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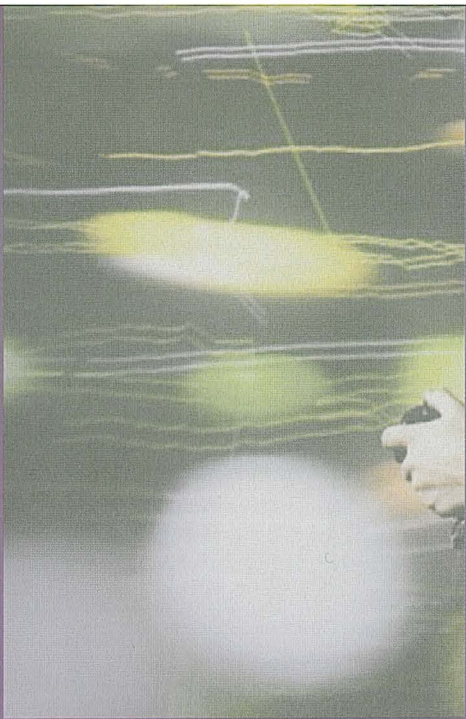

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A Multi-nodal Ring Finite Element for Analysis of Pipe Deflection

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Abstract: *The main objective of this work is to present a numerical formulation to solve the problem of the deformation analysis of thin-walled circular cylindrical pipes under concentrated loads. The solution is based on a displacement field entirely defined from a set of trigonometric functions where the amplitudes are assigned as nodal parameters in a multi-nodal finite element. With this formulation it is possible to provide an easy alternative tool when compared with a complex finite shell or solid element modelling for the same type of applications. The present work permits to examine the deflection of pipe rings subjected to lateral (transverse) static loading conditions. Several case studies presented have been compared and discussed with numerical analyses results obtained with a shell element from Ansys® programme.*

Keywords: *multi-nodal ring; pipe; deflection; trigonometric functions.*

1. INTRODUCTION

Piping structures have high technological importance in different fields of applications, as fluid transport, energy production and chemical processes. Advanced structural applications, as nuclear power production, aeronautics and aerospace industry, must withstand high stress levels and extreme temperature environment, this demanding exigent safety design standard and skilful inspection steps for certification of fitness for the purpose of these types of structures. These piping elements exhibit complex deformations fields given their toroidal geometry and the multiplicity of the configuration of external loads. The performance of piping structures in its ability to support load is typically assessed by the measure of deflection from its initial shape. This characteristic controls the design limitation on flexible structures and helps to identify structural problems associated with other performance criteria.

The study of the flexibility and stress state of piping structures subjected to generalized forces has been an area of interest of many engineers and physicists, given the high interest of the theme in many structural applications. Important innovative contributions for this research was registered by Theodore von Karman [1], who proposed the first really effective solution-based on a Fourier expansion displacement field and a variational method. Vigness [2] generalized the solution and proposed an experimental procedure to verify the derived bending equations. G. Thomson [3] worked with an analytical formulation using doubly-defined trigonometric series functions to develop the displacement field and performed many experimental studies. Öry and Wilczek[4] presented an economical method based on transfer matrix techniques to define the stress and strain calculation. The technique of expanding the displacement and load field trigonometric functions has been followed until the emergence and continuous development of finite elements. The use of trigonometric functions combined with the current algebraic shape functions used in the development of finite elements, in the approach to the solution of problems in structural mechanics has known encouraging contributions [4, 5]. News models used for stress and displacements fields determination, under mechanical or thermal loads, were formulated using numerical techniques with new finite elements [6-12]. Studies on circular cylindrical shells under concentrated forces have been presented by many researchers using curved beam, shell and solid [13, 14]. The particular case of the formulation of cylindrical shells using ring elements has a straightforward contribution from Oñate [15], where this author used a combined formulation for the displacement field dealing with Fourier series along the circumference and algebraic shape functions along the axial direction. Karamanos [16] investigated the instability caused by pressure for the two and three dimensional tube under two opposite concentrated radial loads using a nonlinear finite element tools and experimental validation. The study of the effect of radial loads on cylindrical shells has an important role in the engineering design and

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installation conditions of pressure vessels, having cylindrical shape. Furthermore the response of oil and gas pipelines under transverse load conditions, caused by trawl gears or anchors is also important due the possible catastrophic damage [17], as well as, lateral impact loading on tubular members of offshore platforms. Relevant literature reveals that investigations increased on the study of deformation of tubular sections due to concentrated and uniformly distributed lateral or transverse loads [16-19].

