GREENHOUSE AIR TEMPERATURE OPTIMAL FUZZY CONTROLLER

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Abstract: A new scheme of fuzzy optimal control for the temperature of an Agriculture Greenhouse is presented. The proposed method is based on the Pontryagin’s Minimum Principle (PMP) that is used to train an adaptive fuzzy inference system to estimate values for the optimal co-state variables. This work shows that it is possible to successfully control a greenhouse by using these techniques. A method is presented to control the greenhouse air temperature achieving significant energy savings by minimizing a quadratic performance index selected for the desired operating conditions. This approach allows finding a solution to the optimal control problem on-line by training the system, which can be used on a closed-loop control strategy. Successful simulations results for the controlled system are presented.

Keywords: Control, Optimal, Fuzzy

1. INTRODUCTION

The agricultural crop sector is being rapidly transformed into an industry of major importance that must rely heavily on advanced crop management techniques and intelligent control systems, essential components of the new generation of plant factories. One of present challenges in this industrial sector is about the need for better management, improved product quality and reduced production cost. Intensive agriculture is proved to be energy inefficient and unsustainable in long run if not managed efficiently. The main aim is increasing the yield per unit of land and labour up to optimum value. The use of standard control strategies was not capable to supply the tools for the improvement of this reality. The proliferation of new computational techniques, based on Computational Intelligence Techniques, provides new solutions that made this objective realizable, with new software with increased functionalities. A wealth of research efforts is being focused today on providing intelligent computer-based management systems, driven by optimal or sub-optimal methods. The climate greenhouse control in order to improve the development of a specific cultivation and to minimize the production costs is becoming increasingly important for the growers. So, greenhouse crop production management systems are becoming increasingly sophisticated and are using many of the advanced methodologies and tools of industrial automation, modern control theory and Computational Intelligence (CI). The knowledge components, necessary to deploy CI in crop production, include a variety of forms such as models, fuzzy reasoning, evolutionary algorithms as well as implementation platforms such as networked and robotic systems. Computational Intelligence, also known as Artificial Intelligence (AI), is the science that attempts to replicate human intelligence on computers. Both procedural and knowledge-based (declarative) programmings are used to perform tasks normally done by human experts or consultants. To meet the increasing complexity of agricultural systems, it is essential to address the issues of their management with increasingly sophisticated methodologies. This has led to the promising field of Precision Agriculture, where the goal is to improve the efficiency of operation as

