

Jânio Monteiro · António João Silva ·  
António Mortal · Jaime Aníbal ·  
Manuela Moreira da Silva · Miguel Oliveira ·  
Nelson Sousa *Editors*

# INCREaSE 2019

Proceedings of the 2nd International  
Congress on Engineering and  
Sustainability in the XXI Century



**INCREaSE 2019**  
International Congress on Engineering  
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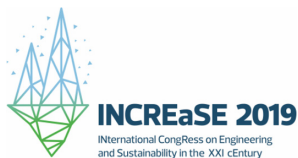


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Editors

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*Editors*

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# Two-Phase Bubble Flow: Experimental and Numerical Challenges

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**Abstract.** A water and air mixture model is validated via experimental data in a newly rebuild laboratory apparatus where the main variable compared was the pressure gradient along the pipe in a bubble flow pattern. The experimental apparatus presents a constant circular cross-section of 0.032 m of diameter and 3.815 m of length. The flow set-up was ascendant co-current and 30 pairs of superficial velocities of liquid-gas were measured. A numerical solution, using the mixture model, was implemented to predict the pressure gradient and void fraction of the two-phase flow for a one dimensional, steady-state, isothermal, no phase transition, no mass transfer and constant specific mass and viscosity conditions. The pressure gradient numerical results of the mixture model agreed with experimental data within an relative error envelope inferior to 2%.

**Keywords:** Two-phase flow · Mixture model · Pressure gradient · Two-phase flow experimental apparatus

## 1 Introduction

Two-phase flows of gas-liquid mixtures in a vertical pipes has an extensive application in the chemical, petroleum and energy production industries.

In nuclear reactors, for instance, an important parameter is the drop pressure

which can, significantly, affect the safety of nuclear reactors. The accident Loss of Coolant Accidents (LOCA) are mainly due to small crevasses created in the reactor's wall [1].

Another application for two-phase flow study is the air-lift pump. The air-lift pumps have several advantages over the other pumps, to mention a few they do not have any moving parts and do not need lubrication or suffering from wearing out problem, which means low maintenance are required. Air-lift pumps can be used for lifting corrosive and/or toxic substances in chemical industries, sandy or salty water and viscous liquids like hydro-carbons in oil industry [2]. Air-lift pumps are finding increasing use when a compressed air is readily available as a source of renewable energy for water pumping applications [3].

Other important and fundamental parameter for two-phase flow modeling is the void fraction. The void fraction is essential for calculation of many parameters, such as two-phase mixture densities, viscosities, actual velocities of each phase and the frictional pressure-drop. There are some experimental methods to obtain the void fraction: electrical impedance tomography [4,5]; double wire mesh sensors [6] and quick-closing ball valve technique [7].

There are 4 main types of two-phase flow classified according to the increase in the gas superficial velocity [8]: bubble, slug, churn and annular flow. The experimental apparatus that will be presented allows the study of these four types of flows. However, in this paper we present the study of an upward flow of air and water in steady-state, one-dimensional on a vertical pipe with constant diameter. This work addresses 2 vectors of the bubble flow regime and pattern: the experimental challenges related to the study of two-phase flows; the verification of the simplified numerical model [9,10] against the experimental results.

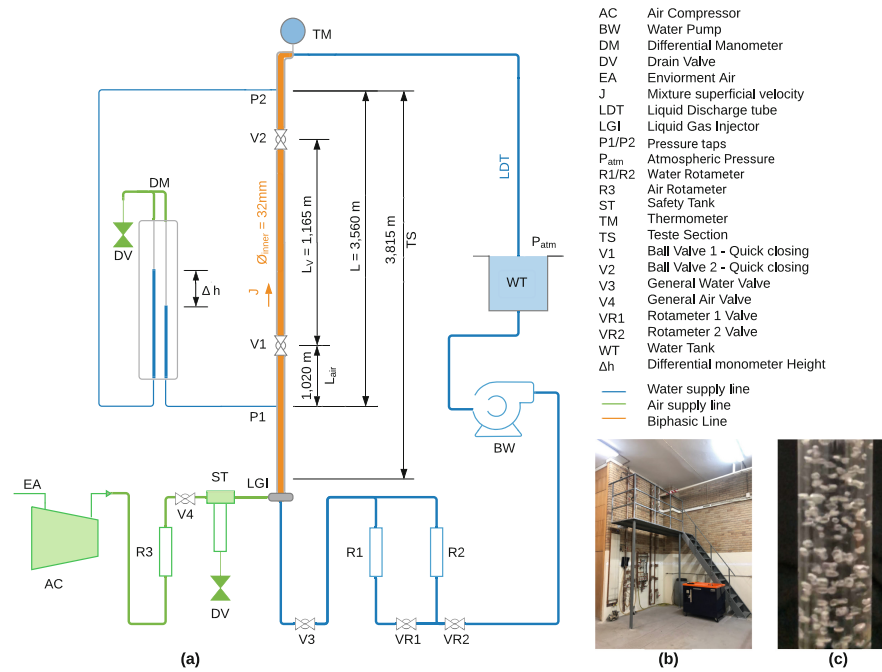
The bubble flow regime occurs when the gas phase is approximately uniformly distributed in the form of discrete bubble in a continuous liquid phase [8]. The relevant parameters for the evaluation of this type of flow are the pressure gradient  $\Delta P/L$  and the void fraction  $\alpha$ , defined as the drop pressure between two distanced points and the gas fraction in the pipe. These variables will be addressed in Sects. 2.1 and 2.2.