The aim of this study is to develop an alternative numerical method for pipe ring analysis when used in buried sewerage piping or when submitted to mechanical loads generated during the transportation to the plant site or during the installation procedures. The present work extends the alternative formulation, based on a multi-nodal ring finite element with two nodal sections, used to simulate concentrated loads in the referred situations, previous presented and experimentally validated by Fonseca [11]. In this work, several numerical results obtained by ring element will be compared with other shell element from Ansys® programme. The studied cases permits to examine the deflection of pipe rings subjected to transverse static loads. The multi-nodal ring element presents a good accuracy for the displacement field under generalized loads in different structural applications. The displacement calculation at any transverse section is easy, given the simplicity of the involved algorithms in the formulation. The main advantage is associated to the generation of simple meshes with low number of elements, remarkable economy in degrees of freedom and in computational time.

2. A MULTI-NODAL RING FINITE ELEMENT FORMULATION

The basic kinematics assumptions refer to the deformation of a thin shell as used in a small-defection analysis, as referred in [11]. The assumptions in the problem formulation of in-plane bending are: The shell is thin, this meaning that the normal to the shell surface does not distort and the transverse section is inextensible, not including pressure effects. The strain analysis and curvature geometry field of a non-symmetrically deformed cylindrical shell may be founded on the behaviour of plates and shells of revolution. Kinematics expressions relating the midsurface strains are expressed by (1-3) and the curvatures and twist by (4-5).

$$\varepsilon_{SS} = \frac{\partial u}{\partial s},\ \varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial v}{\partial \theta} + w\right) \text{ and } \gamma_{s\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial s} \quad (1\text{-}3)$$

$$\chi_{\theta\theta} = \frac{1}{r^2}\left(-\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2}\right) \text{ and } \chi_{s\theta} = \frac{2}{r}\left(-\frac{\partial v}{\partial s} + \frac{\partial^2 w}{\partial s \partial \theta}\right) \quad (4\text{-}5)$$

where ε_{SS} is the longitudinal membrane strain, $\varepsilon_{\theta\theta}$ is the meridional curvature from ovalization, $\gamma_{s\theta}$ the shear strain, $\chi_{\theta\theta}$ is the meridional curvature variation from ovalization and $\chi_{s\theta}$ the twist variation.

A simplified theory is useful under certain conditions and is applicable to a variety of shells forms. However, we shall deal only with the inextensional deformation of circular cylindrical shells. This theory is often preferred when shell structures resist loading principally through bending action. Such cases include a cylinder subjected to loads without axial symmetry and confined to a small circumferential portion and there is considerable bending caused by changes in curvature, but no stretching of midsurface length. Deformations of these types are thus described as inextensional referred in [20]. In this shell theory, the midsurface in-plane strain components given by equations (1-3) are taken to be zero. We realise from equation (1) that (u) depends on θ and equation (2) leads to:

$$w = -\partial v / \partial \theta \quad (6)$$

The geometric parameters considered for this element definition are: The length of the pipe s, the wall thickness t and the mean section radius of the pipe r. Figure 1 shows the essential parameters and the degrees of freedom used to define the finite element, as thin-walled circular cylindrical pipes.

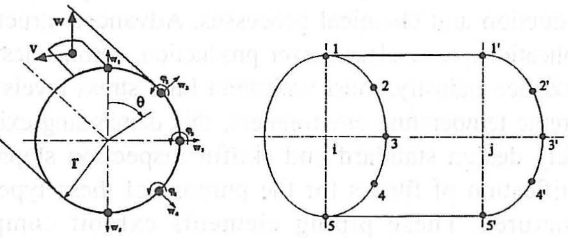

Figure 1: Multi-nodal Ring Finite Element, Nodal Arrangement and Degrees of Freedom

The displacement field proposed characterises a uniform ovalization in the section and a variation along the shell length. To obtain the shape functions a displacement field has been considered in the normal and radial direction.