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well as the quality and consistency of products by compensating the vagueness and uncertainty of the environment. This objective can only be achieved by applying advanced information and optimal control technologies to production management of the greenhouse processes. Moreover, the possibility of incorporation of linguistic information, supplied from an expert, into optimal control learning methodologies is a gratefully task. During the last decades, many control strategies based on classical and modern control have been tested to control those systems but, unfortunately, with limited results. Interactions between the internal and external variables and the complexity of the phenomena (multivariable, nonlinear, no stationary) are such that it is often difficult to implement the conventional techniques of regulation. Moreover, these methods induce choices to simplify assumptions on the models, incomplete measurements of physical variables, disturbances on the parameters, atmospheric interference, and climates sub processes which are not envisaged in the model. To achieve efficient production in greenhouses it is necessary to employ optimal control strategies to adjust the growth conditions in a way that high and good quality yields are generated at low expense and low environmental load. In this way, several climate variables, such as the air temperature, humidity and CO₂ concentration, must be optimally controlled by actuating on the heating, the ventilation and the CO₂ injection systems according to specified criteria’s. The structure and parameters of this control system must be found from a dynamic optimization problem that has been formulated as a minimize (or maximize) problem, expressing trough an explicit desired optimal criterion and taking in account the physical and physiological limitations involved in the processes. Modern theory of optimal control is based on two principal results: Pontryagin Maximum (or Minimum) Principle (PMP) and Bellman’s method of dynamical programming. However, It is well known that exact analytical solutions exists only for specific classes of optimal control problems, such as, linear systems with quadratic integral cost function. But so far, few results can provide an effective way of optimal control design for general nonlinear systems. Actually fuzzy logic represents one of the most important techniques dealing with nonlinearities. Additionally, the fuzzy inference systems emerged as one of the most useful approaches to collect human knowledge and expertise on control and to transform the collected knowledge into a basis for developing controllers (Chuen-Lee, 1990a; Chuen-Lee, 1990b; Wang, 1997). We present an alternative approach to nonlinear optimal control based on a fuzzy logic system and on PMP. According to Pontryagin’s Minimum Principle the co-state variables play a key role in finding the optimal control. However, looking at the problem on a fuzzy logic ground, the co-state variables appears to behave like the output of an expert system that knows which sequence of values will minimize the cost function. In this approach, the (adaptive) fuzzy inference system can be used to generate actuator values, but its primary function is to generate estimates of the co-state variables. A number of stable and optimal fuzzy controllers were developed for linear systems by using the PMP with quadratic cost function. Wang (Wang, 1998) developed the optimal fuzzy controller for linear time-invariant systems. Based on the conventional linear quadratic optimal control theory, Wu and Lin (Wu and Lin, 2000a) presented a design method of the optimal controllers for continuous and discrete-time fuzzy systems. Later, Wu and Lin (Wu and Lin, 2000a) developed a design scheme of the optimal fuzzy controller under finite or infinite-horizon by using the calculus-of-variation method. Moreover, Wu and Lin (Wu and Lin, 2000a; Wu and Lin, 2000b) presented local and global approaches of optimal and stable fuzzy controller design methods for both continuous and discrete-time fuzzy systems under both finite and infinite horizons by applying traditional linear optimal control theory. This study considers the application of optimal control strategy to the air temperature inside a greenhouse. The purpose of this work is to implement an optimal control algorithm, using the learning optimal strategy described in more detail in (P. Salgado, 2007). In this article, a Fuzzy Model (FM) is used as the nonlinear controller of the co-state variables of the optimal control problem. The FM is trained to directly minimize the performance index subjected to plant outputs, states and inputs. The optimization is carried out using a gradient scheme that is computed employing the recently developed concept of convergence of state and co-state optimal trajectories.

2. THE FUZZY INFERENCE SYSTEM

Several structures and learning algorithms are capable of implementing fuzzy inference engine and can be used as co-state variable controller. Without any loss of generalization, the used fuzzy system results from the following algorithm.

Consider a system \( y = f(x) \), where \( y \) is the output variable and \( x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n \) is the input vector. Let \( U = [\alpha_1, \beta_1] \times \cdots \times [\alpha_n, \beta_n] \) be the domain of input vector \( x \). The problem to solve is the following: consider the input-output data pairs \( (x_k, y_k(i)) \), \( k = 1, 2, \cdots, n_p \), where \( x_k \in U \) and \( y_k(i) \in V = \mathbb{R} \) is the derivative value, with \( i = 1, 2, \cdots, r \). This data is assumed to be generated by an unknown nonlinear function \( y = f(x) \) and our objective is to design a fuzzy system \( g(x) \) based on these input-output pairs that approximates the unknown function \( f(x) \). Typically, the expert knowledge expressed in a verbal form is translated into a collection of if-then rules of the type:
functions. The fuzzy rule antecedent space is partitioned by a grid of triangular membership sets of the training data where the membership functions characterized, respectively, by fuzzy membership functions \( A_i^j \) and \( C_{i_1,\ldots,i_n} \), and the index set is defined by:

\[
I = \{i_1, i_2, \ldots, i_n | i_j = 1, 2, \ldots, N_j\},
\]

with \( j = 1, 2, \ldots, n \). Fuzzy system \( g(x) \) is constructed following the steps:

**Step 1:** Partition of the input space - For each \( j, j = 1, \ldots, n \) define \( N_j \) fuzzy sets in \( [\alpha_j, \beta_j] \) using the following triangular membership functions:

\[
A_i^j (x) = \mu (x; a_j^i, b_j^i, c_j^i), r = 1, \ldots, N_j,
\]

\[
\alpha_j^i = x_j^i < \ldots < c_j^i = \beta_j^i, \mu (x; a_j^i, b_j^i, c_j^i)
\]

a triangular membership given by \( \mu (x; a_j^i, b_j^i, c_j^i) = (x-a)/(b-a) \), for \( a \leq x \leq b \); \( \mu (x; a_j^i, b_j^i, c_j^i) = (c-x)/(c-b) \), for \( b \leq x \leq c \); 0 otherwise and \( a < b < c \). After completing this task, the domain space is partitioned by a grid of triangular membership functions. The fuzzy rule antecedent \( R_{i_1,\ldots,i_n} \) can be viewed as the fuzzy set \( A_{i_1,\ldots,i_n} \times A_{i_1,\ldots,i_n} \in U \), with membership functions \( A_{i_1,\ldots,i_n} \times A_{i_1,\ldots,i_n} \in A_{i_1} \times \cdots \times A_{i_n} \). \( g(x) \) is the min or product \( T \)-norm operator and \( i_1, \ldots, i_n \in I \).

**Step 2:** Learning of the rule base - For each antecedent with index \( i_1, \ldots, i_n \in I \), find the subsets of the training data where the membership function \( A_{i_1,\ldots,i_n} \times A_{i_1,\ldots,i_n} \) is not null. If the number of points found is not zero, then rule \( R_{i_1,\ldots,i_n} \) is added to the rule base, represented by a table of indexes: \( RB = \{i_1, \ldots, i_n \in I : A_{i_1,\ldots,i_n} \times A_{i_1,\ldots,i_n} > 0\} \), with \( k = 1, \ldots, n_p \).

**Step 3:** The fuzzy system - Here it is assumed to use the singleton fuzzifier, the product inference engine and the centre-average defuzzifier. The fuzzy system can thus be represented by:

\[
g (x, \theta) = p^T (x) \cdot \theta,
\]

where \( p(x) = [p_1(x), \ldots, p_M(x)]^T \) and \( \theta = [\theta_1, \ldots, \theta_M]^T \) are vectors of the fuzzy basis functions (FBF’s) and the constant consequent constituents, respectively. \( \theta_i \) is the point in \( P \) at which \( C_i \) \((i)\) achieves its maximum value and \( l \in RB \) is the index of the rule. Each fuzzy basis function (FBF) of the fuzzy system is given by \( p_i (x) = A_i (x) / \sum_{l=1}^M A_l (x) \).

There are three main reasons for using the FM described as a basic building block for adaptive fuzzy controllers or identification systems:

- These fuzzy logic systems are constructed from fuzzy IF-THEN rules using specific fuzzy inference, fuzzification, and defuzzification strategies, which allow the incorporation of information from human experts into controllers. In optimal control context, this information can be expressed through the co-state variable of the optimization process:

- It has been showed in other works (Ying, 1994; J. A. Dickerson, 1996; Castro and Delgado, 1996; Wang and Mendel, 1992) that fuzzy logic systems are universal function approximators;

- The structure and the parameters of FM described by equation 3, will be adapted based on the training information. Various well established structural and parametric learning methods could be used to create and adjust the parameters of the membership functions of the rules (Ying, 1994) in order to make this adaptation.

### 3. THE OPTIMAL CONTROL ALGORITHM

Consider the nonlinear discrete dynamic system as:

\[
x_{k+1} = g (x_k) + h (x_k) u_k,
\]

where \( g : \mathbb{R}^n \to \mathbb{R}^n \) and \( h : \mathbb{R}^n \to \mathbb{R}^n \) are continuous over \( \mathbb{R}^n \). Assume that \( g_k + h_k u_k \) is Lipschitz continuous on a set \( U \subseteq \mathbb{R}^n \) containing the origin, and that system is stabilized in the sense that there exists a continuous control on \( U \) that asymptotically stabilizes the system. It is desired to find a sequence of \( u_k \), which minimizes the cost function:

\[
J = \Phi (N, x_N) + \sum_{k=1}^{N-1} L_k^k (x_k, u_k),
\]

where

\[
L_k^k (x_k, u_k) = (x_k - r_k)^T R_k (x_k - r_k) + u_k^T Q_k u_k,
\]

and

\[
\Phi (N, x_N) = (x_N - r_N)^T R_N (x_N - r_N).
\]