The experimental procedures and problems to obtain bubble flow accurate measurements are similar to the other types of flows. These challenges are also discussed in this paper in Sect. 4.3.

## 2 Experimental Procedure

The experiments were performed in a vertical acrylic pipe with a constant circular cross-section of 0.032 m and 3.815 m of length, named bubble column. The pressure difference was measured with a differential manometer, DM, connected to 2 pressure taps, P1 and P2, located 3.560 m apart: at the base and the top of the bubble column. The experimental apparatus is shown schematically in Fig. 1.

The bubble column have also two ball valves distanced 1.165 m apart to measure the average void fraction, V1 and V2 in Fig. 1. The blue line in Fig. 1



**Fig. 1.** (a) Schematic representation of the experimental apparatus, (b) Experimental apparatus, (c) Experimental bubble flow.

represents the water circuit. Water is introduced at the base of the column via a centrifugal pump BW. The flow rate is regulated by a ball valve VR1 or VR2, and measured by a rotameter R1 (KDG - Houdec S7E-60) or R2 (RDG - Houdec S18xE500), with a 3.00% accuracy. The air, represented by green line in Fig. 1, is supplied by a air compressor AC, with maximum pressure of  $6.00 \times 10^5 Pa$ . The air flow rate is controlled by a rotameter R3 (KDG-1100 - 2C-150 - ruby ball), which allows a max and min flow rate, respectively, of  $1.00 \times 10^{-5}$  and  $1.67 \times 10^{-5} m^3/s$  with an accuracy of 5.00%. The gas was introduced in the base of the column through a sparging device, LGI. This device was made with porous nylon material, allowing the formation of small-diameter ( $\sim 0.003 m$ ) bubbles and proper mixing with the water. In order to prevent water from entering the air circuit by accident and hampered the measurement of the air flow, a safety tank, ST, was built.

Thirty pairs of superficial velocities were tested, which were 6 superficial liquid velocities,  $J_L$ , between  $1.720 \times 10^{-2}$  and  $10.360 \times 10^{-2} m/s$  with 5 superficial gas velocities,  $J_G$ , between  $1.243 \times 10^{-2}$  and  $2.072 \times 10^{-2} m/s$ . For each pair of superficial velocities, the pressure gradient,  $\Delta P/L$ , and the void fraction,  $\alpha$ , were measured. The temperature at which the experiments were performed varied between 17.5 and 18.5 °C, measured in the end of the test section, TS, by a thermometer, TM.

## 2.1 Experimental Drop Pressure

For each pair of superficial velocities  $J_L$  and  $J_G$ , and after steady state was reached, the value of the water height difference was recorded in the differential manometer,  $\Delta h = h_2 - h_1$ . The difference of pressure,  $\Delta P$ , or pressure gradient,  $\Delta P/L$ , its derived from the hydrostatic equation, Eq. (1).

$$\frac{P_1 - P_2}{L} = \frac{\Delta P}{L} = \rho_L g - \frac{\rho_L g \Delta h}{L} \quad (1)$$

where  $L$  is the distance between the two taps in the column,  $\rho_L$  is the water density and  $g$  is the acceleration of gravity.

## 2.2 Experimental Void Fraction

The void fraction was obtained by two methods: the first through the quickly closing of the valves  $V_1$  and  $V_2$ ; the second using the relation from [11], recurring to the experimental terminal rise bubble velocity in a stagnant medium.

The first method is simple and functional [12], the ball valves were used to completely stop the two-phase flow. Once the two valves are closed simultaneously, it is possible to measure the gas height ( $L_{gas}$ ), and then calculate the void fraction as the ratio of heights between the gas and the total height ( $L_V$ ), Eq. (2).

$$\alpha_{exp} = \frac{L_{gas}}{L_V} \quad (2)$$

This method is known as the *quick-closing valves* method.

The second method used the relation of Zuber [11], Eq. (3), modified by the works from Nicklin [13]. This is known as the *Zuber/Nicklin relation*, and is obtained from the rearrangement of the kinematic slip law:

$$\alpha_{exp} = \frac{J_G}{J + V_{\infty,B}} \quad (3)$$

where  $V_{\infty,B}$  is the terminal bubble velocity rise in a stagnant medium, experimentally measured through the length  $L_{air}$  (shown in Fig. 1) and with the aid of a chronometer.  $J = J_G + J_L$  is the superficial mixture velocity.

# 3 Numerical Procedure

## 3.1 Mixture Model

The mixture model is based on the simplification of the two-fluid model, which results from the application of the concept of mixing in a two-phase flow, i.e., the components of the mixture are treated as a single fluid with specific rheological characteristics [9]. An important aspect of the mixture model is the reduction in the number of transport equations required for the formulation as compared to the two-fluid model [9].

There are two variables to be determined in the mixture model: one relative to the void fraction and the terms that involve the surface tensions of the mixture. Therefore, it is necessary to introduce models for the determination of these closing parameters, which in turn depend on the flow pattern, in the case of bubble flow.

