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The prediction of fatigue life using the $k_1$-$k_2$ relationship

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**ABSTRACT:** Fatigue resistance is used in the analysis and design of pavements to predict their life cycle. It is evaluated through time consuming laboratory tests, mainly when performed at very low strain levels. At low strain levels the testing time can last more than one day. Due to the heterogeneity of the material, a large number of samples are tested during days or weeks. The results of fatigue tests are expressed in terms of the number of cycles for the tensile strain level applied. Two constants ($k_1$ and $k_2$) obtained from a statistical analysis take part in this relationship. To know these two constants, at least two fatigue tests are needed, performed at different strain levels. $k_1$ and $k_2$ can be correlated and, in this case, the relationship between the fatigue life and the strain level has only one constant, which can be evaluated using the results obtained by a fatigue test. This paper presents the evaluation of the $k_1$ and $k_2$ relationship for Portuguese mixtures based on the results of 32 different asphalt mixtures composed by four different types of aggregate gradations.

1 **INTRODUCTION**

Fatigue resistance is used in the analysis and design of pavements to predict the pavement life. It is evaluated through laboratory tests which are time consuming when tested at low and very low strain levels. This type of test can last for more than one day. Due to the heterogeneity of the material, a large number of samples are tested during days or even weeks. The results of fatigue tests are expressed in terms of the number of cycles for the tensile strain level applied. Two constants ($k_1$ and $k_2$), obtained from a statistical analysis, are involved in this relationship. These constants correspond to the interception and slope of the fatigue line in the log-log scale. Two fatigue tests, which are performed at different strain levels, are needed to obtain those constants. For each strain level a certain number of specimens were tested to ensure a correct characterization of fatigue.

There are some studies where the coefficients of the fatigue law ($k_1$ and $k_2$) are correlated and, in this case, the relationship between the fatigue life and the strain level has only one constant which can be evaluated using the results of the fatigue test for one strain level.

This relationship, and consequently, a fatigue life law with only one parameter, if existing, can be used for preliminary studies and obtained with very short fatigue tests which last no more than a couple of hours.

However, this approach does not replace the normal procedure to characterize the fatigue life of an asphalt mixture, where at least two strain levels are tested.

This paper presents the evaluation of the $k_1$ and $k_2$ relationship for Portuguese mixtures based on the results of 32 different asphalt mixes composed by four different types of aggregate gradations. The 32 mixtures include 3 wearing course mixes, 9 mixes used in binder course layers, 16 mixtures used in base layers and 6 mixtures with high stiffness binder (10/20 penetration bitumen). The mixtures for binders and base layers use conventional bitumen, whereas the mixtures for wearing courses use both conventional and modified binders. The binder content is identical in each group of mixtures.
2 FATIGUE

The fatigue resistance of an asphalt mixture refers to its ability to withstand repeated bending without reaching fracture. Fatigue, a common form of distress in asphalt pavements, manifests itself in the form of cracking from repeated traffic loading. It is important to have measurements of the fatigue characteristics of specific mixtures over a range of traffic and environmental conditions, so that fatigue considerations can be incorporated into the process of designing asphalt pavements. The fatigue characteristics of asphalt mixtures are usually expressed as relationships between the initial stress or strain and the number of load repetitions to failure, determined by using repeated flexure, direct tension, or diametral tests performed at several stress or strain levels (Tayebali et al., 1994).

Fatigue tests are carried out in two modes, controlled strain and controlled stress. In the controlled strain mode, the strain is kept constant by decreasing the stress during the test, whereas in the controlled stress mode the stress is maintained constant, what increases the strain during the test. In general, controlled stress testing has been related to relatively thick pavement constructions where high stiffness is the fundamental parameter that strengthens fatigue life. Controlled strain testing, on the other hand, has been associated with thin conventional flexible pavements where the elastic recovery properties of the material have a fundamental effect on its fatigue life (Artamendi et al., 2004).

The fatigue behaviour of a specific mixture can be characterized by the slope and relative level of the stress or strain versus the number of load repetitions to failure (N) and can be defined by a relationship of the following form proposed by Monismith et al. (1971), in Equation 1:

\[ N = k_1 \times \left( \frac{1}{\varepsilon_t;\sigma_t} \right)^{k_2} \]  

(1)

Where \( N \) is the number of repetitions to failure; \( \varepsilon_t;\sigma_t \) are tensile strain and stress applied; \( k_1 \) and \( k_2 \) are experimentally determined coefficients.

The coefficients \( k_1 \) and \( k_2 \) correspond to the intercept and slope of the fatigue line in the log-log scale. At least, two fatigue tests are needed to obtain them, mainly performed at different strain levels. For each strain level a certain number of specimens were tested to ensure a correct characterization of fatigue.

However, Monismith and Salam (1972) state that Pell suggests that there exists a relationship between \( k_1 \) and \( k_2 \) in the form:

\[ k_2 = f(\log(k_1)) \]  

(2)

Lytton et al. (1993) also stated that, based on fracture mechanics, parameter \( k_1 \) is a function of \( k_2, A \) (a parameter that defines the cracking propagation rate), and \( E \) (stiffness modulus).

More recently, Zhou et al. (2007) state that \( k_1 \) and \( k_2 \) are correlated by the following relationship:

\[ k_1 = 10^{6.97 -3.20k_2} - 0.873 \]  

(3)

3 ASPHALT MIXTURES

This study was carried out by analyzing 32 different asphalt mixtures tested through four-point bending tests at a temperature of 20ºC and a frequency of 10 Hz. The 32 mixtures included 3 wearing course mixtures, 9 mixtures used in binder course layers, 16 mixtures used in base layers and 6 mixtures with high stiffness binder (10/20 penetration bitumen). The mixtures for binder and base layers use conventional bitumen, whereas the mixtures for wearing courses use both conventional and modified binders. The binder content is identical in each group of mixtures.