The desired state at \( k \) sample, \( R_k \) and \( Q_k \) are matrices that allow to weight attainment of the desired state versus control effort. The control problem is to find the control \( u^*_k \) sequence that minimizes the above criterion or cost function \( J \). In interval \([1, N]\) is the prescribed control time interval, \( \Phi (N, x_N) \) is the cost on the final state value \( x_N \), and \( L_k^k (x_k, u_k) \) is the cost on both state and command at instant \( k < N \). The solution for this problem given by PMP is as follows. One defines the sequence of Hamiltonian functions \( H^k \):

\[
H^k = L^k + \lambda^T_{k+1} f^k,
\]

where \( \lambda_k \in \mathbb{R}^n \) is a vector of Lagrange multipliers. Accordingly to common usage one will designate \( \lambda_k \) as the co-state variables. The optimal sequence \( u^*_k \)
that minimizes the criterion of equation 5 is found by solving simultaneously the following equations:

\[ \lambda_k = A_k \lambda_{k+1} + R_k (x_k - r_k), \]

\[ u_k = -Q_k^{-1} B_k \lambda_{k+1}, \]

\[ \lambda_N = R_N (x_N - r_N), \]

where \( A_k = (\partial f^k / \partial x_k)^T \) and \( B_k = (\partial f^k / \partial u_k)^T \).

The equation 10 may be taken as an optimal feedback control law, if the optimal value of the co-state variable, \( \lambda_{k+1}^* \), is known at time \( k \). Now the approach proposed in this paper may be made explicit. One takes \( \lambda_{k+1} \) as the output of a fuzzy inference system \( \mathbf{A} \) that at instant \( k \) generates an estimate of \( \lambda_{k+1}^* \), having as inputs the observed state \( x_k \) and the time to go \( N - k \): \( \lambda_{k+1} \approx \lambda_{k+1}^* = \mathbf{A}(x_k, N - k) \). This gives the feedback control law \( u_k = -Q_k^{-1} h(x_k) \mathbf{A}(x_k, N - k) \), which by incorporation of the \( h \) function into the fuzzy inference system can be streamlined to:

\[ u_k = -Q_k^{-1} \Lambda h(x_k, N - k), \]

From equation 4 and 10 we have:

\[ x_{k+1} = g_k - H_k \lambda_{k+1}, \]

where \( H_k = h(x_k)Q_k^{-1} [h(x_k) \mathbf{T} \mathbf{g}(x_k)] \).

In general, finding solutions is not an easy task due to the equations interdependence, which implies the use of forward and backward time sequences. If, by adaptation or learning through fuzzy inference system, along successive runs or training iterations of the system from \( k = 0 \) to \( N \), the estimates converge to the optimal ones, then any of the control laws becomes optimal. Additionally to this problem, we are confronted in many real applications with limitations in the actuators values and in the measuring sensors. Generally, the admissible values of actuations are bounded:

\[ u_{\text{min}} \leq u_k \leq u_{\text{max}}, \]

When this happens, the restriction defined on equation 14 must be taken in account in the formulation of the optimal problem by addition of a restriction equation. Alternatively, we adopt a weight to penalize the cost function when the value of actuations violates the limiters, by using a more strong penalized matrix \( \tilde{Q}_k > Q_k \). So, in these cases of violations of restriction of actuators we have:

\[ u_k = -\tilde{Q}_k^{-1} h(x_k) \lambda_{k+1}, \]

In this circumstance the cost function of equation 6 is rewrited as:

\[ L^k (x_k, u_k) = (x_k - r_k)^T R_k (x_k - r_k) + \lambda_{k+1}^T \cdot (h^T(x_k)Q_k^{-1} h(x_k)) \lambda_{k+1}, \]

