral confinement of the sleepers. It is important to note that even if the ballast appears to be in good condition, its actual resistance may be severely compromised, with losses of more than 50% in total lateral stiffness in situations of complete fouling. The analysis of the ballast condition, therefore, must consider its internal behavior and not just its surface appearance [15].

3.1.5 Lateral and Longitudinal Resistance

Lateral and longitudinal resistance are fundamental components for the stability of the railroad, especially in continuously welded rails, where thermal and dynamic forces can compromise the structural integrity of the track. Lateral resistance acts to prevent the transverse displacement of sleepers and rails and is essential in preventing thermal buckling, while longitudinal resistance limits the axial movement of rails and is critical in mitigating the effects of thermal expansion and stresses induced by train braking or acceleration.

Lateral resistance consists of three main mechanisms of interaction between the sleeper and the ballast: friction at the base of the sleeper, friction on the side faces (crib ballast), and passive pressure exerted by the ballast shoulders. Classic studies have shown that about 65% of the total lateral resistance of the track comes from the ballast, 25% from the fasteners, and only 10% from the rails [17]. Tests such as Single Tie Push Tests (STPT) have been widely used to quantify this resistance, allowing the behavior of each component to be evaluated and more realistic numerical models to be calibrated. Analysis of the progressive degradation of ballast, especially due to fine particles, subgrade intrusion, or inadequate compaction, can reduce the lateral resistance of the track by more than 50%, even when there are no visual signs of failure. This highlights the need for careful monitoring of ballast quality over time [15].

Complementing this analysis, longitudinal resistance has gained prominence in recent studies due to its critical influence on the integrity of fasteners and the distribution of thermal stresses along the track. According to Dersch et al. [26], longitudinal resistance is often neglected in comparison to lateral resistance, despite being responsible for many types of failures in fastening systems, such as clip breakage, rail seat deterioration, and failures in direct fastening systems. In their research with Track Panel Pull Tests (TPPT), it was found that panels with concrete sleepers had 20% more longitudinal strength than wooden sleepers, and that the condition of the ballast (compacted vs. loose) can affect longitudinal stiffness by up to 80%.

These tests also revealed that the contribution of each component of the system is significant: approximately 65% of the longitudinal resistance is provided by the ballast between the ties (crib ballast), 30% by the base of the ties, and 5% by the shoulders of the ballast. This shows that both the geometry and the compaction state of the ballast directly influence the axial response of the track, and are essential factors for numerical analysis and safety design. In addition, the type of fastener, its arrangement, and the anchoring force influence the relative sliding between the rail and the sleeper, defining the operational limits of the track under longitudinal actions. However, the longitudinal resistance between the rail and the sleeper must be at least 15 kN per complete fastening (i.e., per sleeper), ensuring that it is significantly greater than the longitudinal shear resistance of the sleeper in the ballast bed. This is equivalent to approximately 5 kN per side (half the sleeper), and ensures that, in situations of high axial stresses, such as those induced by extreme thermal variations, the fastening system remains functional, and longitudinal movement is absorbed by the interaction of the sleeper with the ballast, and not by failure of the fasteners [1].

Thus, the integration of lateral and longitudinal resistance effects becomes indispensable in modeling the structural behavior of railway tracks, especially in curved sections with superelevation, where vertical, centrifugal, and thermal forces act in combination.

Understanding these mechanisms allows for the improvement of numerical models, the establishment of more accurate operating limits, and the implementation of more effective maintenance strategies.

In the case of lateral resistance, figure 3.5, the curve reflects the combined contribution of friction at the base and lateral faces of the sleeper, as well as the passive pressure of the ballast shoulders, with the initial stiffness being strongly influenced by the degree of compaction and integrity of the ballast. In the longitudinal resistance, figure 3.6, the graph highlights the axial response of the track, highlighting the role of the fastener anchorage and the interaction between sleeper and ballast, especially in the ballast between the sleepers. Both graphs exhibit linear behavior in the initial phase, characterizing a constant stiffness typical of the interactions between the sleeper and ballast before significant movements or degradation occur.

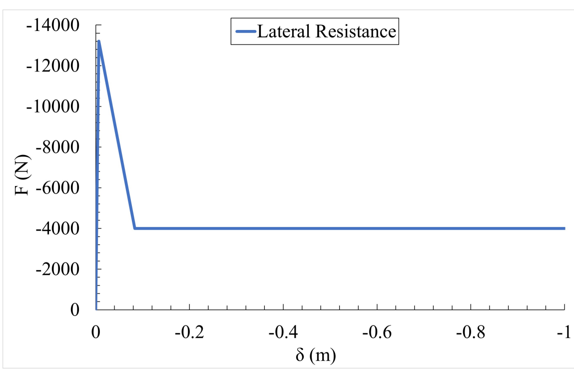

Figure 3.5: Lateral Resistance

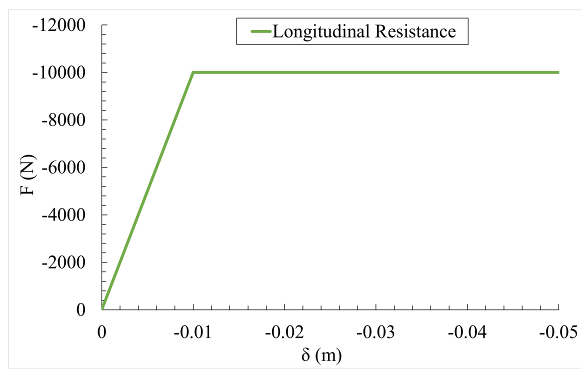

Figure 3.6: Longitudinal Resistance

3.2 Numerical Model Description

The numerical model developed in this study aims to analyze the lateral stability of continuously welded rails (CWR) installed on curves with superelevation, subject to thermal variations. The modeling was performed in the ANSYS APDL, using three-dimensional elements to accurately represent the main components of the permanent way, including rails, sleepers, fasteners, and ballast.

The nonlinear behavior of the system was considered in terms of both material properties and the resistance offered by the rail-sleeper-ballast interfaces. The analyses were based on actual railway operating conditions, incorporating initial geometric imperfections and temperature variations. The formulation of the problem allowed the evaluation of the critical buckling temperature and the effects of parameters such as fastener stiffness, sleeper type, curvature, cant, and ballast resistance, as will be detailed in the following sections.

3.2.1 Geometry and Track Configuration

The numerical model was developed based on a typical curved railway track configuration with continuously welded rails (CWR), considering a section with a radius of 350 m and an arc with a length of 200 m, as seen in the representation in figure 3.7, values that are representative of segments critical to lateral stability. The choice of curved segments allows for a more realistic assessment of the effects of superelevation and centrifugal forces induced by passing trains, which are particularly relevant for the analysis of thermal buckling.

