Failure analysis of biocomposite sandwich pipe under internal pressure – Application for high pressure gas transportation pipelines MEDGAZ

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ABSTRACT
In this paper, analytical and 3D numerical models are developed to investigate the mechanical behavior of sandwich pipe under internal pressure loading. The suggested models provide an exact solution for stresses, strains and displacement on the sandwich pipe, which is made of epoxy material for the core layer and reinforced materials with an alternate-ply for the skin layers. The aim of this analysis is to evaluate the potential applications of jute and pineapple leaf fiber (PALF) bio-fibers in order to replace glass synthetic fibers generally employed in sandwich pipes. In this subject, a failure analytical analysis was developed using TSAI-WU criterion. The results of stress, strain and displacement distribution through the thickness are presented for the analytical and numerical models. The comparison between the both models results show a very good agreement. In order to increase the rigidity of a biocomposite sandwich and reduce the gap compared with a synthetic sandwich, a gradual reinforcing of layer numbers was chosen, which permitted the best behavior. The ultimate pressure and safety factors obtained by increasing biocomposite layers are significant for composite transportation pressure pipelines, especially for sandwich pipe based on PALF/epoxy.

1. Introduction

The world’s interest in gas and oil is climbing steadily, which are definitely the most essential sources of energy on our planet [1]. In the natural gas transportation industry, there will definitely be a rising demand for research, development, and innovation activities. Using composite structures for natural gas transportation systems such as flowlines, gathering pipelines, and distribution pipelines continue to grow [2]. In comparison to many industries, such as building construction, automobiles, and aircraft, where composite materials have been widely used for decades [3], the gas industry has been slow to adopt composites, despite all of composites’ benefits and their high potential for use in pipe systems [4]. These structures are becoming more prevalent in the gas industry. Gas transportation infrastructure exists between North Africa and Europe, carrying gas from Algeria and Libya to Europe through Italy and Spain. The volume of gas transported is around 63.5 bcm annually.

Pipelines are essential for natural gas and hydrogen transportation across national and international borders. The steel pipes are installed in various environments and have suffered various damage problems [5]. The most prevalent steel pipe defects are: corrosion damage [6–19], cracks [6–8,13,17,20–22], durability [8,13,23,24], maintenance costs [8,14,15,25], buckling and burst piping problems [26].

The usage of composite structures, including pipelines unidirectional multilayer, now represents one of the greatest solutions in the industry, especially for the transport and storage of fluids such as gas, water, and oil [2,27]. Composite pipes are widely recognized and applied in the oil and gas industry due to their benefits of corrosion resistance and structural flexibility can be custom designed, high durability, no need for coating, easy and fast installation, and low maintenance costs [26,
Not all pipes are manufactured identically. Therefore, the material choice for usage in pipeline systems and hydrocarbon transportation may impact operational costs, maintenance charges, lead time, total cost of ownership, compliance, ease of installation, and many more. There are several composite pipe configurations, including multi-layered cylindrical pipes and sandwich pipes, which are applied in tubular structures.

Multi-layered composite tubes are used in gas and hydrogen transportation due to their high specific strength and stiffness. That’s why scientists have performed different research on thick or thin multi-layered composite cylinders [31–36].

While, glass fiber and steel pipes have been widely used for the transportation of oil and gas products also natural gas, sandwich pipes have been explored as one solution to the high-pressure challenges associated with natural gas and hydrogen transport [37,38].

Sandwich pipes are light, versatile composite structures that are composed of three components: the core, internal pipe, and inner pipe [37–41]. The last two are dubbed skin.

Sandwich tubes can be circular [1,39,42–45], square [46,47], rectangular [48], or hexagonal [49] in shape.

Synthetic fibers are increasingly being replaced by natural fibers, particularly autochthonous materials [50]. Sadeghian et al. [51] have tested the flexural behavior of a sandwich composite made of flax fiber composite with a cork core and glass fiber composite with a polypropylene core. They discovered that sandwich composites consisting of natural flax fibers and natural cork core materials performed similarly to their counterparts made of synthetic glass fibers and synthetic honeycomb core materials in the investigation.

