Hierarchization Process by Possibilistic
Fuzzy Clustering of Fuzzy Rules

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Abstract
This paper presents a possibilistic fuzzy clustering algorithm that is applied to a multidimensional fuzzy set or fuzzy rules. This method can be used to decompose the fuzzy system into an hierarchical structure. The methodology presented leads to a fuzzy partition of the fuzzy rules, one for each cluster, which corresponds to a new set of fuzzy sub-systems. This technique is tested to organize the fuzzy model into a new and more comprehensive structure.

Fuzzy Clustering of Fuzzy Rules

Step 1 – Sets: X={x₁, ..., xₙ} of points; ℋ={R₁, ..., Rₘ} of rules with relevance rᵢ(xₖ), c is the number of clusters. Initialize U(0).

Step 2 – On the rᵗʰ iteration, with r = 0, 1, 2, ..., compute:
ψᵢ⁽ⁿ⁾ = \( \frac{\sum_{k=1}^{M} wᵢ^{n} \sum_{i=1}^{n} rᵢ(xₖ) xₖ}{\sum_{i=1}^{M} wᵢ^{n} \sum_{k=1}^{n} rᵢ(xₖ)} \)

Step 3 – Compute the new partition matrix U(r+1):
\( uᵢ^{n+1} = \frac{1}{1 + \left( \sum_{k=1}^{n} rᵢ(xₖ) Dₙ \right) / \eta} \) \( rᵢ(xₖ) \)

where \( Dₙ = \| xᵢ - vᵢ \| + \alpha \| \beta - \sum_{j=1}^{c} wᵢ^{j} zᵢ, (xₖ) \| \).

Step 4 – Compute the clusters offsets, h⁽ⁿ⁾:
\( hᵢ⁽ⁿ⁾ = - \sum_{j=1}^{c} \sum_{k=1}^{n} wᵢ^{j} rᵢ(xₖ) zᵢ, (xₖ) / \sum_{i=1}^{M} wᵢ^{n} rᵢ \) \( i=1, ..., c \) \( (6) \)

Step 5 – Compute the new partition matrix W(r+1):
For each cluster i minimizes:
\( Wᵢ⁽ⁿ+1⁾ = \text{Min} \sum_{k=1}^{n} \left[ rᵢ(xₖ) - hᵢ⁽ⁿ+1⁾ - \sum_{j=1}^{c} wᵢ^{j} zᵢ, (xₖ) \right] ^{2} \) \( i=1, ..., c \) \( (7) \)

where \( Qᵢ^{n} \) is the weigh matrix of the iᵗʰ cluster problem with non-null elements \( Qᵢ^{n} = uᵢ^{n} rᵢ(xₖ) \).

Step 6 – If \( \| U(r+1) - U(r) \| < \varepsilon \) then the process ends. Otherwise let r = r + 1 and go to step 2.

Hierarchization Priority Structure (HPS)