An alternative form of the mixture model, introduced by [10], proposes mass and momentum conservation equations for each phase of the mixture in terms of the primitive variables of the problem, rather than using the properties of the mixture and the diffusion velocity. This alternative form was developed for studies on two-phase flows in the steady-state and transient regime [10]. In this model, it is necessary to know the slip ratio between the phases for the closing of the system equations.

The main equations of the mixture model are based on the modified drift flux model [10]. The mass conservation equations of the gas and liquid phases are defined in Eqs. (4) and (5), respectively [10].

$$\frac{\partial}{\partial t}[\rho_G \alpha_G] + \frac{\partial}{\partial z}[\rho_G \alpha_G U_G] = \Gamma, \quad (4)$$

$$\frac{\partial}{\partial t}[\rho_L R_S] + \frac{\partial}{\partial z}[\rho_L R_S U_L] = -\Gamma, \quad (5)$$

where  $U_G = J_G/\alpha_G$  and  $U_L = J_L/R_S$  are the velocities of the gas and liquid phases weighted by the volumetric fraction of each phase, with  $\alpha$  the void fraction and  $R_S$  the liquid fraction,  $Q_G$  and  $Q_L$  are respectively the volumetric flow rates of the gas and liquid phases. The variable  $\Gamma$  is the interfacial mass transfer condition.

The equation of momentum conservation of the mixture, Eq. (6), is defined by [10]:

$$\begin{aligned} & \frac{\partial}{\partial t}[\rho_G \alpha_G U_G + \rho_L R_S U_L] + \frac{\partial}{\partial z}[C_{V,G} \rho_G \alpha_G (U_G)^2 + C_{V,L} \rho_L R_S (U_L)^2] \\ & = -\frac{\partial}{\partial z}P - \frac{f}{2D}[\rho_G \alpha_G + \rho_L R_S]J^2 - [\rho_G \alpha_G + \rho_L R_S]g.\sin(\theta) \end{aligned} \quad (6)$$

where  $C_{V,G}$  and  $C_{V,L}$  are respectively the coefficients of covariance of the velocities of the gas and liquid phase,  $\rho_G$  is the density of the gas,  $f$  is the friction factor,  $D$  is the inner pipe diameter,  $P$  is the mixture pressure,  $\theta$  is the column slop ( $90^\circ$ ) and  $z$  is the axial coordinate along the pipe, varying from 0 to  $L$ . For turbulent flows the velocity and concentration profiles are approximately flat, such as the coefficient of covariance of the phase velocity is approximately equal to unity,  $C_{V,k} \cong 1$ . Thus, this coefficient is omitted from the momentum equation conversions in the formulations of the models defined in the sequence [9].

Considering the boundary conditions, the mass conservation equations of the gas and liquid phases, in the absence of mass transfer,  $\Gamma = 0$ , are reduced to:

$$\frac{d}{dz}[\rho_G \alpha_G U_G] = 0 \quad \therefore \quad \rho_G \alpha_G U_G = G_G = cte. \quad (7)$$

$$\frac{d}{dz}[\rho_L R_S U_L] = 0 \quad \therefore \quad \rho_L R_S U_L = G_L = cte. \quad (8)$$

with the mass flow  $G_G$  and  $G_L$  of each phase remaining constant along the tube.

The simplification of Eq. (6), momentum conservation of the mixture for steady-state flow and  $\sin(90^\circ) = 1$  is given by:

$$\frac{d}{dz} \left[ \sum_{k=G,L} C_{V,k} \rho_k \alpha_k (U_k)^2 \right] = -\frac{d}{dz} P - \frac{f}{2D} [\rho_G \alpha_G + \rho_L R_S] J^2 - [\rho_G \alpha_G + \rho_L R_S] g \quad (9)$$

Gathering the terms with the gradient in the axial direction, the Eq. (10):

$$\frac{d}{dz} \left[ P + \sum_{k=G,L} C_{V,k} \rho_k \alpha_k (U_k)^2 \right] = -\frac{f}{2D} [\rho_G \alpha_G + \rho_L R_S] J^2 - [\rho_G \alpha_G + \rho_L R_S] g \quad (10)$$

The Eq. (10) can be written in terms of phase velocities,  $U_G$  and  $U_L$ , with  $\alpha_G + R_S = 1$  and  $\rho_s = \rho_G + \rho_L$ , so in a more simplified way:

$$\frac{d}{dz} [P + \alpha \rho_G (U_G)^2 + (1 - \alpha) \rho_L (U_L)^2] = -\tau_{W,S} - \rho_s g \quad (11)$$

where  $\tau_{W,S}$  is frictional wall force the mixture per volume unit of the mixture. In the Eq. (11) shows the formulation of the mixture model, where there are in this Eq. 3 variables to be determined:  $P$ ,  $\alpha$  and  $\tau_{W,S}$ . And these variables are dependent on the flow pattern, in this case, bubble flow.