At the end of 20th and 21st centuries, the researcher and industrials have increased the use of natural fibers instead the synthetic ones because of the environmental and climatic issue, their lightness and goods mechanical properties compared to glass fibers in the fields of automobiles, locomotives, marine structures, and commercial applications [52,53]. Environmental protection challenges have recently received more attention globally. The fact that natural fibers are based on renewable plants and can be easily biodegraded has encouraged additional study into this sector to increase the use of eco-friendly products. Natural fiber composite materials are such an appropriate material that they can replace synthetic composite for many practical applications where we need high strength and low density. Pineapple leaf fiber (PALF) [54–56], palm, hemp [57,58], alfa, jute [42,52,59], cotton, bamboo, flax [60], silk [46,48], kenaf [43,49,61], and many others have been used in recent studies and the development of natural fiber composite structures. Table 1 compares natural and glass fibers and demonstrates in plain terms whether one has advantages over the other in several situations.

New bio-sandwich pipelines are a vital feature of today’s and tomorrow’s energy and industrial sectors, allowing for the transportation of different energy sources such as hydrogen as well as natural gas. Using natural fibers as the skin of the sandwich structures may provide a multifunctional, environmentally friendly structure. Several research efforts are being conducted to replace synthetic skins with composites made from natural fibers [51,60,65–67].

Boria et al. [68] studied the response to low-velocity impact of sandwich structures with thin flax/epoxy face-sheets and an agglomerated cork core by using a non-linear dynamic FE model solved using LS-DYNA. Their findings demonstrated that agglomerated cork can be a renewable alternative to standard synthetic foam materials, as well as control damage extension and provide high energy absorption. An analytical model of a hybrid vessel with the goal of building an enhanced high hydrogen pressure storage vessel was created by Hocine et al. [69]. This analytical model provides a precise solution for stresses and strains on the hybrid model’s cylinder section subjected to thermo-mechanical static loading and hydrogen leakage. For all this reasons, the present paper is focused to realize an analytical and numerical behavior study of bio composite sandwich pipe under internal pressure.

2. Problem description

The aim of this analysis is to evaluate the potential applications of eco-composites based on natural fibers such as jute and PALF bio-fibers in order to replace synthetic skins fibers of sandwich pipes in the natural gas pipeline industry. A failure analysis of biocomposite sandwich pipe under internal pressure, where the skins are based on bio-composite fibers is established. Analytical and 3D numerical models are developed which provides an exact solution for stresses, strains and displacement through the thickness of skins and core.

3. Presentation of analytical and FEM models

Considering a sandwich composite made up of three adjacent hollow cylinders that correspond to the two multilayered skins (upper and lower) and the epoxy core from inside to outside, of component of the transportation pipelines under internal pressure P (See Fig. 1), where the radius r is as follows: \( n \leq r \leq r_0 \).

3.1. Analytical model

For the analytical model, all displacements, strains, and stresses are independent of \( \Theta \) for the axisymmetric sandwich pipe. The radial displacements \( r \) are independent of axial displacements. Z as well as axial coordinates is not dependent on the radial ones. Therefore, the displacements are:

\[
U_r = U_r(r), U_\theta = U_\theta(r, z), U_z = U_z(Z)
\]  

The strain-displacement relations can be expressed as:

![Fig. 1. Geometry of the sandwich pipe and wall stress components.](image-url)
The general stress-strain relationship for each k-component exposed to axisymmetric thermo-mechanical loading [69] is shown below:

\[
\begin{align*}
\sigma_r &= C_{11} \varepsilon_r + C_{12} \varepsilon_\theta \quad \sigma_\theta &= C_{12} \varepsilon_r + C_{33} \varepsilon_\theta \\
\tau_{r\theta} &= C_{44} \varepsilon_r + C_{45} \varepsilon_\theta \\
\tau_{r\phi} &= C_{66} \varepsilon_r + C_{56} \varepsilon_\theta
\end{align*}
\]

The local balancing equations are as below in each k-component:

\[
d \frac{d \epsilon_r^{(k)}}{dr} + \frac{\sigma_r^{(k)}}{r} = 0
\]

The following differential equation is constructed by substituting the expression of radial and hoop stresses from equation (3) into equation (4) and applying equation (2):

\[
d^2 \frac{U_r^{(k)}}{dr^2} + \frac{1}{r} \frac{dU_r^{(k)}}{dr} - \frac{N_r^{(k)}}{r} U_r^{(k)} = \left[ N_r^{(k)} \Delta T \right] \varepsilon_r + N_\theta^{(k)} \varepsilon_\theta
\]

