Abstract—Today’s competitive and volatile market requires flexibility, quality and efficiency from the logistics operations. In this context, warehouses are an important link of the logistic chain and warehouse management plays an important role over customer’s service. Throughout this work we analyze a mathematical model aiming to support warehouse management decisions. A case study is used for that purpose and the model jointly identifies product allocation to the functional areas in the warehouse, as well as the size of each area. This case study also evaluates the performance of the model when real data is used. Model is solved using LINGO 9.0 mixed-integer commercial solver, and potential savings achieved using the proposed technique are discussed.

Index Terms— Warehouse Operations, Facilities Planning and Design, Mixed-integer programming, Case study.

I. INTRODUCTION

WAREHOUSE management is, nowadays, a great challenge in the field of supply chain management. Inventory level management, warehouse design, warehouse operations and space optimization and costumers’ requirements are some examples of important challenges in this context. Warehouses must be flexible structures to provide quality, efficiency and effectiveness of the logistics operations in a very demanding, competitive and uncertain market.

On the other hand, modern supply chain management principles compel companies to reduce or eliminate inventory levels. Additionally a warehouse requires labour, capital and information technologies, which are expensive resources. So, why do we still need warehousing? According to Bartholdi and Hackman [3] there are four main reasons why warehouses are useful: to consolidate products in order to reduce transportation costs and to provide customer service; to take advantage of economies of scale; to provide value-added processing and to reduce response time. Thus, warehouses will continue to be an important node at the logistic network.

In distribution logistics where shorts lead-times and flexibility are essentials warehouse design and operations become more important and complex. Furthermore, the ever-increasing variety of products and the constant changes in customer demand has placed a tremendous emphasis on the ability to establish efficient logistic operations. Warehouse design and operations will be determinant to have quality, efficiency and flexibility.

Typically, a design runs from a functional description, through a technical specification, to equipment selection and determination of the layout. A layout must be modular, adaptable, compact, accessible and flexible and must be capable to respond to changing conditions, to improve space utilization and to reduce congestion and movement. For these reasons, warehouse design is a highly complex task, where, sometimes, trade-offs have to be made between conflicting objectives.

According to Hassan [13] an important aspect of designing a warehouse is its layout. The design of the layout should be concerned with the arrangement of the functional areas, determining the number and location of input/output (I/O) points, determining the number of aisles, their dimensions and orientation, estimating space requirements, designing the flow pattern, and defining picking zones.

The majority of scientific research studies address isolated problems. However, most real problems are unfortunately not well-defined and often cannot be reduced to multiple isolated sub-problems. Therefore, design often requires a mixture of analytical skills and creativity. Anyhow, research aiming an integration of various models and methods is badly needed in order to develop a methodology for systematic warehouse design (Rouwenhorst et al. [23]).

In this paper we tackle problems found during the redesign process of a warehouse in a Portuguese company. In particular, we adapt a mixed-integer programming model to support two warehouse management decisions. The model, developed by Heragu [14], jointly determines the size of functional areas and allocates products to them. The results of applying this model to this case study are analyzed and potential savings achieved are discussed.
II. WAREHOUSES

Warehousing is concerned with all the material handling activities that take place within the warehouse. They include the receiving of goods, storage, order-picking, accumulation and sorting and shipping. Basically, we can distinguish two types of warehouses: distribution warehouses and production warehouses. According to Berg [5], a *distribution warehouse* is a warehouse in which products from different suppliers are collected (and sometimes assembled) for delivery to a number of customers. A *production warehouse* is used for the storage of raw materials, semi-finished products and finished products in a production facility.

There are many activities that occur at a warehouse. Typically, distribution warehouses receive products - Stock Keeping Units (SKU) - from suppliers, unload products from the transport carrier; store products, receive orders from costumers, assemble orders, repackage SKU and ship them to their final destination. Frequently, products arrive packaged on large scale units and are packaged and shipped on small units. For example, SKU may arrive in full pallets but must be shipped in cases.