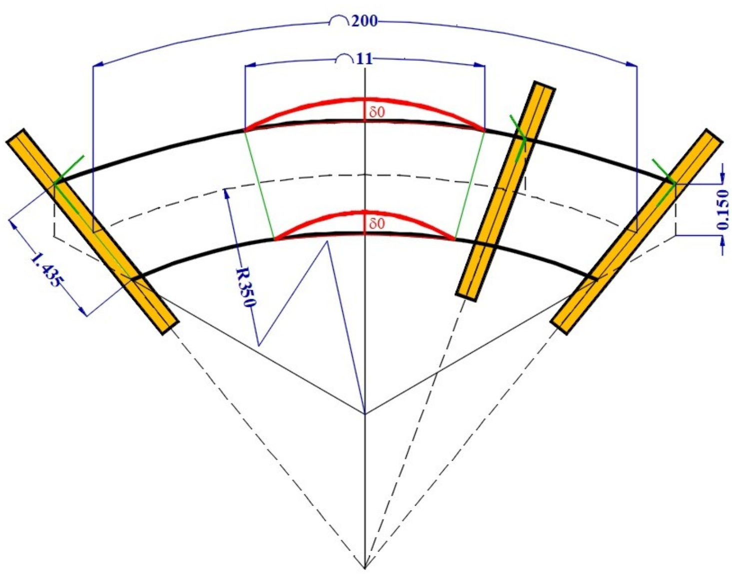

Figure 3.7: Representation of the system with a radius of 350 meters.

The modeled track has a total length of 200 meters, sufficient to capture the modes of lateral instability that develop along sections with amplified deformations. The curved layout implies an asymmetrical distribution of stresses on the rails, requiring special attention to the representation of internal forces [38], figure 3.8, lateral ballast resistance, fastener stiffness, and sleeper geometry. These aspects are fundamental for correctly assessing the critical behavior of the track in the face of thermal variations and dynamic loads.

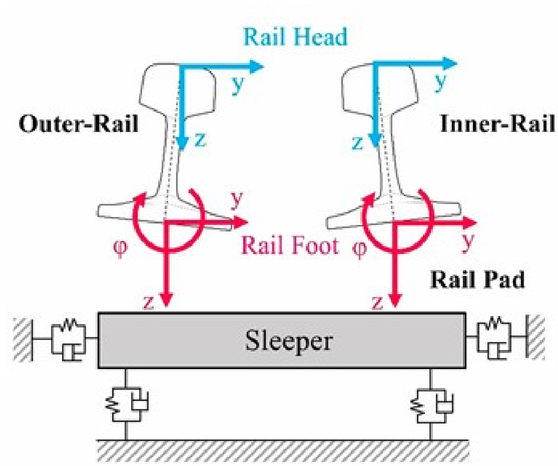

Figure 3.8: Forces occurring on a rail [38].

When developing the railway model in ANSYS, it was necessary to create a local coordinate system for each point on the track, located on the sleepers, as can be seen in figure 3.9. These local coordinate systems were essential to ensure that the orientation of the elements and the application of resistances occurred correctly along the geometry of the railway.

Unlike the ANSYS global coordinate system, which remains fixed, the local systems were defined based on the position and inclination of each sleeper, allowing the curvature and superelevation of the track to be accurately tracked. Without this definition, the application of structural links and elements could be misaligned, compromising the physical consistency and validity of the results obtained in the simulation.

This approach ensured that the modeled railroad respected its actual geometry, allowing for a more accurate analysis of structural behavior under different loading conditions.

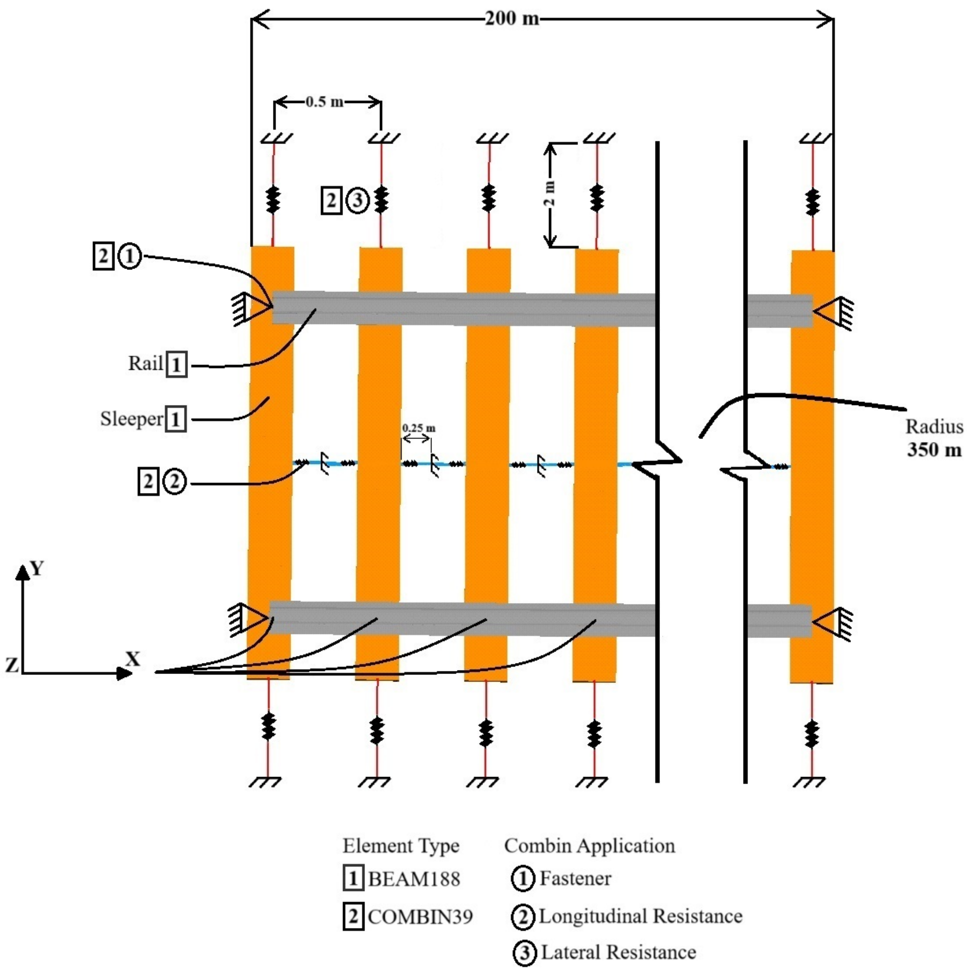

Figure 3.9: Geometry of the model

3.2.2 Cant and Curvature Definition

The cant is the vertical distance difference between the outer rail and the inner rail on a curve. This inclination has the main function of balancing the centrifugal forces acting on the train when traveling on curves, reducing lateral forces on the track and increasing the comfort and safety of the operation [39]. Typical cant values, such as 150 mm, are widely adopted on high-speed and medium-speed lines to ensure stable and comfortable operation.

Curves with a smaller radius have greater curvature, increasing dynamic forces and the need for greater cant to compensate for lateral forces. The combination of curvature and cant directly influences the stability of the railway system, affecting everything from rail wear to the risk of thermal instability in continuously welded rails.

Thus, the precise definition of cant and curvature, as well as the appropriate choice of superelevation value, are fundamental to ensuring the safety, comfort, and durability of the railway. The choice of 150 mm cant, figure 3.10, represents a technical compromise that meets most operating conditions, aligning with regulatory criteria and good railway engineering practices [40].