As represented in figure 1 the element present five nodes under symmetric conditions and a total of eight degrees of freedom per ring section. Consequently, only a half of a ring section is needed to consider when trigono-metric even functions are used. The initial radial displacement used to define the ovalization effect must be calculated using the trigonometric polynomial approxi-mation. A formulation based on trigonometric functions is used and eight parameters are necessary to define the transversal displacement field approximated by equation 7.

$$w(s,\theta)=\sum_{i=2}^{9} a_{i-1}\, i\cos i\theta + S\sum_{i=2}^{9} b_{i-1} i\sin i\theta \qquad (7)$$

Using simple differential equation from beam bending theory the transverse displacement can be calculated:

$$w(s,\theta)=-\sum_{i=2}^{9} a_{i-1}\sin i\theta - S\sum_{i=2}^{9} b_{i-1}\cos i\theta \qquad (8)$$

The rotation field is considered using the derivative function of the transverse displacement field:

$$\varphi(s,\theta)=\frac{1}{r}\frac{\partial w}{\partial\theta}=-\frac{1}{r}\sum_{i=2}^{9} a_{i-1}\, i^2\sin i\theta-\frac{S}{r}\sum_{i=2}^{9} b_{i-1}\, i^2\cos i\theta \qquad (9)$$

The unknown parameters a_i and b_i are determined by imposing boundary conditions according to the ring element section i and j considered, resulting a system of equations to be solved.

The degrees of freedom considered in the proposed multi-nodal ring element are: Node 1 and 5 are the transversal displacement and their derivatives vanish, for nodes 2 at 4 have a radial displacement and one derivative function.

A system with 16 equations is solved for i and j section of the ring element and shapes functions are obtained:

$$\{\delta\}=[B']\{a_i\} \qquad (10)$$

with $\{\delta\}^T=\{W_{1i}\ W_{2i}\ \varphi_{2i}\ W_{3i}\ \varphi_{3i}\ W_{4i}\ \varphi_{4i}\ W_{5i}\ W_{1j}\ W_{2j}\ \varphi_{2j}\ W_{3j}\ \varphi_{3j}\ W_{4j}\ \varphi_{4j}\ W_{5j}\}^T$ which $\{\delta\}$ represents the global displacement field for transversal and rotation degrees of freedom in the semi nodal ring and $[B']$ is constant matrix that results from the imposed boundary conditions.

For node 1 the transversal displacement is equal to one and all others equal to zero. The first shape function appears and is called N_{1i}. With imposed equation (6) we determine the shape function BN_{1i}. The same has been used for all others nodes. The unknown constants $\{a_i\}$ are determined inverting the system equation (10). The first shape function is represented by the equation (11). With these conditions, a new shape functions are determined and the generic local displacements field for in-plane finite element formulation are given by equations (12) and (13).

$$N_{1i}=\left[\frac{3}{16}\cos(2\theta)+\frac{5}{12}\cos(3\theta)+\frac{1}{8}\cos(4\theta)+\frac{3}{32}\cos(5\theta)+\frac{1}{16}\cos(6\theta)+\frac{9}{64}\cos(7\theta)+\frac{1}{8}\cos(8\theta)+\frac{7}{64}\cos(9\theta)\right]- \qquad (11)$$

$$-s\left[\frac{3}{16}\frac{\cos(2\theta)}{L}+\frac{5}{32}\frac{\cos(3\theta)}{L}+\frac{1}{8}\frac{\cos(4\theta)}{L}+\frac{3}{32}\frac{\cos(5\theta)}{L}+\frac{1}{16}\frac{\cos(6\theta)}{L}+\frac{9}{64}\frac{\cos(7\theta)}{L}+\frac{1}{8}\frac{\cos(8\theta)}{L}+\frac{7}{64}\frac{\cos(9\theta)}{L}\right]$$

$$v(s,\theta)=BN_{1i}W_{1i}+BN_{2i}W_{2i}+BN'_{2i}\varphi_{2i}+BN_{3i}W_{3i}+BN'_{3i}\varphi_{3i}+BN_{4i}+BN'_{4i}\varphi_{4i}+BN_{5i}W_{5i}+ \qquad (12)$$
$$+BN_{1j}W_{1j}+BN_{2j}W_{2j}+BN'_{2j}\varphi_{2j}+BN_{3j}W_{3j}+BN'_{3j}\varphi_{3j}+BN_{4j}W_{4j}+BN'_{4j}\varphi_{4j}+BN_{5j}W_{5j}$$

$$w(s,\theta)=N_{1i}W_{1i}+N_{2i}W_{2i}+N'_{2i}\varphi_{2i}+N_{3i}W_{3i}+N'_{3i}\varphi_{3i}+N_{4i}W_{4i}+N'_{4i}\varphi_{4i}+N_{5i}W_{5i}+ \qquad (13)$$
$$+N_{1j}W_{1j}+N_{2j}W_{2j}+N'_{2j}\varphi_{2j}+N_{3j}W_{3j}+N'_{3j}\varphi_{3j}+N_{4j}W_{4j}+N'_{4j}\varphi_{4j}+N_{5j}W_{5j}$$