Typically, in a warehouse there are several functional areas and flows. Next, we briefly describe some of the most common areas and product flows (Fig. 1):

![Functional structure of a warehouse (Salvendy [24]).](image)

At the receiving area products are unloaded and inspected to verify any quantity and quality inconsistency. Afterwards items are transferred to a storage zone or are placed directly to the shipping area (this is called a cross-docking operation). We can distinguish two types of storage areas: reserve storage area and forward or picking area. The reserve area is where products stay until they are required by costumers’ orders. The picking area is a relatively small area typically used to store fast movers products. Most of the flows between areas are the result of replenishment processes. Order picking is one of the most important functions in most warehouses. SKU are retrieved from their storage positions based on customers’ orders and moved to the accumulation and sorting area or directly to the shipment area. The picked units are then grouped by customer order, packaged and stacked on the right unit load and transferred to the shipping area.

The design of a warehouse is a highly complex problem. It includes a large number of interrelated decisions involving warehouses processes, warehouse resources and warehouses organizations (Heragu [14]). Rouwenhorst *et al.* [23] classify management decisions concerning warehousing into strategic decisions, tactical decisions and operational decisions. Strategic decisions are long term decisions and always mean high investments. The two main issues are concerned with the design of the process flow and with the selection of the types of warehousing systems. Tactical management decisions are medium term decisions based on the outcomes of the strategic decisions. The tactical decisions have a lower impact than the strategic decisions, but still require some investments and should therefore not be reconsidered too often. At the operational level, processes have to be carried out within the constraints set by the strategic and tactical decisions made at the higher levels. At this level, the concern includes the operational policies such as storage policies and picking operations.

Gray *et al.* [11] developed an integrated approach to the design and operation of a typical order-consolidation warehouse. This approach included warehouse layout, equipment and technology selection, item location, zoning, picker routing, pick generation list and order batching. Due to the complexity of the overall problem, they developed a multi-stage hierarchical decision approach. The hierarchical approach used a sequence of coordinated mathematical models to evaluate the major economic trade-offs and to reduce the decision space to a few alternatives. They also used simulation technique for validation and fine tuning of the resulting design and operating policies.

After determining warehouse location and its size, layout decisions must include areas definition and what size should be allocated to each functional area. Although addressing this problem is a strategic decision problem, it is strongly associated upon some tactical problems such as how the items will be distributed among the functional areas. Thus a joint solution is desirable. However, the approach usually adopted is to solve the problems sequentially by generating multiples alternatives for the functional area size problem and then determine how the products can be allocated for each of the alternatives. Heragu [14] developed a higher-level model that jointly determines the functional areas size and the product allocation in a way that minimizes the total material handling cost.

A case study is presented in the next section. The use of this case example allowed us to evaluate the model’s performance when real data was used and simultaneously redesign the warehouse.
III. CASE STUDY

F&F, located at Guimarães, north of Portugal, manufactures and distributes house appliances to about 1200 customers around the country (hypermarkets (85%), retail outlets (10%), hotels (1%), and others (1%)) and also some customers in Spain and Africa (3%). Over 25% of the products are acquired locally (including cutlery supplied from a factory owned by F&F), 20-30% are shipped from China and the remaining products are supplied from European countries. F&F processes an average of 40 customer orders per day (20 items per order, on average). The company warehouse is a 4000 m² facility. Currently the firm owns and utilizes approximately 1200 m² of warehouse storage space in a 3 km distant facility.

Items in the main warehouse are stored in pallets over 4000 m² of storage aisles (with 4 storage levels in the vertical direction), corresponding to 2000 storage spaces. The facility is divided into two main sections: picking area and assembly area.

A team of 15 workers are involved in warehouse operations which include receiving, storage, picking consolidation, sorting and shipping and some value-added activities (labelling, repackaging, etc). Receiving, storage operations and picking operations were usually carried out during the morning period. During the afternoon, while picking operations are still in progress, orders were checked for completeness (including searching for missing items) for 3 teams and then packed and prepared for shipping. F&F outsource the transportation activity and therefore it is performed by a logistics provider.