To solve the set of equations resulting from framing a discrete optimal control problem under PMP, an off-line optimization method is usually applied. Here, one proposes a learning algorithm based on an approximate gradient descent method that, during the training iterations, progressively refines the accuracy of the co-state fuzzy estimator. This strategy reduces the necessary computing time and memory, avoiding the calculations of the exact adjoint and the directional derivatives of the cost functional. Below the first algorithm of this method is given in the theoretical form. The implementation of the algorithm will be described in future work. From equation 11 it follows that for optimal trajectories one must necessarily have \( \lambda_N = R_N (x_N - r_N) \) so \( \lambda_N^* = R_N (x_N^* - r_N) \). Let \( E = \lambda_N - R_N (x_N - r_N) \) be the error or difference between the end value of the state and the co-state variable trajectories. As noted above, for optimal trajectories \( x^*(k) \) and \( \lambda^*(k) \), it is a necessary condition that \( \lambda_N^* = R_N (x_N^* - r_N) \) or \( E = 0 \). It is also possible to prove that this is a sufficient condition. If \( E \to 0 \), then \( x_k \to x_k^* \) and \( \lambda_k \to \lambda_k^* \), i.e. the trajectories of the state and co-state variables converge to the optimal ones. It follows that to attain optimal state trajectories, it is necessary that the error \( E \) converge to zero. This objective is achieved by adjusting the final \( \lambda_N \) co-state variables in order to minimize:

\[ E^2 = (\lambda_N - R_N (x_N - r_N))^T \cdot (\lambda_N - R_N (x_N - r_N)) \]

The gradient descent algorithm was employed to determine the adjustments to the final co-state value:

\[ \lambda_{N+1}^q = \lambda_N^q - 2 \alpha \partial E^q / \partial \lambda_N^q \]

where, \( q = 0, 1, 2, \ldots \) is the training iteration number and \( \alpha \) is a scalar step-size variable. For all \( q \):

\[ \partial E / \partial \lambda_N = I - R_N \partial x_N / \partial \lambda_N \]

where \( I \) is the identity matrix.

The summands at the right side of equation 19 can be solved iteratively as:

\[ \partial x_{k+1} / \partial \lambda_N = A_k \partial x_k / \partial \lambda_N - H_k \partial \lambda_{k+1} / \partial \lambda_N \]

with \( \partial x_0 / \partial \lambda_N = 0 \) and \( \partial \lambda_N / \partial \lambda_N = I \).
From equations 9 and 13, the equation 21 can be rewritten as:

\[
\frac{\partial \lambda_k}{\partial \lambda_N} = (I + R_k H_k)^{-1} A_k \frac{\partial \lambda_{k+1}}{\partial \lambda_N} \tag{22}
\]

From the new value of \(\lambda_{N+1}^k\) a new backward co-state trajectory is computed through equation 9. With the new value of co-state variable \(\lambda_{N+1}^k\), for \(k = 1, \cdots, N\), a new state trajectory is also computed. As far as \(E \to 0\) the trajectories of the state and co-state variables will converge to the optimal ones. However, looking at the problem on fuzzy logic grounds, the co-state variables appears to behave like the output of an expert system that knows which sequence of values will minimize the cost function. This can be understood as a control strategy based on an adaptive fuzzy inference system, which generates at each time \(k\), in the control time interval, an estimated value for the co-state variable at time \(k+1\). A training iteration can be defined as a sequence of control actions from \(k = 0\) to \(k = N\). Then, along successive training iterations the fuzzy inference system rules may be changed in order to generate estimates converging to the true optimal values of the co-state variables, tracking the adaptation of co-state variables of equation 18. This implies the convergence of the state variables values to the optimal values. Changing the rules of the fuzzy inference can be achieved with learning algorithms, which take the final input error between state and co-state variables. In fact, it can be shown that, under the quadratic version of criterion equation 5, if the error goes to zero then the state and co-state trajectories will converge to the optimal.

The open-loop trajectory learning scheme was obtained through numerical solution of the boundary value problem, as described above. Then, a FM is trained to approximate the closed-loop relationship between the system states and the computed input. This closed-loop controller is expected to be more robust to plant and initial state perturbation.