All other shape functions are determined in the same form. This formulation presents a simple formula for calculation of the displacement field for the tangential (v) and transversal (w) displacement under a shell element:

$$\{v\ w\}^T=[N]\times\{\delta\} \qquad (14)$$

As referred previously, the mechanical deformation model considers that the pipe undergoes a semi-membrane strain field by the equations (1-5).

The typical use of the principle of virtual work leads to the system of algebraic equations. Having solved the system of algebraic equations, the displacement field is obtained for all the nodes of the multi-nodal ring finite element.

3. PIPE RING ANALYSIS DUE CONCENTRATED LOADS

Concentrated loads can cause a pipe deflection, with bending or twisting of the pipe surface. This is an important item included in the engineering design procedures. The problem presented here, refers to a ring structure with rigid ends, subjected to concentrated load, figure 2. The load is equal (P = 1000 kN) and it is applied at the middle (case 1) or at the end of the pipe length (case 2).

The material is assumed to be isotropic. For material properties the elastic modulus was taken (E = 200GPa), Poisson's ratio (ν = 0.3) and yield stress (σ_y = 560MPa). Different geometric configurations are considered in all pipe rings studied and defined in table 1. For each one two different lengths were considered (L = 200, 400 mm). In this study the ring flexibility in all cases is equal to $D_m/t \leq 95$.

A total of 24 numerical simulations were obtained using the multi-nodal ring finite element represented in figure 1 for different studied cases. Table 2 represent the results of the vertical displacement using different meshes for multi-nodal ring element. This finite element performs well even in the coarse mesh configurations.

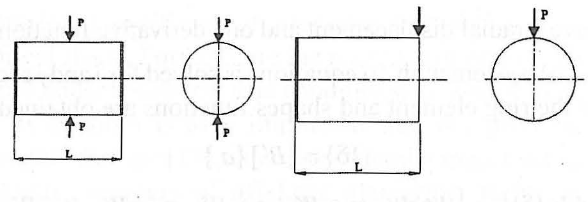

Figure 2: Case Study 1 and 2

Table 1
Pipe Dimensions

Mean Radius(R_m), [mm]	*Wall Thickness (t), [mm]*	*t/Rm*
	3.35	0.021
159.945	3.96	0.025
	4.57	0.029

Table 2
Vertical Displacement with Different Multi-nodal Ring Meshes, δ [*m*]

Cases of study	*Length*	*Number of elements*	*$t/R_m = 0.021$*	*$t/R_m = 0.025$*	*$t/R_m = 0.029$*
		4	0.0027	0.0016	0.0011
	200mm	2	0.0027	0.0017	0.0011
Case 1		8	0.0049	0.0030	0.0019
	400mm	4	0.0049	0.0030	0.0019
		4	0.0140	0.0087	0.0057
	200mm	2	0.0150	0.0088	0.0057
Case 2		8	0.0230	0.0140	0.0090
	400mm	4	0.0230	0.0140	0.0090

Also of 12 numerical simulations are presented using Ansys®, and a finite Shell 63 element was used, with 4 nodes and 6 degrees of freedom per each node: translations in the *x*, *y*, and *z* nodal directions and rotations. The deformed shapes are linear in both in-plane directions. Whenever applicable, boundary conditions for symmetry were used due the shell geometry and the load system. Different meshes were used in Ansys® programme, 16 and 32 elements when pipe length is equal to 200 or 400mm, respectively. With multi-nodal ring, 4 and 8 elements were used for the same conditions. The maximum displacement obtained with the multi-nodal ring element and shell element for each study case is represented in table 3.

In all studied cases the results agree when shell thickness is thin or moderated. Figure 3 and figure 4 represent the comparison of the maximum vertical displacement between the multi-nodal ring and shell element.