The company was facing increasing difficulties in remaining competitive due to the high operation costs and low levels of productivity to maintain existing throughput rate (output) as result of changes in customers demand pattern over the last years: increasing need of extra labour hours, constant need of outsourcing labelling and repackaging tasks. Additionally a decrease in service level was also reported as a result of i) difficulty in meeting customers demands of shorter delivery times; ii) high level of errors in order processing (some as result of stock-out situations)

Finally, a major problem was the increasing need of extra storage space. This problem was mainly due to the increasing variety of products available and to large inventory levels required to face long lead-times (supplier lead-times can be up to 4 months long).

An ABC Pareto’s curve showed that, in the last few years, only 43% of the existing SKUs were processed (positive turnover) and from the total set of existing customers only 8 of them were responsible for 70% of the sales.

The lack of space and the lack of operations efficiency were mainly due to the large number of obsolete items and really slow moving stock items which were eating up square foot after square foot with pallets laying down the aisles preventing pickers from reaching stock positions.

The study also showed that the largest portion of the warehouse operations time was required to find the products (10%) and to walk through the entire warehouse to process an order (60%). This was the result of several factors such: the lack of any arrangement of items on picking route lists, the absence of an efficient layout of items and the lack of an integrated information system.

Furthermore, inefficient operations planning and the incapacity to establish demand forecasts was the source of wrong inventory management policies. It was also observed that codification was not used, some of the fast movers’ items were located on difficult access positions and the different functional areas were not well defined.

The described situation confirmed the need of a redesign of the warehouse. Priorities were the elimination of obsolete items, sizing the functional areas and allocating products to the areas.

As already mentioned we used a mixed-integer programming model, developed by Heragu [14], to redesign the warehouse. This model addresses sizing functional areas and allocates items within the areas. The model uses data readily available to the warehouse manager.

IV. Decision Model

The model, adapted from Heragu [14], includes the following functional areas: receiving, shipping, reserve and picking. Thus, the three following material flow patterns are possible (Fig. 2):

- Flow 1 refers to a pattern that represents a typical warehouse operation. Items are stored in a reserve area and order picking is performed as required. It is assumed that, typically, only those items that remain for long periods of time or items used for replenishment will be allocated to this area.
- Flow 2 refers to items stored initially in the reserve area and then moved to the picking area. This pattern is considered for fast picking operations, order consolidation or even to perform value-added operations.
- Flow 3 refers to items that go directly to the picking area.

The mathematical mixed-integer programming model that
determines the flow pattern for each product and consequently the size of each of the functional areas within the warehouse is:

$$
\min \sum_{j=1}^{n} \left( H_j x_{ij} \right) + \sum_{j=1}^{n} \left( C_j Q_i/2 X_j \right) \\
\sum_{j=1}^{n} X_j = 1 \quad \forall i, j
$$

$$
\sum_{j=1}^{n} \left( Q_i/2 X_j \right) + \sum_{j=1}^{n} \left( p_i Q_i/2 X_j \right) \leq \beta TS \\
\sum_{j=1}^{n} \left( (1-p_i) Q_i/2 X_j \right) + \sum_{j=1}^{n} \left( Q_i/2 X_j \right) \leq \gamma TS \\
\beta + \gamma \leq 1 \\
LL_r \leq \beta TS \leq UL_r \\
LL_p \leq \gamma TS \leq UL_p \\
\beta, \gamma \geq 0 \\
X_j = 0 \text{ or } X_j = 1 \quad \forall i, j
$$

Where the following notation is used:

- $X_j$: proportion of available space assigned to reserve area
- $\beta$: proportion of available space assigned to picking area
- $\gamma$: annual demand of item $i$
- $H_j$: handling cost of a unit load of item $i$ in flow $j$
- $C_j$: storing cost of a unit load of item $i$ in flow $j$
- $Q_i/2$: average number of unit loads in inventory
- $TS$: total available storage space
- $p_i$: average proportion of time a unit load of item $i$ spends in reserve area if item is assigned to flow 2
- $LL_p$, $UL_p$: lower and upper storage space limit for the picking area
- $LL_r$, $UL_r$: lower and upper storage space limit for the reserve area

The above objective function minimizes the handling cost of each product and the corresponding storage cost. The model’s integrity is observed by ensuring that each item is assigned to only one type of material flow pattern and by keeping the space allocated to reserve and picking areas within the limits imposed. The output of this model will play an important role in the process of redesigning the warehouse.