### 3.2 Void Fraction

The void fraction according to the flow pattern, in the case bubble flow, was determined from the volumetric balance as a function of the gas superficial velocity and the velocity of the dispersed bubbles. This void fraction model  $\alpha$  is obtained from the slip kinematic law proposed by [11], Eq. (12):

$$\alpha = J_G / U_B \quad (12)$$

where  $U_B$  is the bubbles velocity. According to the proposal of [13], which assumes that the bubbles move with the superficial velocity of the mixture  $J$ , plus the local slip speed  $V_{\infty,B}$  [9].

$$U_B = C_{0,B} J + V_{\infty,B} \quad (13)$$

So that  $V_{\infty,B} = C_{\infty,B} (\Delta \rho D g / \rho_L)^{1/2}$ .

For dispersed bubbles, the distribution parameter  $C_{0,B}$  and the dimensionless local slip velocity  $C_{\infty,B}$  are defined in accordance with Eqs. (14) and



(15) respectively. Considering the dispersed bubble regime, which results from the effects of surface tension, viscosity, inertia, and thrust [14].

$$C_{0,B} = 1.0 + 0.2 \left( 1 - \sqrt{\frac{\rho_G}{\rho_L}} \right) \quad (14)$$

$$C_{\infty,B} = \sqrt{2}(R_S)^n Eo^{-1/4} \quad (15)$$

where  $n = 0$  for the agitated bubble regime and  $n = 7/4$  for a distocided bubble regime and considering the saturation condition  $R_S = 1 - \alpha$ . The variable  $Eo$  is the Eötvös number defined by  $Eo = \Delta\rho D^2 g / \sigma$ , with  $\Delta\rho = \rho_L - \rho_G$ .

Therefore, Eq. (12) can be written as a function of the volumetric fraction of liquid, such that:

$$f(R_S) = 1.0 - R_S - \frac{J_G}{(C_{0,B}J + V_{\infty,B})} = 0 \quad (16)$$

where  $f(R_S)$  is an implicit function of  $R_S$ , once  $V_{\infty,B}$  is dependent of  $C_{\infty,B}$ , but  $C_{\infty,B}$  is dependent of  $R_S$  as shown on Eq. (15).

Therefore, the Eq. (16) is solved iteratively using a solution method of non-linear equations, such as the secant method, at each step of integration  $\Delta z$  from mixture model, in order to determine  $R_S$  and other related variables.

### 3.3 Friction Force Model

The homogeneous model was used to estimate the value of  $\tau_{W,S}$ , since the flow pattern is limited to the bubble flow, because in this model it is considered a homogeneous mixture, applying the definition of a mixture viscosity and realizing the direct application of the single-phase friction factors.

Whereas  $\tau_{W,S}$  the wall friction force per unit volume, it can be defined by the shear stress of the mixture  $\tau_W$  multiplied by the ratio of the perimeter  $S$  and the cross section area of the pipe  $A$ .

$$\tau_{W,S} = \frac{\tau_w S}{A} = \frac{2C_{f,s}\rho_s J^2}{D} \quad (17)$$

where  $C_{f,s}$  is the friction factor of the mixture Eq. (18) as a function of the Reynolds number of the mixture,  $Re_s$ , as defined in Eq. (19). These two variables are mixture dependent: the specific mass  $\rho_s$  and viscosity  $\mu_s$  of the mixture are, respectively, defined by Eqs. (20) and (21) according to [15].

$$C_{f,s} = \left\{ -3.6 \log \left[ \left( \frac{\varepsilon}{3.7D} \right)^{1,11} + \frac{6.9}{Re_s} \right] \right\}^{-2} \quad (18)$$

where  $\varepsilon$  is the absolute roughness of the pipe.

$$Re_s = \frac{\rho_s J D}{\mu_s} \quad (19)$$

$$\rho_s = \alpha \rho_G + (1 - \alpha) \rho_L \quad (20)$$

$$\mu_s = \alpha \mu_G + (1 - \alpha) \mu_L \quad (21)$$

### 3.4 Mixture Model Solution

The mixture model solution for a isothermal and steady state two-phase flow in bubble flow pattern is shown. By solving Eq. (10) the model is automatically solved. However, Eq. (10) was simplified to a reduced way of an ordinary, non-homogeneous differential equation:

$$\frac{d}{dz}[\Psi(z)] = -\tau_{W,S}(z) - \rho(z).g.\sin(\theta) \quad (22)$$

where  $\Psi(z) = P(z) + G_G U_G(z) + G_L U_L(z)$  is an implicit function of  $P(z)$ . The variables of Eq. (22) are also dependent on  $P(z)$  as shown in Eqs. (23), (24) and (25):

$$\rho(z) = \alpha(z)\rho_G(L)[P(z)/P(L)] + \rho_L[1 - \alpha(z)] \quad (23)$$

$$U_G(z) = J_G(L)[P(L)/P(z)]/\alpha(z) \quad (24)$$

$$U_L(z) = J_L/[1 - \alpha(z)] \quad (25)$$

where  $L$  is the position in the outlet section of the column where the variables are known. Therefore, the solution of Eq. (22) is conditioned in the solution of the equations that define  $\alpha(z)$  and  $\tau_{W,S}(z)$  which are also dependent on  $P(z)$ .

