MODELING AN AGGREGATE PRODUCTION PLANNING

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BACKGROUND

Production planning is an essential and complex activity inside any company which requires simultaneous cooperation between everyone responsible for the decision-making process [1]. Most of the time, small companies cannot afford to customize, implement and train people to use an Advanced Planning and Scheduling (APS) software that is available in the market. Thus, one feasible solution for these companies is to develop their own simplified tools to support the decision-making process [2]. When solving problems involving medium-term planning, more specifically aggregate production planning, it is possible to create an integer linear programming (LP) mathematical model and use it to find the optimal solution [3].

OBJECTIVES

The main objective of this work is to present and develop an integer linear programming mathematical model for aggregate production planning, which is included in the medium-range plans, considering a case study of a real company.

Additionally, this work aims to show that a simple tool based on integer linear programming is very effective for elaborating aggregate production planning, especially considering small companies with limited budgets, proving that they can create their own decision support system.

METHODOLOGY

Integer linear programming is a method that uses a set of mathematical linear functions to describe a certain problem and then solve it to obtain an optimal result expressed in integer numbers [4].

In this proposed model, the objective function is the profit P, which depends on the quantities X_{ij} of product i processed in machine j, times its margin of profit C_i, given in euros. Since the objective function represents the profit, one wants it to be maximized.

To limit the values that X_{ij} can assume, a set of constraints have to be determined. In this model, it is considered three constraints: (i) the available raw material M_{max}, given in total units capable of being produced; (ii) the usage of all machines T_j during the period, given in amount of work shifts needed, based on the average quantity T_{ij} of products i manufactured per shift when processed in the machine j; (iii) the demand for every product D_i, given in units.

The case study considered is about a company that fits the scenario summarized in Table 1.

Table 1. Company's simplified scenario

Working days	Shifts/day	Nº of Products	Nº of Machines
65	2	5	3

Each product is manufactured at a different rate, depending on the machine, as shown in Table 2.

Table 2. Production distribution between products and machines

Product	Machine 01	Machine 02	Machine 03
Model 01	10 units/shift	Not produced	18 units/shift
Model 02	15 units/shift	Not produced	21 units/shift
Model 03	20 units/shift	Not produced	18 units/shift
Model 04	Not produced	28 units/shift	33 units/shift
Model 05	Not produced	19 units/shift	21 units/shift

The company knows the demand and the profit margin for each product, as presented in Table 3.

Table 3. Demand and profit margin per product

Product	Demand	Profit per unit
Model 01	1500 units	€ 23,00
Model 02	1300 units	€ 22,00
Model 03	700 units	€ 21,00
Model 04	950 units	€ 19,00
Model 05	1500 units	€ 20,00

Finally, after considering the product distribution, the integer linear programming model can be written as follows.

$$\text{Maximize } P = \sum_{i=1}^{5}\sum_{j=1}^{3} C_i X_{ij}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{i=1}^{5}\sum_{j=1}^{3} X_{ij} \le M_{max} \\
& \frac{X_{11}}{T_{11}} + \frac{X_{21}}{T_{21}} + \frac{X_{31}}{T_{31}} \le T_1 \\
& \frac{X_{42}}{T_{42}} + \frac{X_{52}}{T_{52}} \le T_2 \\
& \frac{X_{13}}{T_{13}} + \frac{X_{23}}{T_{23}} + \frac{X_{33}}{T_{33}} + \frac{X_{43}}{T_{43}} + \frac{X_{53}}{T_{53}} \le T_3 \\
& X_{11} + X_{13} \ge D_1 \\
& X_{21} + X_{23} \ge D_2 \\
& X_{31} + X_{33} \ge D_3 \\
& X_{42} + X_{43} \ge D_4 \\
& X_{52} + X_{53} \ge D_5 \\
& X_{12} = X_{22} = X_{32} = X_{41} = X_{51} = 0 \\
& X_{ij} \in Z | X_{ij} \ge 0
\end{aligned}$$

RESULTS AND CONCLUSIONS

First of all, it is important to state that this model is oversimplified. In reality, much more variables must be taken into consideration, such as inventory management; raw material storage and logistics; employee costs; among others.

When involving up to 8000 variables, an integer programming problem can be solved through Excel using the Solver add-on. Therefore, since the model described is relatively simple, it was imported into Excel and then solved. The results are shown in Figure 1.

Aggregate Planning					
	Product 1	Product 2	Product 3	Product 4	Product 5
Machine 01:	0	843	1476	-	-
Machine 02:	-	-	-	950	1825
Machine 03:	1948	457	0	0	0
Sum:	1948	1300	1476	950	1825
Demand:	1500	1300	700	950	1500
Surplus:	448	0	776	0	325

Fig. 1. Results of the integer programming model

As can be seen, the maximum profit possible is € 158.950,00, which corresponds to € 37.735.50 a month. All machines are considered to operate almost at their maximum, once their usage is very close to 100%. The only goods that have been manufactured in surplus are products 1, 3, and 5, with 448, 776, and 325 surplus units, respectively. This was made to maximize the usage of every machine because the demand can be satisfied in less than 65 working days; therefore, those surplus items are going to be stored. The fact that only products 1, 3, and 5 are being produced in surplus is because the model identified that in such a way, the profit function is maximized.

It can be concluded that the model satisfies the optimization premise, since, after supplying the demand, the model seeks solutions in which the surplus products are more profitable, and then maximizes their production to ensure that no machine is idle. Furthermore, for all machines that produce the same product at a different rate, it can be seen from the results that the model allocated most of the production of a given product to the most efficient machine.

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IPCA INSTITUTO POLITÉCNICO DO CÁVADO E DO AVE

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2Ai APPLIED ARTIFICIAL INTELLIGENCE LABORATORY

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