Where:

\[
N_r^{(k)} = \frac{C_{11}^{(k)}}{C_{33}^{(k)}} N_r^{(k)} + \frac{C_{12}^{(k)}}{C_{33}^{(k)}} N_\theta^{(k)} + K_1^{(k)} - K_2^{(k)} N_\theta^{(k)} = C_{33}^{(k)} 2 C_{12}^{(k)} 2 C_{33}^{(k)} 2 C_{13}^{(k)}
\]

And:

\[
K_1^{(k)} = a_1^{(k)} C_{11}^{(k)} + a_2^{(k)} C_{12}^{(k)} + a_3^{(k)} C_{13}^{(k)} K_2^{(k)} = a_1^{(k)} C_{12}^{(k)} + a_2^{(k)} C_{22}^{(k)} + a_3^{(k)} C_{33}^{(k)}
\]

The value of \( \beta^{(k)} = \sqrt{N_\theta^{(k)}} \) determines the solution of equation (5), and can be expressed as:

\[
\beta^{(k)} = 1: U_r^{(k)} = D^{(k)} \frac{E^{(k)}}{r + r \ln(r)} + a_4^{(k)} \gamma_r r^2
\]

For \( \beta^{(k)} = 2 \):

\[
U_r^{(k)} = D^{(k)} \frac{E^{(k)}}{r + r \ln(r)} + \left( a_4^{(k)} + a_5^{(k)} \right) \Delta T + a_6^{(k)} \gamma_r r^2
\]

The superscript \( k \) is such as \( k \in [1, w] \) where:

\[
w = n_r + n_\theta + n_i
\]

The continuity conditions for the displacements and stresses in the interfaces lead to:

- The radial displacements’ continuity is determined by:

\[
\forall k [1, w - 1], U_r^{(k)} = U_r^{(k+1)}
\]

- The radial stress continuity is such as:

\[
\forall k [1, w - 1], \sigma_r^{(k)} = \sigma_r^{(k+1)}
\]

- The continuity of the radial stress gives:

\[
\forall k [1, w - 1], \sigma_r^{(k)} = \sigma_r^{(k+1)}
\]

- The axial equilibrium of a cylinder with closed ends is satisfied by the following relation:

\[
2 \pi \sum_{k=1}^{w} \int_{r_{ext}}^{r_{int}} \sigma_r^{(k)} r dr = \pi r^4 p_0
\]

- Because the torque is zero, the expression takes the form:

\[
2 \pi \sum_{k=1}^{w} \int_{r_{ext}}^{r_{int}} \tau_{r\theta} r^2 dr = 0
\]

The equations that follow make it possible to express the vectors’ deformations and stresses in the reference fiber:

\[
\begin{align*}
\epsilon' &= T_r \epsilon \\
\sigma' &= T_r \sigma
\end{align*}
\]

And:

\[
T_r = \begin{bmatrix}
\cos^2 \phi & \sin^2 \phi & 0 & 0 & 0 & 2 \sin \phi \cos \phi \\
\sin^2 \phi & \cos^2 \phi & 0 & 0 & 0 & -2 \sin \phi \cos \phi \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \phi & -\sin \phi & 0 \\
0 & 0 & 0 & \sin \phi & \cos \phi & 0 \\
-\sin \phi \cos \phi & \sin \phi \cos \phi & 0 & 0 & 0 & \cos^2 \phi - \sin^2 \phi
\end{bmatrix}
\]

The Tsai-Wu criterion is introduced in the analytical model in order to evaluate the sandwich pipe strength:

\[
F_{11} \left( \sigma_{11}^{(k)} \right)^2 + F_{22} \left( \sigma_{22}^{(k)} \right)^2 + F_{16} \left( \sigma_{16}^{(k)} \right)^2 + 2 F_{12} \sigma_{12}^{(k)} \sigma_{16}^{(k)} + F_1 \sigma_{11}^{(k)} + F_2 \sigma_{22}^{(k)} \leq 1
\]