The solution of Eq. (22) is numerical integration using a Runge-Kutta fourth-order (*RK-4*) routine, programmed on *MatLab*. The integration step  $\Delta z$  used was  $D/4$ , where it was shown sufficient for the solution of the model. The variable  $P(z)$  is determined by the iterative solution of the implicit equation  $f(P)$ , defined by:

$$f(P) = P + G_G(U_G)_\alpha + G_L(U_L)_\alpha - \Psi = 0 \quad (26)$$

The solution of the Eq. (26) can be obtained through the secant method, at each step of integration  $\Delta z$ . In others words, to solve the Eq. (22) it's necessary to solve at same time the Eq. (26), which means that for each sub-step of the *RK-4*, the Eq. (26) must be solved. On *Matlab* both methods were implemented/codified for a simultaneous solution, not resorting to any *Matlab* library function.

The mixture model solution is shown in Fig. 2. The input data is assigned at the beginning of the model, with the fluid properties and the geometrical conditions of the pipe. The flow pattern is determined. If the flow is a bubble flow, then the void fraction and friction force are calculated according to Eqs. (16) and (17). Other pattern flow beyond bubble flow will not be addressed in this article. The final computing is the pressure gradient, given by Eqs. (22) and (26).

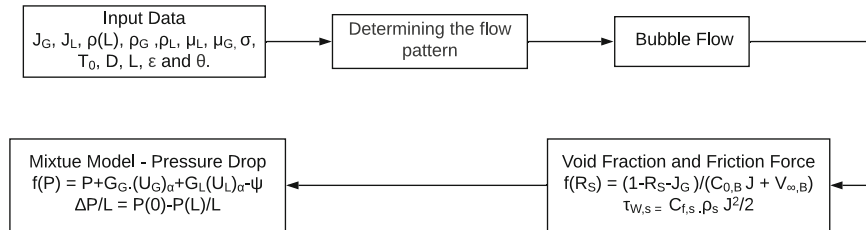


Fig. 2. Mixture model solution flow chart.

The boundary conditions for the solution of the equations are the superficial velocities of the phases  $J_L$ ,  $J_G$  and the mixture pressure  $P$ . It is also necessary to define the flow temperature  $T_0$  and the properties of fluids as the specific masses  $\rho_G$  and  $\rho_L$ , the viscosities  $\mu_G$  and  $\mu_L$  and the interfacial tension  $\sigma$ . For the geometry of the pipe it is necessary to define the inner diameter  $D$ , the length  $L$  and the absolute roughness,  $\varepsilon$ .

## 4 Results and Discussion

The experimental and theoretical study to validate the mixture model with 30 experimental data was performed for different gas and liquid superficial velocities: 6 superficial velocities of liquid  $J_L$  varying from  $1.720 \times 10^{-2}$  to  $10.360 \times 10^{-2}$  m/s; 5 superficial velocities of gas  $J_G$  varying from  $1.234 \times 10^{-2}$  to  $2.072 \times 10^{-2}$  m/s.

### 4.1 Void Fraction

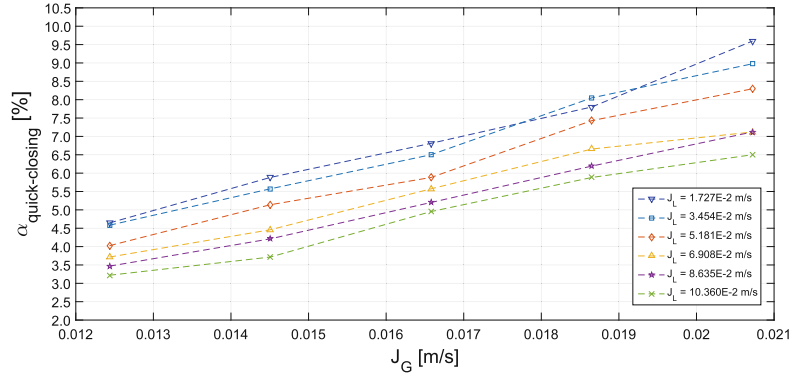
The void fraction analyses were made comparing the two experimental methods described in Sect. 2.2 for the *quick-closing valves* method and *Zuber/Nicklin relation* against the numerical results from the mixture model presented in Sect. 3.2.

The experimental void fraction data show for both methods, Figs. 3 and 4, respectively, for *quick-closing valves* method and *Zuber/Nicklin relation* that when the liquid flow is constant then the void fraction increases with the increase in gas input. For the opposite setup, fixed gas input and increasing liquid flow, the void fraction decreases as the superficial velocity  $J$  of the mixture increases. This is a simple consequence of the higher fluid velocity in the pipe: the faster it moves, the faster it will carry the bubbles upwards. Such two-flow peculiarity was mentioned by [16] and was also observed in all experimental and numerical results. The model accuracy analysis is based on the relative deviation ( $\delta_\Phi$ ), Eq. (27), for the 30 experimental points performed.