### A. Application to the F&F case study

In this case study, an estimate for annual demand and for the average number of unit loads in stock was calculated based on data from the past two and a half years. The estimate for average proportion of time a unit load remains in reserve area if the item is assigned to material flow 2 was given by the warehouse manager.

All items are stored in pallet unit loads. Thus, considering a 1 m$^3$ average volume pallet, we have a storing cost of 20€ per pallet per year to the reserve area and a cost of 35€ per pallet per year to the picking area. In general, the storing cost is given by

$$
C_{ij}[\text{€/year}] = \begin{cases} 
20 & j = 1 \\
20 \times p_i + 35 \times (1 - p_i) & j = 2 \\
35 & j = 3
\end{cases}
$$

The cost of handling a unit load of each item depends on the size of the item, its handling characteristics, as well as the material-handling systems used in the respective flow pattern. Thus, handling costs tend to be more expensive for items allocated to the reserve area then to items allocated to the picking area. Assuming items arrive on pallets and picking is done in case unit loads we defined an average time for receiving a pallet of 1 min. We also considered an average picking time of 1.5 min/carton to items assigned to flow 1, 0.5 min to items allocated to flow 2 and flow 3 and an average replenishment time of 1 min to items at flow 2. Assuming a cost of a picker of 25€/hour the handling cost is given by

$$
H_{ij}[\text{€/pallet}] = \begin{cases} 
0.417 + 0.625 \times \text{(cartons/pallets)} & j = 1 \\
0.833 + 0.208 \times \text{(cartons/pallets)} & j = 2 \\
0.417 + 0.208 \times \text{(cartons/pallets)} & j = 3
\end{cases}
$$

We solved the model using LINGO 9.0 – a commercial branch-and-bound based mixed-integer programming. Results are presented in next section.

### B. Results

The model was solved including the 1500 items with positive turnover, an average inventory level of 2257 unit loads and 2000 stock locations.

Results produced by the model defined a reserve area with 880 stock location units (44% of total space) and a picking area with 300 stock locations (15% of total space). Over 43% of items were assigned to Flow 1, 45% were allocated to Flow 2 and only 12% of items to Flow 3 (Fig. 3).

![Fig. 3. Model’s optimal solution.](image)
As a result of this study F&F decided to redesign their warehouse: storage areas were identified and products were allocated to the areas. Within the picking area items with flows types 2 and 3 were located at lower levels improving picking operations of fast movers products. Replenishment stock and items with low rotation were allocated at the upper levels.

Additionally, other measures were also suggested aiming to improve warehouse operations performance such as: codification of stock positions, development of an information system and disposal of obsolete stock.

V. CONCLUSIONS

The design of a warehouse is a complex problem due to the large number of interrelated decisions sometimes with conflicting objectives. To have a single decision model where all the decisions concerning warehouse design are integrated is still a challenging objective. This paper deals with warehouse operation problems faced by a company that manufactures and distributes house appliances. A wide and complex set of problems were identified and some solutions have already been implemented. In particular, this research addresses the redesign of the distribution warehouse. The mixed-integer programming model used considered both functional area size determination and item assignment problems. The approach revealed to be very useful in defining the new layout given that a significant reduction (about 40%) of the necessary storage area was achieved. Therefore, there was no need of extra storage space.

Another advantage is that input data requirement are usually readily available in most warehouses and the fact of the model considers realistic constraints.

However, many others decisions are not included. The output of this model may be taken as a starting point for further detailed warehouse operations decisions. Despite of the implemented solutions future research is needed on the storage policy and on the order picking process. Afterwards it will be possible to compare other performance measures such as the time required to find the products and the time required to process an order.

REFERENCES