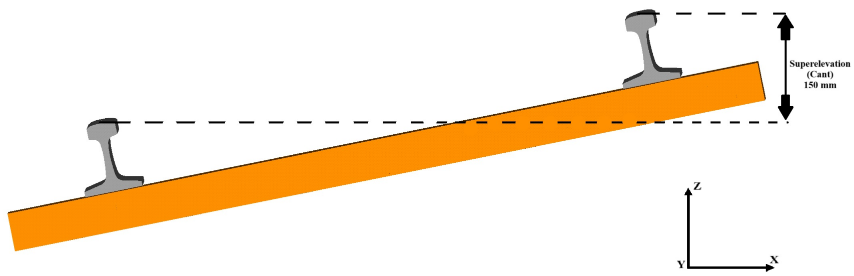

Figure 3.10: Geometry of the model

3.2.3 Initial Imperfections and Misalignments

Initial imperfections and misalignments are fundamental aspects to be considered in the structural analysis of railway systems, especially in continuously welded rails. These imperfections represent unavoidable geometric deviations, present since the manufacture, assembly, or laying of the track, which can significantly influence the behavior of the structure under thermal and dynamic loads. Small initial imperfections act as stress amplifiers, reducing the critical buckling capacity and altering the instability modes predicted by ideal models.

In the present study, the initial imperfection was applied in the central region of the track section, with a total length of 11 meters, aiming to represent a typical initial curvature due to real variations in the installation and laying of the system. This modeling is essential to capture the most realistic responses of the rail, since localized imperfections act as critical points for the onset of thermal buckling. The formula used to describe the imperfection is given by:

$$\delta_0 = \frac{L_0}{a} \sin\left(\frac{\pi x}{L_0}\right) \tag{3.3}$$

where δ_0 is the amplitude of the imperfection as a function of position x along the length L_0 of the imperfection (11 meters), and a is an adjustment parameter that defines the magnitude of the deviation. This sinusoidal form is commonly used in stability analysis to represent moderate and symmetrical imperfections, as recommended in structural standards such as Eurocode 3 [34].

The imperfections were determined for a equal to 270, 500 and 1000, as can be seen in the table 3.3, where the maximum imperfection found was 40mm. The values for the 4 types of profiles used in this work were the same.

Track Radius [m]	**a**	**Sleeper Material**	**Rail Gauge [mm]**	**Maximum Imperfection [mm]**
350	270	Wood	1435	40
350	500	Wood	1435	22
350	1000	Wood	1435	11

Table 3.3: Initial Imperfection Specifications

In addition to geometric imperfections, relative misalignments between track components, such as between rails and sleepers, also influence the stiffness and stress distribution along the railway system. Misalignments can cause stress concentrations that accelerate failure and affect operational safety [41]. Thus, the inclusion of these imperfections in the numerical model, as adopted in this work, provides greater realism in the evaluation of thermal buckling and contributes to the definition of safer criteria for railway design and maintenance.

3.2.4 Boundary Conditions and Constraints

The precise definition of boundary conditions and constraints is essential to ensure the physical representativeness of the numerical model, especially in simulations of continuously welded rail stability. Boundary conditions directly influence buckling modes, permissible displacements, and stress redistribution along the track. In this study, an approach based on the direct connection between the rail and sleeper nodes was adopted, using coupling constraints applied at the position of the fasteners.

The coupling constraint was used to ensure that the rail and sleeper nodes had identical movements in the X, Y, and Z translation directions, as well as in the rotations around the X and Z axes. This condition adequately simulates the joint behavior of the rail-sleeper system in the presence of rigid fasteners, ensuring that both elements deform coherently in the main directions. However, relative rotation between the rail and the sleeper was allowed on the Y axis, using an interface finite element, enabling the representation of the torsional stiffness behavior of the fasteners, which is essential for evaluating the effects of cant and asymmetric loads on curves.

In addition, all nodes in the model had their translation in the Z direction restricted, preventing vertical movements of the track and simulating the support provided by the ballast and underlying structure. At the ends of the rails, complete restrictions were applied to all degrees of freedom, except for rotation around the Z axis. This condition allows free global rotation of the track relative to the horizontal plane, which is consistent with lateral buckling analyses, without imposing artificial locks that could distort the structural response of the model.

Despite the creation of multiple local coordinate systems per cross member, all boundary conditions and couplings were applied based on the ANSYS global coordinate system, this system can be seen in Figure 3.10. This choice was necessary to ensure consistency in the definition of degrees of freedom and to facilitate control of the constraints imposed on the model.

These boundary conditions were defined to balance physical fidelity and numerical stability, ensuring that the model is capable of accurately capturing the dominant modes of thermal instability expected in CWR tracks subjected to critical loads.

3.2.5 Element Types and Mesh Characteristics

The appropriate definition of element types and mesh characteristics is essential to ensure accuracy and numerical stability in finite element simulations, especially in thermal instability studies on continuously welded rails (CWR). In this work, two main types of finite elements were used: BEAM188 and COMBIN39, both available in the ANSYS Mechanical APDL platform.

The BEAM188 element is a three-dimensional beam element based on Timoshenko's theory, which considers both bending and transverse shear effects. Each node has six degrees of freedom (three translations and three rotations) and a warping [42], making it particularly suitable for modeling slender elements subjected to combined bending, torsion, and compression loads, such as railroad rails and sleepers. In addition, BEAM188 employs cubic interpolation functions and full Gauss integration, offering greater accuracy in capturing buckling modes and localized deformations along the track. The key options that were applied in Ansys for this type of element are shown in Table 3.4.

Key Options	**Value**	**-**
$K1$	1	Seven degrees of freedom per node (including warping)
$K2$	1	Rigid
$K3$	2	Cubic form
$K4$	2	Include both
$K6$	0	At integration points
$K7$	2	All Section points
$K9$	3	All Section node

Table 3.4: Beam key options

The COMBIN39 is a unidirectional spring element with nonlinear force-displacement behavior, used to represent the resistance offered by ballast and fasteners. This element can be configured to simulate different mechanical behaviors, including longitudinal, lateral, and torsional stiffness [43]. To model the lateral and longitudinal resistance of the ballast, COMBIN39 was defined with three degrees of translational freedom per node, while the torsional resistance of the fasteners was represented by the degree of rotational freedom around the Y axis (ROTY). This approach allows capturing the nonlinearity of the track support system and the coupling effects between structural components.The key options that were applied in Ansys for this type of element are shown in Table 3.5.

Application	**Key Options**	**Value**	**-**
Torsional Resistance	$K3$	5	ROTY
Lateral Resistance	$K3$	1	UX
Longitudinal Resistance	$K3$	3	UZ

Table 3.5: Combin key options

Unlike the boundary conditions, the BEAM and COMBIN elements used to model the rails and connections between components were defined based on these local coordinate systems, these systems can be seen in Figure 3.9 rather than the global system. This allowed each element to correctly follow the slope and spatial positioning of each sleeper, ensuring an accurate representation of the structure.