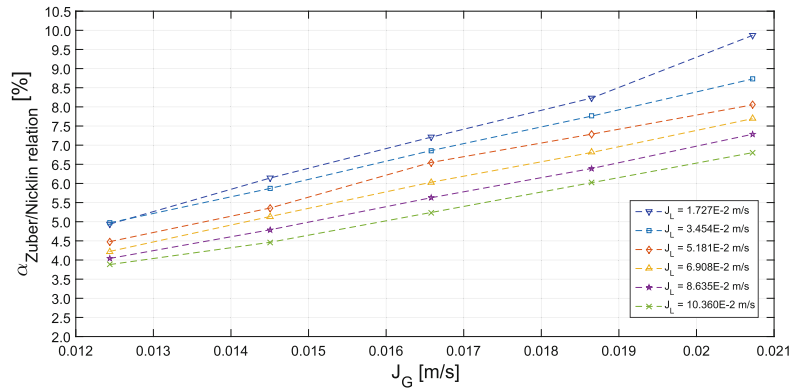
$$\delta_\Phi = 100 \left[ \frac{\Phi_{num} - \Phi_{exp}}{\Phi_{exp}} \right] \quad (27)$$

where  $\Phi$  is a auxiliary variable represented as  $(\Delta P/L)$  or  $(\alpha)$ .

These results are presented in Fig. 5 for the *quick-closing valves* method and Fig. 6 for *Zuber/Nicklin relation*. For the numerical results against the *closing valves* method, Fig. 5, the mean error was 3.32%; the mean minimal error for the lowest gas velocity of  $1.451 \times 10^{-2}$  m/s was 0.46% and mean maximal error was 9.52% for the highest gas velocity of  $2.072 \times 10^{-2}$  m/s. The standard deviation of the relative error was 7.53% and the method shows a large variation of error according to the increase in gas velocity. For lower gas velocities the numerical method underestimates the results in more than 5% while for larger values of gas velocity it overestimates in excess of 13%. This problem may be from the



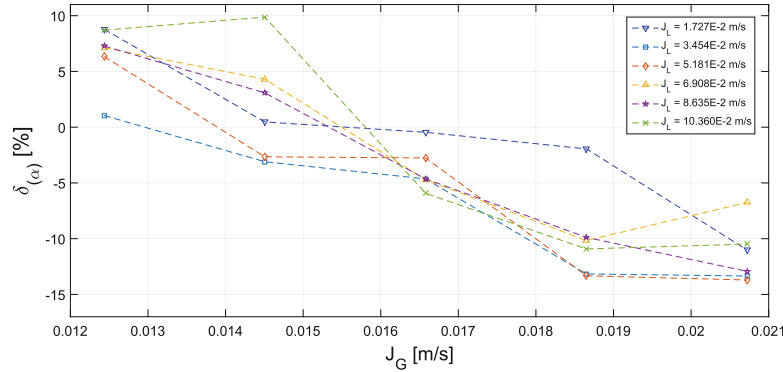
**Fig. 3.** Experimental results of void fraction for all  $J_L$  tested for *quick closing valves* method.



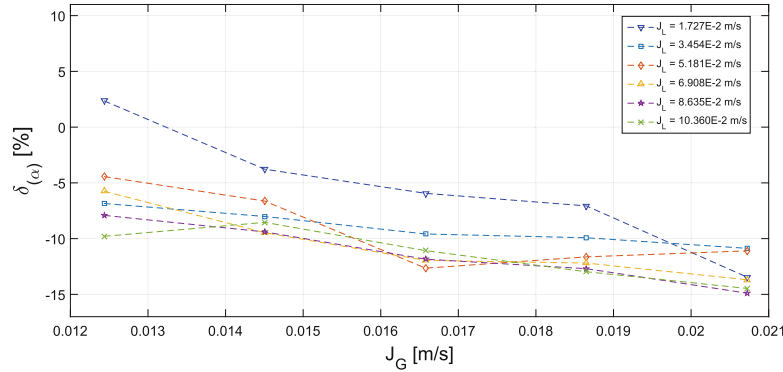
**Fig. 4.** Experimental results of void fraction for all  $J_L$  tested for *Zuber/Nicklin relation*.

difficulty in ensuring the simultaneous manual closing of valves located more than 1.165 m apart. The numerical results against the *Zuber/Nicklin relation* method, Fig. 6, presented a mean error of 9.54%. The error standard deviation was less than half of the *quick-closing valves* method, 3.67% against 7.53%. Apart from one reading, all the numerical results overestimate the void fraction and the dispersion of the prediction is lower.

The bubble distribution parameter, required by this method, was considered  $C_{0,B} = 1$  for the experimental values, while it was individually calculated for every numerical type of flow, presenting an average value of  $C_{0,B,num} = 1.2$ . The experimental value of  $C_{0,B} = 1$  is recommended by [10], and in fact, by calculating the void fraction for the experimental values using the mean numerical values of  $C_{0,B,num} = 1.2$ , yielded much larger error values.