With:

\[
F_{11} = \frac{1}{\sigma_{11}}; F_{22} = \frac{1}{\sigma_{22}}; F_{16} = \frac{1}{\sigma_{16}}; F_{12} = \frac{1}{\sqrt{\sigma_{11} \sigma_{22} \sigma_{16}}} \quad F_{16} = \frac{1}{\sigma_{16}}
\]

The solutions are obtained by using the MATLAB numerical code. The results are represented as functions of the non-dimensional ratio \( R = \frac{r_{ext} - r_{int}}{r_{ext} - r_{int}} \)

3.2. FEM model

In order to validate our analytical approach, finite element simulations with ANSYS 18.1 and 3D elements are used in our study; Solid186 for the isotropic material (epoxy) and layered Solid186 for the
orthotropic material (composites). A higher-order 3D element with quadratic displacement behavior is the ANSYS SOLID 186 element [70], which is shown in Fig. 2. The element contains 20 nodes, each with three degrees of freedom (translations in the three nodal directions). The element may be used as a structural solid or a layered solid element and is suitable for meshing irregular solids. The numerical model presents a uniform mesh of the global geometry, characterized by: 43,911 nodes and 8640 quadratic elements. Fig. 3 illustrates the entire FE model as well as its characteristics.

3.3. Geometry, materials and loading

The sandwich pipe configuration has an inner radius of 60 mm, a core-layer thickness of 10 mm, a skin-layer thickness of 0.27 mm and a length of 100 mm.

In the current study, two types of bio-skin material based on jute and pineapple leaf fibers compared to glass fibers were carried out. The skin materials characteristics are shown in Table 2. The core epoxy is shown in Table 3. The used layered skin stacking sequences are listed in Table 4.

The internal pressure applied to the sandwich pipe is 6 MPa. This pressure threshold was chosen in order to remain within the elastic range.

4. Analytical and FEM analysis

4.1. Analytical and numerical confrontation results

To validate the accuracy of our results, we performed a mesh convergence on the hoop stress of jute/epoxy along a non-dimensional radial distance. We have clearly arrived at a convergent solution.

Fig. 2. The meshing element used for finite element models.

Fig. 3. The sandwich pipe structure. (a) The geometry, (b) the mesh, (c) boundary conditions, and (d) loading of the numerical model.

Fig. 4. Analysis of mesh convergence for finite elements of hoop stress distributions versus R.

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4.1. Analytical and numerical confrontation results

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Fig. 4 summarizes the results of the convergence analyses based on adjusting the number of elements in the mesh. Before developing the analytical results, a comparison is made between the analytical and FEM results. Fig. 5 represents the comparison between both analytical (developed using Matlab) and numerical results obtained by means of commercial software (ANSYS 18.1) for the radial displacement, hoop and axial stress. This comparison shows a very good agreement and demonstrates that the responses of the structures are relatively well anticipated.

A comparison of stress, strain and radial displacement distributions through the wall thickness in the sandwich pipes is established, between synthetic and natural composite materials design as glass/epoxy, PALF/epoxy and jute/epoxy for the same stacking sequence as shown in Table 4.

The hoop stress distributions over the wall thickness of the sandwich pipe components (the inner and outer skins, as well as the core) using the color FE contour presented in Fig. 6.

The inner skin has the maximum stress distribution values compared to the outer skin and core. The hoop stress presented in Fig. 7 (b) is a tensile state. The skin layers are subjected to significantly greater stresses which are about ten times compared to the core layer one. Moreover, the stress of inner skin is more than double compared to the outer skin. The jute/epoxy and PALF/epoxy sandwiches are less sensitive compared to glass/epoxy, where the stress magnitude is lower. The same remarks characterize the axial stress distribution, with a compression state being recorded in the core for all materials compared. The ratio between hoop stress and axial stress which is about ten reflects the internal pressure applied to the structure, and it can lead to a discontinuity between the core and the outer skin.

4.2. Analytical analysis results

Fig. 7(a) shows the distribution of radial stress through the thickness of the sandwich pipe. For the three materials, a compressive radial stress increases from the inner skin to the outer through the core of the sandwich pipe. The maximum radial stress of −6 MPa is found at the inner skin which decries to zero at the outer skin, which reflects the boundary conditions applied.

The findings of the hoop and axial stress distributions between the core and the skins demonstrate a large variation for the multi-layered composite based on glass compared to natural fibers PALF and jute because these observations reveal the large anisotropic properties of glass.