As for the mesh, a refined discretization was adopted to ensure accuracy in the prediction of buckling modes and critical displacements. The rails were discretized with BEAM188 elements with a length of 0.083 m, while the sleepers were modeled with a length of 0.11 m. This discretization was kept uniform for both straight and curved sections, ensuring consistency in the analyses. The choice of element length was based on recommendations in the literature and convergence tests, balancing accuracy and computational cost.

The rail profiles modeled were S30, UIC50, 132RE, and 136RE. The modeling in ANSYS was performed based on AREMA specifications [44] and the corresponding standards for each type of profile. The mesh was generated manually to ensure greater control over the geometry and quality of the elements. These four profiles were modeled to enable comparative and parametric studies in later stages of the analysis.

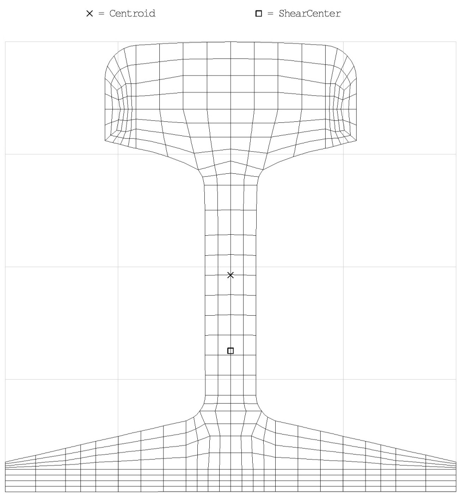

Figure 3.11: S30 Mesh

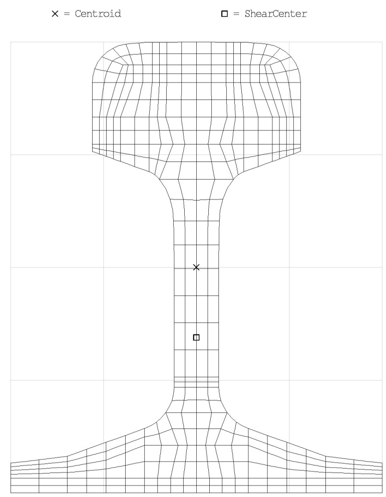

Figure 3.12: UIC50 Mesh

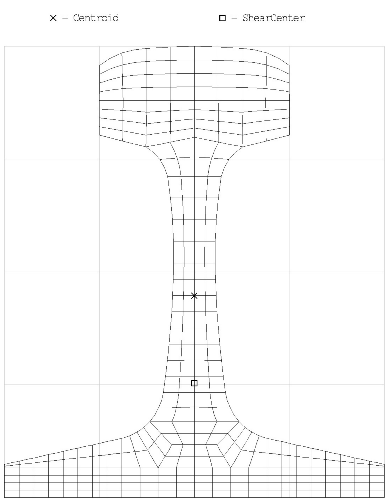

Figure 3.13: 132RE Mesh

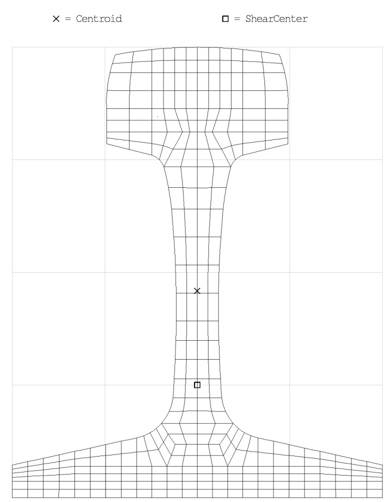

Figure 3.14: 136RE Mesh

3.3 Model Implementation

The numerical implementation of the model is one of the fundamental pillars for the thermal stability analysis of continuously welded rails. In this study, a three-dimensional model was developed using the finite element method to simulate the structural response of the railway under different temperature variations, considering initial imperfections, curvature, and cant. The model was implemented in the ANSYS Mechanical APDL, which offers flexibility in the definition of specific structural elements and allows advanced control of geometric, material, and contact nonlinearities.

The modeling explicitly considered the main components of the track structure: rails, sleepers, fasteners, and ballast. For this purpose, appropriate elements were chosen to represent the structural behavior of each component, as well as their interactions. The analysis was conducted in a nonlinear manner, using robust load control strategies to capture the post-critical behavior of thermal buckling.

The representation of the resistance offered by the ballast and fasteners, COMBIN39 elements were used, positioned at distances representative of the track geometry. In the case of lateral resistance, the elements were connected between nodes at a distance of 2 meters, simulating the interaction of the sleeper with the ballast. In the case of longitudinal resistance, the elements were inserted at a distance of 0.25 meters between the nodes, corresponding to half the spacing between consecutive sleepers. Torsional resistance, representing the torsional behavior of the fasteners, was modeled with COMBIN39 elements connecting nodes at a distance of only 0.001 meter, simulating the fastening of the rail. These distances are indicated with dimensions in figure 3.9, which illustrates the arrangement of the elements.

3.3.1 Software and Solver Used

Numerical modeling was performed using ANSYS Mechanical APDL software, widely used in structural engineering for complex simulations involving geometric and material nonlinearities and boundary conditions. The developed model incorporated a geometric nonlinear analysis with initial imperfections GMNIA (Geometric and Material Nonlinear Imperfection Analysis), essential for the accurate evaluation of the structural behavior of continuously welded rails (CWR) under thermal loads.

The standard solver for nonlinear static analysis was employed, with control based on generalized displacements. To ensure the stability of the solution, convergence criterion based on the infinite displacement norm was defined, adopting a tolerance of 10^{-3} and a reference value of 0.1 m. This criterion was chosen because it is the most appropriate for the nature of the problem, given that the focus of the analysis is on determining the transverse displacements associated with thermal buckling of the track.

In addition, to improve the convergence of the solution in highly nonlinear stages, a numerical stabilization strategy with energy dissipation defined as 10^{-6} was applied. This technique is particularly effective in simulations sensitive to small variations in stiffness, as is the case with thermal buckling, allowing greater robustness in tracking the structural response in post-critical regimes.

3.3.2 Load Cases and Temperature Ranges

To investigate the thermal instability behavior of the railway track, an incremental temperature variation was applied to all nodes of the rail structure. The analysis began with an initial stress-free temperature T_0 of 20°C, representing the installation condition without thermal stresses. The temperature was then gradually increased until it reached 300°C, a value chosen to represent an extreme and conservative thermal loading scenario.

The temperature was applied using the ANSYS table function, allowing the definition of a direct relationship between simulation time and applied temperature. Thus, a thermal ramp was created in which the temperature varied from 20°C to 300°C over 300 time units, with time increments defined as 5 units. This approach allowed precise control over the introduction of thermal loading, facilitating observation of the progressive behavior of the structure.

Considering a linear temperature increase, the rate of temperature change was 0.933 °C per unit of time. Therefore, each time interval of 5 units corresponds to an approximate temperature increase of 4.67 °C. The minimum and maximum time interval sizes defined in the analysis were $1e^{-6}$ and 5 units, which correspond to minimum and maximum temperature increases of approximately 0.000933 °C and 4.67 °C, respectively.

The use of controlled thermal increments is fundamental in analyses involving buckling, as it allows clear capture of the evolution of the system up to the critical point of instability. In addition, application by table ensures compatibility with the GMNIA procedure, adopted in this study to accurately simulate the combined effects of geometry, materials, imperfections, and thermal loading.