**Fig. 5.** Results for relative errors for all  $J_L$  tested for *quick-closing valves* method.

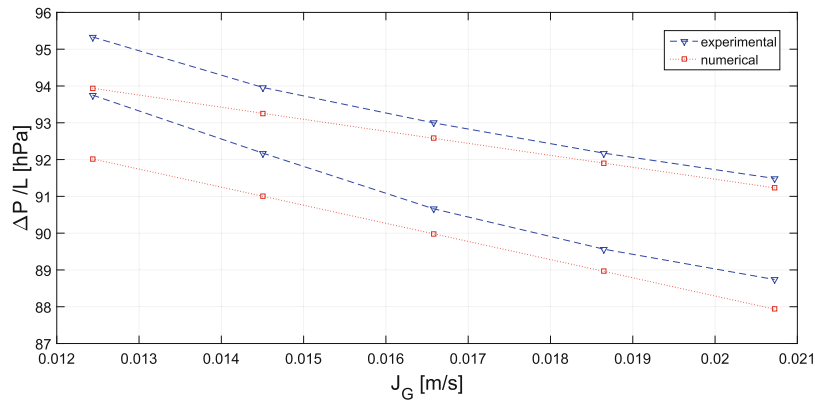


**Fig. 6.** Results for relative errors for all  $J_L$  tested for *Zuber/Nicklin relation*.

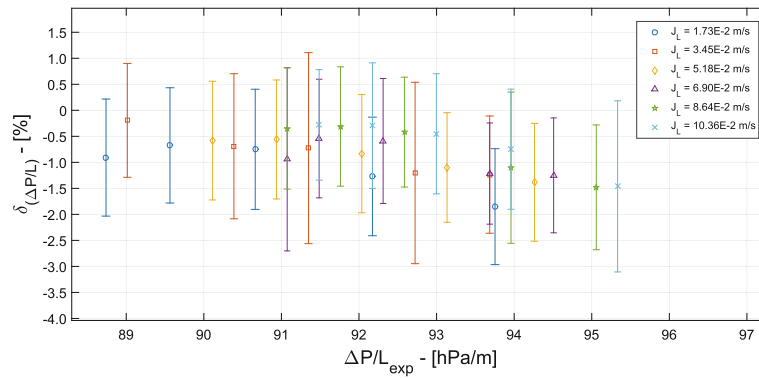
## 4.2 Pressure Gradient

The pressure gradient analysis was made comparing the numerical results against the experimental results with a propagated uncertainty.

The pressure gradient for the lowest and highest fixed superficial liquid velocity and increasing superficial gas velocity are shown in Fig. 7. These curves presents a negative slope, that is, increasing the  $J_G$ , the void fraction also increases and the mixture density decreases also with the pressure gradient. Such feature was observed in all experimental and numerical results. The compared error of the experimental against numerical pressure gradient results, and respective uncertainties of the experimental pressure gradient measurements, are depicted in Fig. 8 for all superficial liquid velocities. The standard deviation of the relative error of the pressure gradient was 9.20%, showing a stable correspondence between experimental and numerical results.



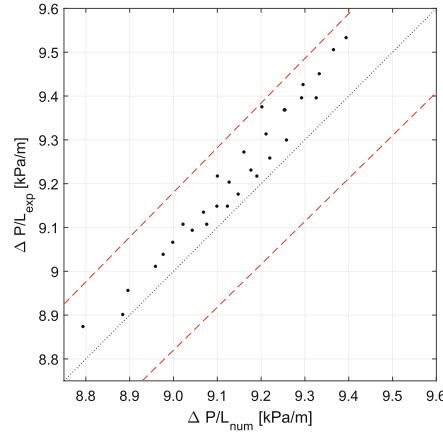
**Fig. 7.** Pressure gradient Experimental vs Numerical.



**Fig. 8.** Relative error from pressure gradient

As shown in Fig. 9, the relative error ( $\delta_{(\Delta P/L)}$ ) for the pressure gradients does not exceed 2.00% and the largest experimental uncertainty was 1.83%. These low values were expected once this flow pattern is strongly influenced by the gravitational force, and dependent by the mixture density and void fraction.

The results were expected since the mixture model tends to make a good prediction of the pressure difference for this type of vertical bubble flow pattern as demonstrated by [9].



**Fig. 9.** Pressure gradient  $\Delta P/L$  comparison for experimental results (y-axis) against numerical results (x-axis), dashed red lines correspond to the upper and lower 2% experimental error.

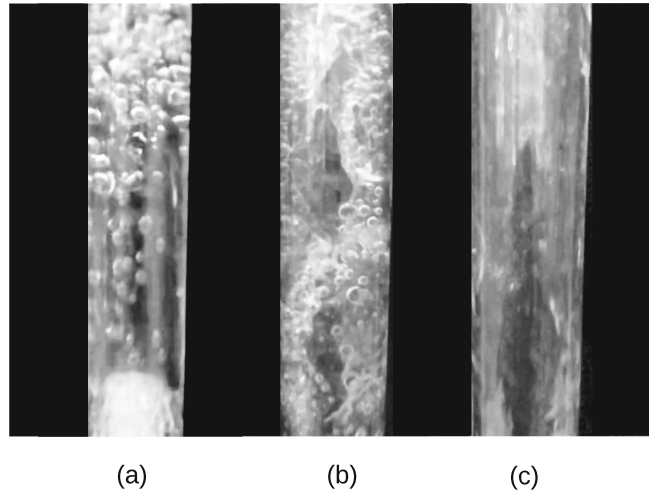
### 4.3 Other Challenges and Improvement Opportunities in the Experimental Apparatus

The *quick-closing valves* method was effective and simple, but its accuracy could be improved with solenoid valves to guarantee the simultaneous closing by only one operator. Also the experimental void fraction determination via the *Zuber/Nicklin relation* method could also be improved by changing the velocity measurement procedure: from a manned chronometer to measure the bubble ascending time to an electronic/optic velocity sensor.

The experimental gradient pressure measurement was simple and reliable when the connection from the hoses between the bubble column and the differential manometer was non-obstructed. Other wise,  $\Delta P/L$  was unreliable.