The hoop stress presented in Fig. 7 (b) is a tensile state. The skin layers are subjected to significantly greater stresses which are about ten times compared to the core layer one. Moreover, the stress of inner skin is more than double compared to the outer skin. The jute/epoxy and PALF/epoxy sandwiches are less sensitive compared to glass/epoxy, where the stress magnitude is lower. The same remarks characterize the axial stress distribution, with a compression state being recorded in the core for all materials compared. The ratio between hoop stress and axial stress which is about ten reflects the internal pressure applied to the structure, and it can lead to a discontinuity between the core and the outer skin.
upper layer of the lower skin and the inner layer of the upper skin, (Fig. 7 (c)).

Fig. 8 shows the shear stresses distribution through the radius which is predominantly carried by the skins, with the sign of shear stress alternating through plies. The shear stress jump occurs depending on the layer fibers orientations between 60 and -60°, while, it is around zero in the core layer. The essential function of the sandwich core is to transmit, through shearing, the mechanical actions from one skin to the other. In terms of shear behavior, the stress magnitudes obtained for the bio-composite are better than those of the synthetic.

Fig. 9 shows a quasi-linear behavior of the variation of radial displacement through the radius. The structure’s stiffness allows for a progressive decrease in radial displacement from one layer to another, which is determined by the material properties of the composite used as well as the stacking sequence of the lower and upper skins. The passage from one layer to another for skins or from skin to core translates the continuity of displacement imposed by the model. The magnitudes of radial displacement show that synthetic structures based on glass/epoxy undergo lower radial swelling compared to that of biocomposite.

The axial strain versus R is constant which reflect the impose boundary conditions and negligible compared to hoop strain due to loading mode (pure internal pressure). While, it depends on the material of the skin and the largest values is obtained for PALF/epoxy, followed by Glass/epoxy and the lowest is attributed to jute/epoxy (Fig. 10 (a)).

The hoop strain versus R is quasi-linear regressive and the maximal values are located at the internal wall of the pipes for the three materials investigated where the damage can be initiated (Fig. 10 (b)).

The pressure tested about 6 MPa represents a ratio of 2–3 of the service pressure, estimated 3 MPa which gives a very good safety margin for using biocomposites for manufacturing sandwich pipes.

Fig. 10 (c) shows the distribution of radial strain through the wall thickness, for the three different types of materials, where discontinuous variation is registered at the interface between the skin and the core and show compression state. The jute/epoxy has the largest radial strain in the internal part of the skin, followed by the PALF/epoxy, and the lowest for the glass/epoxy.

The TSAI-WU criterion allows us to determine the rupture limit of the most stressed internal layers according to the increment of the loading pressure for the material configurations chosen in this work. Sandwich
pipes composed of glass/epoxy and two biocomposites, jute/epoxy and PALF/epoxy, are compared. (See Fig. 11).

It is noticed that gradually reinforcing biocomposite sandwiches by increasing the number of layers from Seq 1 to Seq 4 (see Table 4) increases their rigidity. The results show that for biocomposite sandwich pipe PALF/epoxy, the ultimate pressure is around 39 bars for Seq1, 65 bars for Seq2, 80 bars for Seq3, and finally 90 bars for Seq4. These results are better compared to sandwich pipes based on Jute/epoxy.

The limit pressure for a sandwich tube based on biocomposite reaches 85 bars for Seq4, allowing for a safety factor of 2.83 for PALF/epoxy and 2.66 for jute/epoxy. It is noticed that the overall process is operated at 30 bar of high pressure for hydrogen transportation pipelines [71].

The safety factors obtained for biocomposites are larger than 2.25 reported by Nghiep et al. for storage composite pressure vessels [72].

In other words, these results obtained allow us to have confidence in the substitution of synthetic fibers by natural ones while optimizing the number of layers.

Fig. 12 shows the analytic response of sandwich pipe under pure internal pressure which is in good agreement with the evolution of hoop stress versus axial and hoop strains.

The jute/epoxy has the best results (i.e., having the smallest values of hoop stress, axial and hoop strains) followed by the PALF/epoxy and the glass/epoxy.