3.4 Model Validation

The validation of the developed numerical model is a fundamental step to ensure the reliability and accuracy of the results obtained in the analysis of thermal buckling in continuously welded rails (CWR). For this purpose, the simulation results were compared with data published in the literature, such as the data found by Kish [17], which constitute a consolidated reference in the study of the thermal and structural behavior of railway systems.

The comparison focused on key parameters, such as lateral displacements and critical buckling temperatures, allowing verification of whether the implemented model adequately reproduces the physical phenomena observed experimentally and in previous analyses. The agreement between the numerical results and Kish's data demonstrates that the model correctly captures the interactions between rails, sleepers, fasteners, and ballast, as well as the effect of initial imperfections and component stiffness on the thermal stability of the track.

In addition, the validation reinforces the appropriateness of methodological choices, such as the use of BEAM188 elements to represent structural components and COMBIN39 elements to model the nonlinear characteristics of ballast and fasteners. Thus, the model is robust and consistent, suitable for analyzing different configurations and operating conditions of the railway track.

The accuracy of the model is further supported by Nava [24], who validated a similar finite element approach for curved rails. In his study, Nava obtained good agreement with experimental measurements, with a root mean square error (RMSE) of 2.9°C in lateral displacements and consistent buckling modes. The methodological choices adopted in this work, such as the use of BEAM188 and COMBIN39 elements, are in accordance with those validated by Nava, which reinforces the accuracy of the model in representing the interactions between rail, sleeper, and ballast, as well as in estimating the critical buckling temperature. This study expanded on previous models by incorporating a 150 mm superelevation on the track, providing a more complete representation of the behavior of rails on curves under thermal loads.

3.5 Parametric Study

The parametric study is an essential step in the analysis of thermal buckling of continuously welded rails (CWR), allowing the evaluation of how different variables influence the stability and structural behavior of the railway track. Through the systematic variation of geometric parameters, material properties, and operating conditions, we seek to understand the isolated and combined effects of these factors on the critical buckling temperature, modes of instability, and responses of the railway system.

Cant [mm]	**Track Curvature**	**Fastener Stiffness**	**Ballast Stiffness**	**Maximum Imperfection [mm]**
150	350	Weak or Strong	Weak or Strong	40
				22
				11

Table 3.6: Study Parameters

In this work, 24 simulations were performed, with parameters such as cant, track curvature, fastener and ballast stiffness and the amplitude of imperfections, parameters that are in table 3.6. The choice of these parameters is based on previous studies, such as those by Kish, Navarro, and Pucillo, which highlight the relevance of these factors in the safety and durability of the railway.

The parametric analysis was performed using the numerical model implemented in ANSYS Mechanical APDL, with specific variations of each parameter kept constant to isolate their effects. This procedure makes it possible to identify the operational limits of the system and propose recommendations for design and maintenance that minimize the risk of buckling, especially in critical sections with sharp curvatures and high levels of superelevation.

In addition, the parametric approach helps to understand the complex interactions between the structural elements of the track, allowing a more realistic and comprehensive assessment of the track behavior under variable conditions, contributing to the improvement of predictive models and the operational safety of the railway system.

3.5.1 Influence of Cant and Curvature

The geometric configuration of the railway track, especially with regard to horizontal curvature and cant, has a significant influence on the thermal stability of continuously welded rails (CWR). Curved rails naturally exhibit lateral forces and constraints that affect their susceptibility to buckling when subjected to temperature variations. When combined with cant, which is normally used to balance centrifugal forces during train passage, this geometry modifies the stress distribution and stiffness of the permanent way system.

Curvature imposes an initial geometric imperfection in the longitudinal alignment of the rails, which tends to reduce the critical buckling temperature. The smaller the curve radius, the more pronounced this effect. Sharp curves cause the rails to behave like initially imperfect columns, more prone to lateral instability under thermal expansion. Studies such as those by Kish and Navarro show that tracks with small radius have significantly lower buckling resistance compared to straight sections, especially in high temperature conditions.

Superelevation, in turn, alters the load paths within the track structure by tilting the rail plane. Although its main purpose is to compensate for the centrifugal acceleration of trains on curves, its presence also influences the distribution of lateral pressures between the sleeper and the ballast. When well designed, cant can redistribute part of the vertical load to the lateral direction, increasing the effective lateral confinement of the rails. Recent investigations based on numerical simulations and experimental data, such as those carried out by Chuadchim, indicate that cant can have a stabilizing effect under certain conditions, especially by improving the interaction between the sleeper and the ballast shoulder [21].

However, the effect of cant on lateral resistance and critical buckling temperature is not purely beneficial. Excessive cant values, especially in situations of cant deficiency, such as in cases where the train speed is lower than the equilibrium speed, can result in an increase in the lateral component of the train weight acting in the downward direction, which tends to amplify the lateral forces on the rails. Thus, understanding the balance between the geometric imperfection imposed by the curvature and the stabilizing or destabilizing effects of cant is critical for accurate prediction of thermal buckling.

In the case of this study, a curvature radius of 350 meters is being used, figure 3.7, which is considered a small radius, but due to the chosen curvature angle of 32.74°, the curve is not pronounced. In addition, the cant used is 150 mm, which also influences the increase in temperature where buckling occurs.

3.5.2 Variation of Weak and Strong Resistances

The railroad resists thermal expansion of the rail through components that offer mechanical reactions in the lateral, longitudinal, and torsional directions. These resistances can be classified as weak resistance and strong resistance, depending on the direction and intensity of the restraint offered.

Weak and strong resistance result from a combination of the stiffness of the elements, the friction and confinement provided by the ballast, the properties of the sleepers, and their support conditions. In simple terms:

- Longitudinal resistance restricts expansion or contraction along the rail axis.
- Lateral resistance limits the lateral displacement of the rail and is most critical in lateral buckling.
- Torsional resistance controls the rotation of the rail around its vertical axis and affects the coupled buckling modes.

Lateral, longitudinal, and torsional resistances were parameterized with values varying as shown in Figures 3.15, 3.16, and 3.17, representing conditions ranging from loose systems, where more worn tracks, insufficient settlement, and damaged fasteners are simulated, to well-maintained and reinforced tracks.