It was found that the gas rotameter, R3 in Fig. 1(a), is more accurate when operating in medium range, approximately half of its scale. An observable consequence is that the relative percentage error of  $\Delta P/L$  was the lowest when the gas rotameter was in the middle scale, independently of the liquid flow, Sect. 4.2.

Other types of flow patterns may be obtained in this apparatus. The provided examples of slug flow Fig. 10(a), churn Fig. 10(b) or annular Fig. 10(c) were photographed in the IPB's two-flow experimental apparatus. Furthermore, it is also possible to be prepare the apparatus for downward, counter-current, co-current and U-tube flow research.



**Fig. 10.** Other two-phase flows captured from IPB's apparatus: (a) Slug flow; (b) Churn flow; (c) Annular flow.

## 5 Conclusions

A vertical, upward air-water experimental and numerical study of bubble flow pattern is presented via 2 relevant parameters: the pressure gradient and the void fraction. The two-phase flow conditions were: one dimensional, steady-state, isothermal, no phase transition, no mass transfer and constant specific mass and viscosity.

The mixture model was the numerical solution implemented to obtain the theoretical pressure gradient and void fraction. Its solution is based on analytical equation to predict closing parameters.

The experimental results of the pressure gradient were obtained with a differential manometer and the void fraction was obtained with two methods: *quick-closing valves* method and *Zuber/Nicklin relation*. The comparison of the experimental against numerical parameters was performed for 30 pair of superficial velocities. The relative error for pressure gradient envelope was inferior to 2.00% and the largest experimental uncertainty measured was 1.83%. The results of void fraction with *quick-closing valves* method had a medium relative error of 3.32% and for *Zuber/Nicklin relation* was 9.54%.

The newly rebuild laboratory apparatus is reliable, although it presents conditioning set-up particularities regarding operational procedures and fine-tuning, that have to be accounted for.



## References

1. Huang, F., Takahashi, M., Guo, L.: Pressure wave propagation in air-water bubbly and slug flow. *Prog. Nucl. Energy* **47**, 648–655 (2005)
2. Hanafizadeh, P., Ghanbarzadeh, S., Saidi, M.H.: Visual technique for detection of gas-liquid two-phase flow regime in the airlift pump. *J. Pet. Sci. Eng.* **75**, 327–335 (2011). Elsevier BV
3. Kassab, S.Z., Kandil, H.A., Warda, H.A., Ahmed, W.H.: Air-lift pumps characteristics under two-phase flow conditions. *Int. J. Heat Fluid Flow* **30**, 88–98 (2009)
4. Schlegel, J., et al.: Void fraction and flow regime in adiabatic upward two-phase flow in large diameter vertical pipes. *Nucl. Eng. Des.* **239**(12), 2864–2874 (2009). Elsevier BV
5. Chalgeri, V.S., Jeong, J.H.: Flow patterns of vertically upward and downward air-water two-phase flow in a narrow rectangular channel. *Int. J. Heat Mass Transf.* **128**, 934–953 (2019). Elsevier BV
6. Szalinski, L., et al.: Comparative study of gas-oil and gas-water two-phase flow in a vertical pipe. *Chem. Eng. Sci.* **65**(12), 3836–3848 (2010). Elsevier BV
7. Xue, Y., et al.: Investigation on the void fraction of gas-liquid two-phase flows in vertically-downward pipes. *Int. Commun. Heat Mass Transf.* **77**, 1–8 (2016). Elsevier BV
8. Taitel, Y., Bornea, D., Dukler, A.E.: Modelling flow pattern transitions for steady upward gas-liquid flow in vertical tubes. *AIChE J.* **26**, 345–354 (1980)
9. Lima, L.E.M.: Análise do Modelo de Mistura Aplicado em Escoamentos Isotérmicos Gás-Líquido, Ph.D. thesis, Universidade Estadual de Campinas (2011)
10. Pauchon, C., et al.: Tacite: a transient tool for multiphase pipeline and well simulation. In: Society of Petroleum Engineers (1994)
11. Zuber, N., Findlay, J.A.: Average volumetric concentration in two-phase flow systems. *J. Heat Transf.* **87**(4), 453–468 (1965)
12. Wu, B., Firouzi, M., Mitchell, T., Rufford, T.E., Leonardi, C., Towler, B.: A critical review of flow maps for gas-liquid flows in vertical pipes and annuli. *Chem Eng J.* **326**, 350–377 (2017). <https://doi.org/10.1016/j.cej.2017.05.135>
13. Nicklin, D.J.: Two-phase bubble flow. *Chem. Eng. Sci.* **17**(9), 693–702 (1962). ISSN 0009-2509
14. Ishii, M.: Thermo-Fluid Dynamic Theory of Two-Phase Flow. Eyrolles, Paris, France: [s.n.]. Collection de la Direction des Etudes et Recherches d'Electricite de France (1975)
15. Dukler, A.E., Wicks, M., Cleveland, R.G.: Frictional pressure drop in 2-phase flow: B. An approach through similarity analysis. *AIChE J.* **10**(1), 44–51 (1964). ISSN 0001-1541
16. Wallis, G.B.: One-Dimensional Two-Phase Flow. McGraw-Hill Book, New York (1969). [S.l.]