5. Conclusion

This analysis, based on analytical and numerical methods is established to evaluate the potential applications of eco-composites sandwich pipes under pure internal pressure in order to replace synthetic composite (glass/epoxy). The findings lead to the following conclusions.

- The comparison of the analytical and FEM results of radial displacement, stress and strain reveals a good agreement.
- The hoop and axial stress show that the inner skins are significantly stressed compared to the outer. This can lead to the initiation of progressive damage at the inner wall of the sandwich tube which imposes to increase the number of layers of the internal skin compared to the external one.

The jute/epoxy has the best results (i.e., having the smallest values of hoop stress, axial and hoop strains) followed by the PALF/epoxy and the glass/epoxy.
• The results of shear stress confirm that the core part of the sandwich tube doesn’t contribute to the resistance of shearing state which is supported just by the skin parts.
• In terms of hoop strain, the increase in the number of layers of natural fiber-based skins leads to a superior response of biosandwich compared to synthetic sandwich in terms of stiffness.
• The optimization of the number of natural fiber-based skins layers is essential to quality/price ratio.
• The results of axial strain show that the structure is compressed in the axial direction. Where the jute/epoxy demonstrates a good behavior compared to the glass/epoxy.
• The results of the hoop strain show a swelling of the sandwich tube, which is more noted for the natural fibers composite compared to the synthetic fiber.
• The radial strain results indicate a state of compression of the sandwich tube wall across the thickness, with a concentration at the core.
• The TSAI-WU criterion is used to prevent failure of the sandwich pipe in function of the increasing of internal pressure loading. The gradual reinforcing of bio-composite sandwiches by raising the number of layers elevates their rigidity.
• The pressure studied in this work is a factor of three of the service pressure, which is anticipated to be 30 bars. This affords a very good safety margin when using biocomposites for manufacturing sandwich tubes.
• Substituting synthetic fibers by bio-fibers yields a satisfactory result by increasing the number of layers and can play the same role at this stage of analysis.
• In order to have the optimum analytical and numerical tool design, the damage behavior that might occur in the skin and the core must be incorporated into the model. This behavior will be treated in the forthcoming study.

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Credit author statement


Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

Data availability

Data will be made available on request.

Nomenclature

Roman letters

$D^{(k)}, E^{(k)}$ The integration constants
$F_{11}, F_{12}, F_{22}, F_{33}, F_{1}, F_{2}$ The standard Tsai-Wu parameters
$n_c$ Number of core layers
$n_s$ Number of skin layers
$r$ Radial direction
$r_{ext}^{(k)}$ Outer radius of the layer $k$ [mm]
$r_i$ Inner radius of skins [mm]
$r_o$ Outer radius of skins [mm]
$T_r$ Constraint base change matrices
$T_p$ Deformation basis change matrices
$U_r$ Radial displacements [mm]
$U_z$ Axial displacements [mm]
$U_\theta$ Circumferential displacements [mm]
$w$ Number of composite layers
$Z$ Axial direction

Greek letters

$\alpha_r, \alpha_z, \alpha_\theta$ Thermal expansion coefficients [10⁻⁵ °C⁻¹]
$\gamma_{rz}$ Shear strain vector in the plan z-r
$\gamma_{z\theta}$ Shear strain vector in the plan z-\theta
$\gamma_{r\theta}$ Shear strain vector in the plan r-\theta
$\gamma_0$ and $\epsilon_0$ The integration constants
$\Delta T$ Temperature deviation [°C]
$\epsilon_r$ Radial strain vector
$\epsilon_z$ Axial strain vector
$\epsilon_\theta$ Circumferential strain vector
$\bar{\epsilon}$ Deformation vector in the coordinate system
$\Theta$ Circumferential direction
$\sigma_{uu}$ The tensile strengths of the composite material [MPa]
$\sigma_{cu}$ The Compressive strengths of the composite material [MPa]
$\sigma_{yuu}$ In the layer plane shear strength [MPa]
\( \sigma \) Stress vector in the coordinate system [MPa]

\( \sigma_{uu} \) In the direction of the fibers [MPa]

\( \sigma_{vv} \) In the transverse direction [MPa]

\( \tau_{vv} \) Shear strain vector in the plan \( z - r \) [MPa]

\( \tau_{rr} \) Shear strain vector in the plan \( z - \Theta \) [MPa]

\( \phi \) The winding angle (°)

References