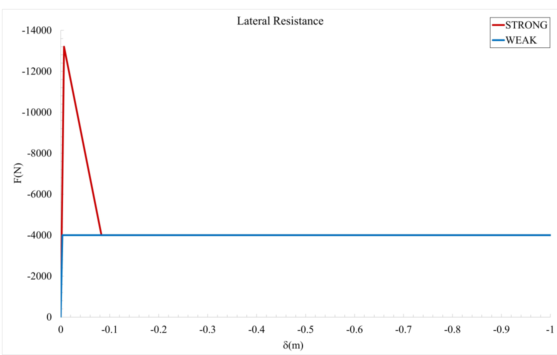

Figure 3.15: Longitudinal Resistance

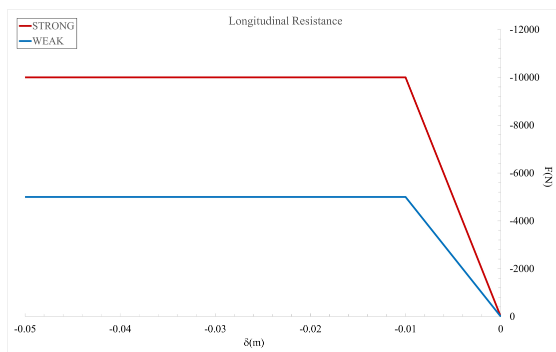

Figure 3.16: Lateral Resistance

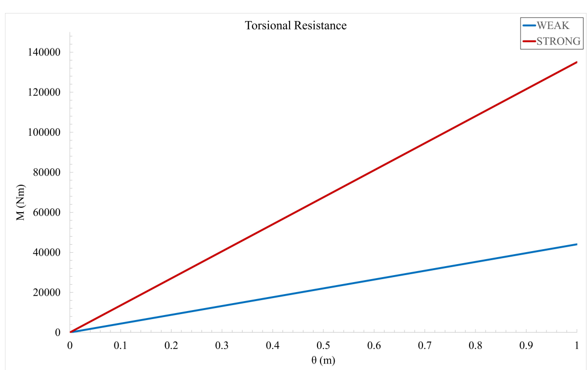

Figure 3.17: Torsional Resistance

The numerical analysis revealed the following effects on thermal buckling:

- Reduction in lateral resistance → drastically decreases the critical buckling temperature, as the rail loses containment against lateral displacements induced by thermal expansion. This is the most sensitive variable, especially in sharp curves.

- Increased lateral resistance → raises the buckling temperature, but with limited gain beyond a certain point, indicating saturation in the lateral restraint effect.

- Reduced longitudinal resistance → facilitates axial movements, which can delay the accumulation of compressive thermal stresses on the rail. However, this can lead to rail migration and the formation of gaps, which also impairs overall stability.

- Excessive increase in longitudinal resistance → Reduces any relief from thermal movement, concentrating stresses on the rail and anticipating buckling in critical areas.

- Torsional resistance → influences more complex buckling modes, such as those with rail rotation combined with lateral displacement. Low torsional resistance allows free rotation and favors unstable modes on inclined rails, those with cant, while high resistance helps keep the rail aligned on the sleeper.

Conditions such as ballast degradation, sleeper deformation and fastener failure simultaneously affect all three types of resistance. It is therefore essential to consider these variations in combination in realistic analysis models.

3.5.3 Effect of Misalignment Amplitude

The presence of initial lateral misalignments in the rail geometry, even if subtle, can cause significant changes in the structural behavior of the track under thermal loads. In the model developed in this study, the effect of misalignment amplitude was analyzed based on the introduction of geometric imperfections along the curved rail, with a radius of 350 m and a superelevation of 150 mm. The track was represented by beam elements, such as BEAM188, and the connections with the fastening-ballast system were modeled by linear springs, using COMBIN39, allowing the coupling between longitudinal forces and lateral displacements to be evaluated.

As the misalignment amplitude increases, a significant reduction in the critical thermal buckling load is observed. This occurs because the rail, already having an initial curvature, requires less deformation energy to reach an unstable mode. In other words, the "pre-deformed" rail responds more sensitively to thermal expansion, initiating lateral displacements even before reaching the critical load of an ideal geometry. This effect is amplified in curved regions, where the axial stress combines with bending moments resulting from the horizontal curvature of the track and the superelevation, creating a more severe stress state.

Another important effect is the redistribution of reactions in the fasteners and ballast. With greater misalignment, the lateral forces absorbed by the springs (representing the resistance of the infrastructure) become more concentrated in certain regions, indicating a risk of loss of effective contact between the sleeper and the ballast. This behavior was observed in simulations based on a progressive increase in temperature.

In addition, misalignment causes asymmetry in the system, generating unbalanced stresses between the inner and outer rails. In the case of tracks with superelevation, this means that the slope designed to balance the dynamic load can become unfavorable in the thermal context, aggravating displacements to the outer side of the curve and reducing the efficiency of the lateral resistance provided by the ballast.

Therefore, the results obtained indicate that the variation in misalignment amplitude is one of the most critical factors in the thermal stability of rails on curved tracks with superelevation. Small geometric imperfections can significantly anticipate buckling, redistribute stresses unevenly, and compromise track integrity, especially when associated with ballast degradation or variation in fastener stiffness. The inclusion of this variable in the modeling is therefore essential for a realistic assessment of track safety under extreme thermal actions.

Chapter 4

Results

This chapter discusses the main results of the numerical analysis conducted to evaluate the thermal stability of continuously welded rails under different structural, support, and track geometry conditions. The modeling was performed without considering vertical loads, focusing solely on the lateral response of the rail to thermal variations. This approach, although simplified, allows the phenomenon of lateral thermal buckling to be isolated, which is often cited as one of the most critical modes of failure in modern railways.

The unstable nature of buckling makes the problem highly non-linear, both geometrically and materially. To this end, the maximum lateral displacement was monitored at the midpoint of the span, more specifically at the upper node of the rail, where the greatest amplitude of deformation occurs. In total, 24 simulations were performed, covering combinations of different rail profiles (136RE, 132RE, UIC50, and S30), element strength conditions (strong and weak), and initial imperfection levels, which were found based on different values of a, table 3.3.

The figures in this chapter, figures 4.1 and 4.12, graphically represent the results obtained through the simulations, revealing patterns consistent with the literature. The results point to three parameters as the most relevant in determining the critical temperature: the initial geometric imperfection (L_o), the lateral stiffness of the foundation, and the rail profile. Variations in these elements were responsible for changes greater than 348.71% in the critical values.