4. THE GREENHOUSE MODEL

The greenhouse climate model describes the dynamic behaviour of the stated variables by the use of differential equations for the air temperature, humidity and CO\(_2\) concentrations that results from a combination of the various physical processes involving heat and mass transfer taking place in the greenhouse and from the greenhouse to the outside air. In this paper, only the temperature variables presented in figure (1) are considered.

The presented physical model has as the main objective validating the proposed diffuse model and the performance of the FCFRA algorithm. The model considers a greenhouse divided into two control system devices (heating and ventilation), the covering material, the air inside, the outside space and the soil beneath the greenhouse. These components are characterized by the heating pipe water temperature, \(T_{\text{heat}}\), for the heating system; inside air temperature, \(T_{\text{in}}\); outside air temperature, \(T_{\text{out}}\) and the soil temperature, \(T_{\text{soil}}\). Other fundamental measured variables of the process are the solar radiation, \(R_{\text{ad}}[\text{W/m}^2]\), and two main control signals (control variables): heating \((0 \leq u_{\text{heat}} \leq 1)\) and ventilation \((0 \leq u_{\text{vent}} \leq 1)\), corresponding to a range from 0 to 100% of the actuator nominal power.

The energy balance to the greenhouse air is affected by the energy supplied by the heating system, \(Q_{T,\text{heat}}\), the energy losses to the outside air due to the conduction through the greenhouse cover and the forced ventilation exchange, \(Q_{T,\text{vent}}[\text{W/m}^2]\), the energy exchange with the soil \(Q_{T,\text{soil}}[\text{W/m}^2]\) and the heat supplied by radiation from the sun, \(Q_{T,\text{Rad}}[\text{W/m}^2]\). These fluxes obey to the following relations:

**Sunlight radiant energy:**

\[
Q_{T,\text{Rad}} = c_{\text{rad}} \cdot R_{\text{ad}}
\]

where the coefficient \(c_{\text{rad}}\) denotes the usual fraction of the solar radiation transformed into heat;

**Energy losses to the outside air:**

\[
Q_{T,\text{vent}} = (c_{\text{vent}} \phi_{\text{vent}} + c_{\text{leak,roof}}) \cdot (T_{\text{in}} - T_{\text{out}})
\]

where \(c_{\text{leak,roof}}\) is the heat transfer coefficient through greenhouse cover. The ventilation flux, \(\phi_{\text{vent}}\), is the sum of natural ventilation \((\phi_{\text{leak,vent}})\) with the forced ventilation \((c_{\text{force,vent}} \cdot u_{\text{vent}})\), and therefore \(\phi_{\text{vent}} = c_{\text{force,vent}} \cdot u_{\text{vent}} + \phi_{\text{leak,vent}}\) and \(c_{\text{vent}}\) is the heat capacity per unit volume of air.

![Fig. 1. Greenhouse climate model structure.](image-url)
Energy transferred from the soil:

\[ Q_{T,\text{soil}} = c_{\text{soil}} \cdot (T_{\text{soil}} - T_{\text{in}}) \]  \hspace{1cm} (25)

where \( c_{\text{soil}} \) is the heat transfer coefficient. In the inside volume, a heat balance equation can be written and solved taking into account the different heat fluxes present here, i.e., the following differential equation:

\[
\frac{dT_{\text{in}}}{dt} = \left( Q_{T,\text{heat}} - Q_{T,\text{out}} + Q_{T,\text{soil}} + Q_{T,\text{rad}} \right) \cdot \frac{1}{C_{\text{cap,T}}}
\]  \hspace{1cm} (26)

where \( C_{\text{cap,T}} \) is the heat capacity of the greenhouse air. The computed parameters of the greenhouse climate model are shown in Table 1. These parameters were obtained by employing a nonlinear optimization strategy to a set of real measured data. The training data are a collection of data recorded between January 20 and February 9, with a sampling interval of 1 minute. The simulated and measured air temperatures were compared during a test period. These curves show a good agreement between measured data and the models output, with negligible error.