Table 3
Comparison of Vertical Displacement, δ [*m*]

Cases of study	*Length*	*$t/R_m = 0.021$*		*$t/R_m = 0.025$*		*$t/R_m = 0.029$*	
		Multi-nodal	*Shell*	*Multi-nodal*	*Shell*	*Multi-nodal*	*Shell*
	200mm	0.0027	0.0021	0.0016	0.0017	0.0011	0.0015
Case 1	400mm	0.0049	0.0041	0.0030	0.0034	0.0019	0.0028
	200mm	0.0140	0.0151	0.0087	0.0079	0.0057	0.0058
Case 2	400mm	0.0230	0.0242	0.0140	0.0151	0.0090	0.0119

In all simulations the vertical displacement are larger when the pipe length increases. Case 2 refers to a ring pipe solution with more deflection. The ring deflection is smaller in all simulations, when the pipe thickness increases. The comparison between multi-nodal ring element and shell element from the finite element code gave similar results. Figure 5 shows the response of all studied cases and considering different lengths, using normalized values for applied load (*p*) and obtained displacement (*x*).

The applied load value *P* is normalized using the following equation:

$$P_* = \sigma_y \frac{t^2}{4}\sqrt{\frac{D_m}{t}} \quad (p = P/P_*) \tag{15}$$

The vertical displacement δ is normalized with respect to the ring pipe radius $R_m (Y = \delta / R_m)$.

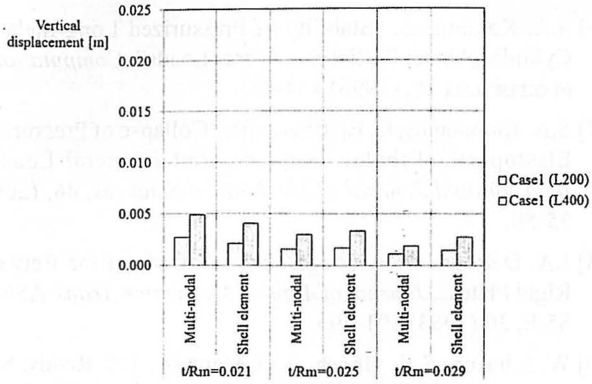

Figure 3: Maximum Displacement δ, case 1.

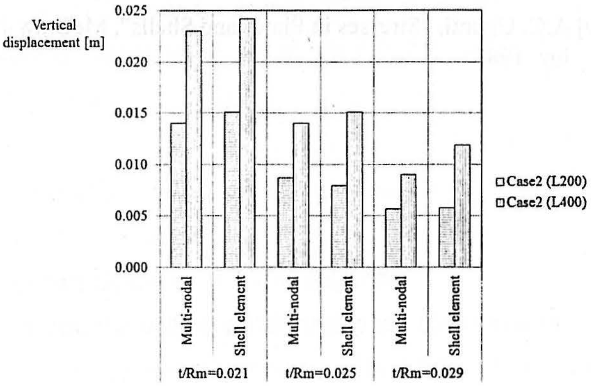

Figure 4: Maximum Displacement δ, case 2.

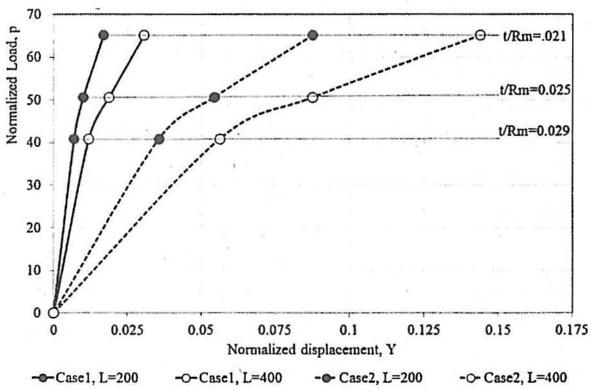

Figure 5: Normalized Load and Displacement, Case 1 and Case 2.

The behaviour of all paths is similar, studied load case 2 reaches greater values of deflection when compared with load case 1. Also, when increased length and decreasing the ratio between thickness and radius, the pipe deflection is higher.

4. CONCLUSION

The presented formulation using a multi-nodal ring element under symmetric conditions makes possible to determine a displacement field under shell surface for in-plane bending in thin-walled cylindrical pipes. The multi-nodal ring element exhibits excellent behaviour in linear bending conditions like shell elements. The numerical results to validate the accuracy of the multi-nodal ring element have shown a good agreement when compared with similar analysis using Ansys®. The presented solution has performed well even with coarse element meshes, and appears as a simple and easy-to-handle alternative to the use of shell elements. In studied cases the vertical displacement are larger when the pipe length increases and the ring deflection is smaller when the pipe thickness increases.

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