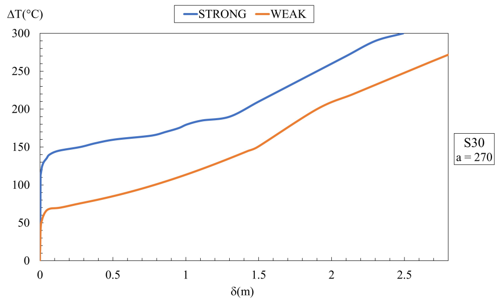

Figure 4.1: S30 buckling curves for imperfection a = 270

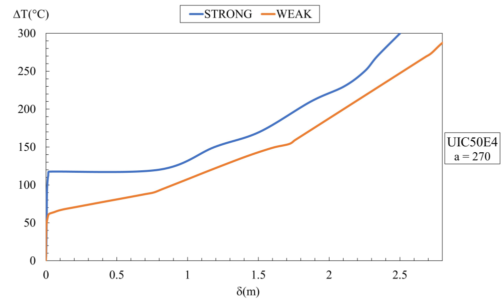

Figure 4.2: UIC50 buckling curves for imperfection a = 270

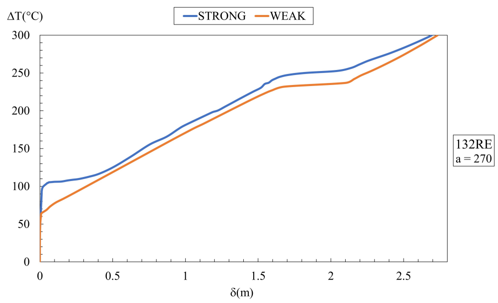

Figure 4.3: 132RE buckling curves for imperfection a = 270

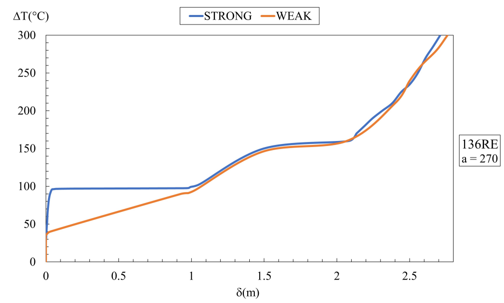

Figure 4.4: 136RE buckling curves for imperfection a = 270

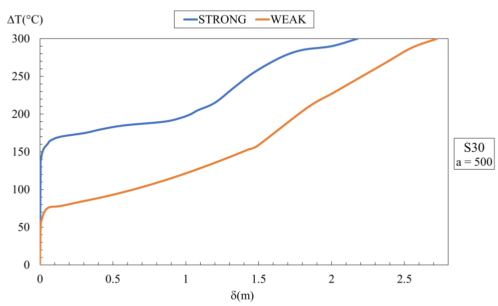

Figure 4.5: S30 buckling curves for imperfection a = 500

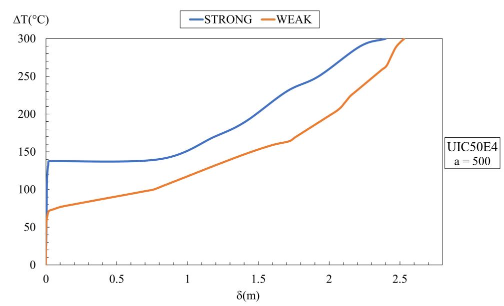

Figure 4.6: UIC50 buckling curves for imperfection a = 500

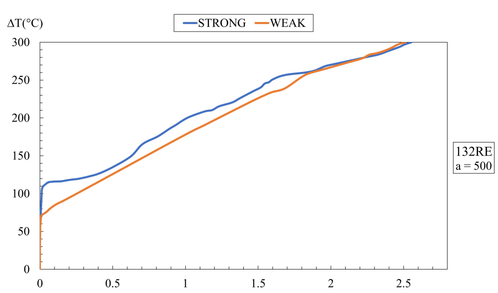

Figure 4.7: 132RE buckling curves for imperfection a = 500

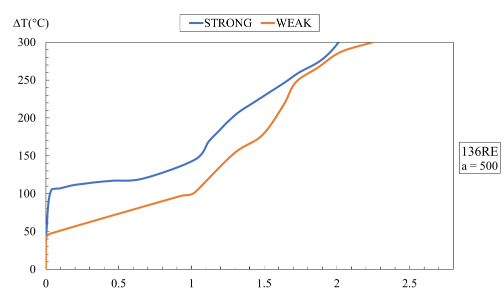

Figure 4.8: 136RE buckling curves for imperfection a = 500

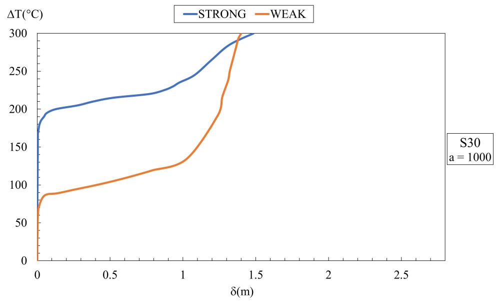

Figure 4.9: S30 buckling curves for imperfection a = 1000

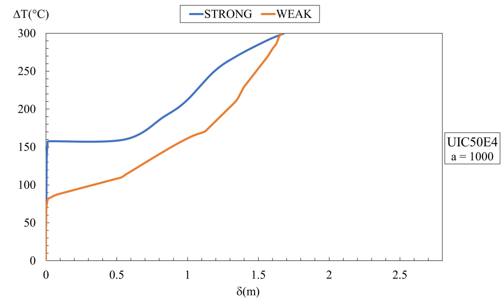

Figure 4.10: UIC50 buckling curves for imperfection a = 1000

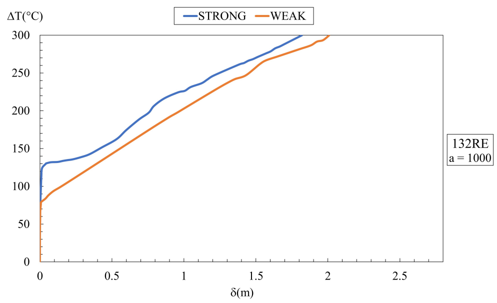

Figure 4.11: 132RE buckling curves for imperfection a = 1000

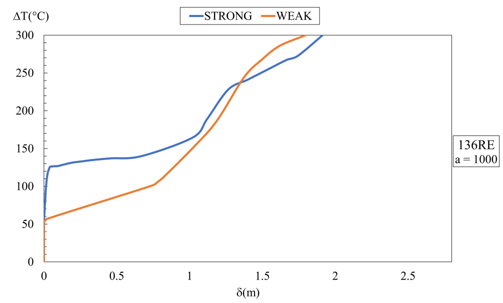

Figure 4.12: 136RE buckling curves for imperfection a = 1000

On rails laid on compacted ballast, which represent a stronger resistance, the critical buckling temperature varied between 96ºC and 141ºC, on railways with imperfections a equal to 270, depending on the rail profile. In situations with degraded or loose ballast, which represent a weaker resistance, the critical delta temperature dropped to values close to 38 ºC.

Another factor analyzed was the type of rail profile. Four profiles were compared: 136RE, 132RE, UIC50, and S30. Table 4.1 shows the average critical temperatures obtained for each one under standard conditions (curved track, strong ballast, and imperfection a = 270).

The comparative analysis between the rail profiles revealed different behaviors in relation to thermal buckling. The S30 profile, despite being the lightest and traditionally used in low-traffic lines, presented the best results in terms of thermal stability, with critical temperatures higher than those observed in the 132RE and 136RE profiles.

Track Configuration	Imperfection L_θ		
	$\alpha = 270$ (reference)	$\alpha = 500$ (variation)	$\alpha = 1000$ (variation)
S30 - Strong Resistance	143 °C (-)	169 °C (18%)	195 °C (36%)
S30 - Weak Resistance	68 °C (-)	77 °C (13%)	88 °C (29%)
UIC50 - Strong Resistance	118 °C (-)	137 °C (16%)	149 °C (26%)
UIC50 - Weak Resistance	62 °C (-)	74 °C (19%)	81 °C (30%)
132RE - Strong Resistance	103 °C (-)	114 °C (10%)	131 °C (27%)
132RE - Weak Resistance	62 °C (-)	71 °C (14%)	83 °C (33%)
136RE - Strong Resistance	97 °C (-)	107 °C (10%)	128 °C (31%)
136RE - Weak Resistance	38 °C (-)	43 °C (13%)	57 °C (50%)

Table 4.1: Imperfection, rail type and ballast effect on the critical buckling temperature.