5. SIMULATION RESULTS

The fuzzy optimal controller design objective is to control the greenhouse inside air temperature and humidity, represented here by the state vector \( x(t) \), subjected to a fitness function of equation 5 and limited by a range of possible actuators values of equation 14. The control variables are the heating control \( (0 \leq u_{\text{heat}} \leq 1) \) and the ventilation control \( (0 \leq u_{\text{vent}} \leq 1) \).

The optimal co-state trajectories will be computed and will be used in the adaptation of the co-state fuzzy controller. Moreover, the process output center parameters of fuzzy rules are adjusted; the other parameters of the fuzzy system are kept fixed. It is shown the effectiveness of the learning strategy in the control of the inside air temperature and air humidity, which is a nonlinear dynamical system.

In the training phase of the fuzzy optimal controller, quadratic cost weights, the initial states vector and reference signal vector were \( R = \text{diag}[1, 0.5] \), \( Q = \text{diag}[10, 0.01] \), \( \bar{Q} = \text{diag}[20, 20] \), \( S = 4R \), \( x_0 = [11, 1\, \text{°C}; 0.08 \, \text{kg/m}^2\cdot\text{m}^{-3}]^T \). The reference signals are:

\[
r_k = \{ \text{Temp}_{\text{Ref}}(k) ; \text{RH}_{\text{Ref}}(k) \}, \text{ for } k = 1, \ldots, 1440 \text{ and } T = 1 \text{ minute},
\]

and \( \text{Temp}_{\text{Ref}}(k) \) is given by:

\[
\begin{align*}
12°\text{C, for } 1 \leq k < 500 & \text{ & } 1200 \leq k \leq 1440; \\
28°\text{C, for } 700 \leq k \leq 1000; \\
12 + 0.08(k - 500)^2 & \text{C, for } 500 \leq k < 700; \\
28 - 0.08(k - 1000)^2 & \text{C, for } 1000 \leq k < 1200;
\end{align*}
\]  \hspace{1cm} (27)

and

\[
\text{RH}_{\text{Ref}}(k) : \text{RH}_{\text{Ref}}(k) = 80% \]  \hspace{1cm} (28)

The learning process of section 3 was applied to implement the co-state fuzzy control through 74 fuzzy rules. A set of simulation tests were performed for a predefined reference signals (for temperature equation 27 and absolute/relative humidity equation 28). In each example, a co-state discrete trajectory, initially supplied by the fuzzy system, is improved by the learning algorithm. The resulting trajectory is used to update the fuzzy control weights. This approach goes on continuously to reduce the feed forward control function error during the FM training and generates the optimal control trajectory. The FM supplies the values of co-state variables for each one of 24 hours instances of a day. The intercalary co-state values are obtained for linear interpolations of these values. Figure 2 and 3 shows the controlled greenhouse inside air temperature and absolute humidity for 24 hours of a day (22 of January, 1998), with a measured interval of 1 minute. The hatched lines are the reference signals. Both signals track fairly their references signals taking in account the cost inferences of the heating and ventilations actuators. In Figure 4 is represented the co-state trajectory obtained by referred optimization processes and its correspondent action variables. The behaviour of state variables and actuations are agreed with the restrictions of the problem and the trajectories are optimal with respect to the quadratic performance index.

6. CONCLUSIONS

This work consisted in the implementation of the non-linear quadratic optimal strategy to control the inside air temperature and humidity of an Agricultural Greenhouse. The method described is based on nonlinear optimal strategies to help the fuzzy co-state system to map optimal values of the co-state variables. A learning algorithm interactively adjusts the co-state variable values in an optimal way that is continuously saved in the fuzzy inference system.
Fig. 2. Trajectory of inside air temperature controlled by the optimal fuzzy control (‘-’). The detached is the reference temperature and the cyan line is the outside air temperature.

Fig. 3. Trajectory of inside absolute humidity controlled by the optimal fuzzy control (‘-’). The detached is the reference temperature and the cyan line is the outside air absolute humidity.

Fig. 4. Actuations computed by the optimal control algorithm: $u_{\text{heat}}$ and $u_{\text{vent}}$; Trajectory of co-state variables computed by the proposed optimal control algorithm (‘.’).

7. REFERENCES