This seemingly counterintuitive behavior can be attributed to the lower longitudinal stiffness of the S30 profile, which limits the accumulation of compressive stresses sufficient to initiate buckling.

The American 132RE and 136RE profiles, which are more robust and used on high-capacity lines, exhibited buckling at lower temperatures, with reductions of up to 78% compared to the S30, when placed in the same scenario. This vulnerability may be related to their high slenderness and greater thermal energy storage capacity, which, under unfavorable support conditions, favors the occurrence of instabilities.

The European UIC50 profile showed intermediate performance, with critical temperatures slightly higher than those of the American profiles, but still lower than those observed for the S30. This result reinforces the idea that thermal buckling depends not only on the mass or stiffness of the cross section, but on the interaction between the rail, fasteners, and ballast resistance. Thus, the choice of the most suitable profile for a given application must consider not only the load supported, but also the thermal and structural conditions of the track.

The analysis of the effect of initial imperfections was performed by varying the parameter a of the initial deformation function, adopting values of 270, 500, and 1000. As expected, the increase in imperfection resulted in a significant rise in critical temperature, as it simulates a more advanced state of compression. In extreme scenarios, this increase reached 50%, while under good structural conditions, and when placed in the same scenario, the increase reached 10%, suggesting that sensitivity to imperfection depends directly on the overall stiffness of the system.

Another highly relevant factor identified in the analysis was the presence of cant, especially on curved sections, when compared to previous studies [24] [6]. When correctly dimensioned, contributing significantly to the stabilization of the rail against lateral displacement. In practical terms, this results in higher critical buckling temperatures, as the track becomes less susceptible to instability under the effect of thermal expansion. This influence is more significant when combined with favorable structural conditions, such as compacted ballast and fasteners with high torsional rigidity.

In addition, the cant also influences the buckling mode. On curved tracks without superelevation, a higher incidence of progressive buckling was observed in previous studies, as in the Nava study [24], where profiles 132RE and 136RE were also studied on curves, but without superelevation, Table 4.2 compares these values.

Resistance	**Imperfection**	**Profile**	**Previous Studies (°C)**	**Present Study (°C)**	**Variation (%)**
Strong	270	132RE	68	103	+51.47%
	270	136RE	66	97	+46.97%
	500	132RE	98	114	+16.33%
	500	136RE	95	107	+12.63%
	1000	132RE	118	131	+11.02%
	1000	136RE	119	128	+7.56%
Weak	270	132RE	28	62	+121.43%
	270	136RE	28	38	+35.71%
	500	132RE	37	71	+91.89%
	500	136RE	37	43	+16.22%
	1000	132RE	46	83	+80.43%
	1000	136RE	48	57	+18.75%

Table 4.2: Temperature Change with Cant.

However, in configurations with an adequate cant, the rails exhibited more predictable and stable behavior, often with explosive buckling only at very high temperatures. This not only increases the thermal safety margin, but also provides a more controlled structural response, which is essential for preventive maintenance and railway operation planning in regions with large temperature variations. These results reinforce the importance of cant not only as an element of dynamic comfort, but also as a structural safety feature against thermal instability.

In summary, the results obtained show the complexity of thermal buckling and reinforce that its prevention requires multidisciplinary attention involving geometric design, choice of materials, and maintenance inspections. The use of profiles suitable for railways, rigid sleepers, and continuous maintenance of the ballast emerge as fundamental strategies to ensure track stability in scenarios of global warming and increasingly intense thermal variations.

Chapter 5

Conclusion

This study conducted a detailed numerical analysis of the behavior of continuously welded rails subjected to lateral thermal instability, focusing on buckling induced by temperature variations. The modeling was conducted in static conditions, disregarding vertical loads, which allowed isolating the phenomenon of thermal lateral buckling, recognized as one of the most critical modes of failure in modern railways.

The simulations covered different structural conditions, track geometries, and rail types, totaling 24 simulated scenarios. The maximum lateral response was monitored at the point of greatest beam lateral displacement, where buckling tends to occur first. The results obtained demonstrated a high sensitivity of the critical buckling temperature to three main parameters: initial geometric imperfection, lateral stiffness of the foundation, and rail profile. Together, these factors were responsible for variations of more than 340% in the critical values.

Among the four profiles analyzed (136RE, 132RE, UIC50, and S30), the S30 profile, despite being lighter and traditionally used on lines with lower traffic, presented the best thermal performance, reaching higher critical temperatures. This is due to its lower longitudinal stiffness, which reduces the accumulation of compressive stresses, delaying the onset of buckling. In contrast, the more robust profiles (132RE and 136RE), although ideal for heavy loads, showed lower resistance to thermal buckling under unfavorable support conditions, with drops of up to 78% in critical temperatures compared to S30.

The influence of initial imperfections was also significant. The increase in the initial deformation parameter generated significant reductions in the critical temperature, with a more pronounced impact on rails laid on degraded ballast. This reinforces the need to consider actual geometric deviations of the track in stability analyses.

Another noteworthy aspect was the analysis of the cant, which is often ignored in thermal buckling studies. The results confirmed that the cant acts as a stabilizing element, introducing a vertical component that behaves as lateral precompression, which is especially useful in curved sections. Rails with correctly dimensioned cant presented higher critical temperatures and more stable behavior, with explosive buckling only in extreme scenarios.

These findings not only validate the importance of cant as a dynamic comfort feature, but also position it as an essential factor for structural safety in the face of intense thermal variations. Its proper application, combined with the use of compatible rails, rigid foundations, and imperfection control, constitutes a robust strategy for mitigating instability risks on railways, especially in the face of climate change and increasing extreme temperatures.

In summary, this study confirms the multifactorial nature of thermal buckling and highlights the need for integrated approaches between geometric design, component specification, and preventive maintenance. The thermal stability of rails depends not only on the cross-section or weight of the profiles, but also on the complex interaction between the rail, fasteners, sleepers, and support conditions. It was also observed that the total lateral resistance of the sleeper increased, with a proportional increase in the contribution of the ballast, while the base support became less relevant. The adoption of these practices is vital to ensure the safety and durability of railway infrastructure in an increasingly challenging climate scenario.

5.1 Future Works

The results obtained in this study show the importance of cant in the thermal stability of continuous rails, but also point to the need for more extensive analysis. Future studies should explore different cant configurations, including excessive or insufficient values, and their interaction with other structural parameters, such as ballast stiffness and fastening type. Considering appropriate variations in geometry and superelevation along curves, taking into account the requirements of current regulations, may contribute to a more accurate understanding of the combined effects of cant and lateral buckling.

In addition, future simulations should incorporate vertical loads and dynamic forces, which are absent in the present model, to reflect actual operating conditions on the railroad. The introduction of train weight, speed variations, and interaction forces between wheels and rails will allow for the evaluation of possible synergies or conflicts between thermal buckling and operation-induced instabilities. Experimental tests on different scales can validate numerical models, while research into new grades of materials for sleepers and fastenings can optimize thermal response. Different types of steel for rails can be studied. In addition, the use of geometric imperfections based on actual measurements and the investigation of alternative materials for sleepers and fastenings could enrich the model and guide more effective maintenance practices.

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