Each mixture was tested using the AASHTO T321-03 standard, which establishes testing at least 6 specimens in two or 4 tensile strain levels. Some mixtures were tested using 18 specimens at 3 strain levels, as recommended by the European standard - EN 12697-26.
The results obtained for the 32 mixtures were expressed in the terms represented in Equation 4, which relates fatigue life to the tensile strain applied:

$$N = k_1 \times \left( \frac{1}{\epsilon_t} \right)^{k_2}$$  \hspace{1cm} (4)$$

where $N$ is the number of repetitions to failure; $\epsilon_t$; $\sigma_t$ are tensile strain applied and stress applied; $k_1$ and $k_2$ are experimentally determined coefficients.

The $k_1$ and $k_2$ coefficients, as well as the correlation coefficient ($R^2$) and the $N_{100}$, the number of cycles at a strain level of 100E-6 and the $\epsilon_6$, the strain level for a fatigue life of 1E6, are presented in Table 1.

The analysis of the results from Table 1 allows concluding that the correlation coefficients for all mixtures are extremely high, proving the best fit obtained for the fatigue models. It can be also observed that the mixtures for a specific layer present fatigue behaviors expressed by the different values of the $N_{100}$ and $\epsilon_6$. The only parameter that seems to have a relatively constancy is the slope of the fatigue model ($k_2$).

### Table 1. Fatigue test results

<table>
<thead>
<tr>
<th>Layer</th>
<th>Mixture</th>
<th>Intercept ($k_1$)</th>
<th>Slope ($k_2$)</th>
<th>$R^2$</th>
<th>$N_{100}$</th>
<th>$\epsilon_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wearing course</strong></td>
<td>Mix 02</td>
<td>1.591E+17</td>
<td>4.430</td>
<td>0.984</td>
<td>2.2E+08</td>
<td>338</td>
</tr>
<tr>
<td></td>
<td>Mix 13</td>
<td>2.576E+18</td>
<td>4.762</td>
<td>0.983</td>
<td>7.7E+08</td>
<td>404</td>
</tr>
<tr>
<td></td>
<td>Mix 17</td>
<td>4.784E+17</td>
<td>4.927</td>
<td>0.994</td>
<td>6.7E+07</td>
<td>235</td>
</tr>
<tr>
<td><strong>Binder course</strong></td>
<td>Mix 01</td>
<td>8.371E+18</td>
<td>5.021</td>
<td>0.990</td>
<td>7.6E+08</td>
<td>375</td>
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<tr>
<td></td>
<td>Mix 02</td>
<td>1.591E+17</td>
<td>4.430</td>
<td>0.984</td>
<td>2.2E+08</td>
<td>338</td>
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<tr>
<td></td>
<td>Mix 03</td>
<td>2.076E+17</td>
<td>4.468</td>
<td>1.000</td>
<td>2.4E+08</td>
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</tr>
<tr>
<td></td>
<td>Mix 06</td>
<td>5.050E+18</td>
<td>4.907</td>
<td>0.972</td>
<td>7.8E+08</td>
<td>388</td>
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<td>Mix 09</td>
<td>7.837E+16</td>
<td>4.314</td>
<td>0.997</td>
<td>1.8E+08</td>
<td>335</td>
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<tr>
<td></td>
<td>Mix 13</td>
<td>2.576E+18</td>
<td>4.762</td>
<td>0.983</td>
<td>7.7E+08</td>
<td>404</td>
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<td></td>
<td>Mix 15</td>
<td>6.213E+20</td>
<td>5.604</td>
<td>0.983</td>
<td>3.9E+09</td>
<td>437</td>
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<tr>
<td></td>
<td>Mix 16</td>
<td>1.563E+19</td>
<td>5.399</td>
<td>0.995</td>
<td>2.5E+08</td>
<td>278</td>
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<tr>
<td></td>
<td>Mix 23</td>
<td>1.410E+15</td>
<td>4.067</td>
<td>0.976</td>
<td>1.0E+07</td>
<td>178</td>
</tr>
<tr>
<td><strong>Base course</strong></td>
<td>Mix 04</td>
<td>7.977E+17</td>
<td>4.717</td>
<td>0.980</td>
<td>2.9E+08</td>
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<tr>
<td></td>
<td>Mix 07</td>
<td>4.628E+16</td>
<td>4.316</td>
<td>0.960</td>
<td>1.1E+08</td>
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<tr>
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<td>Mix 08</td>
<td>1.419E+16</td>
<td>4.210</td>
<td>0.996</td>
<td>5.4E+07</td>
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<td>Mix 10</td>
<td>2.823E+17</td>
<td>4.728</td>
<td>0.979</td>
<td>9.9E+07</td>
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<td>Mix 11</td>
<td>1.074E+13</td>
<td>3.311</td>
<td>0.968</td>
<td>2.6E+06</td>
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<tr>
<td></td>
<td>Mix 12</td>
<td>2.353E+15</td>
<td>4.121</td>
<td>0.977</td>
<td>1.3E+07</td>
<td>188</td>
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<tr>
<td></td>
<td>Mix 14</td>
<td>4.297E+16</td>
<td>4.510</td>
<td>0.993</td>
<td>4.1E+07</td>
<td>228</td>
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<td></td>
<td>Mix 18</td>
<td>7.865E+18</td>
<td>5.140</td>
<td>0.995</td>
<td>4.1E+08</td>
<td>323</td>
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<td></td>
<td>Mix 19</td>
<td>2.060E+16</td>
<td>4.268</td>
<td>0.959</td>
<td>6.0E+07</td>
<td>261</td>
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<tr>
<td></td>
<td>Mix 20</td>
<td>1.103E+15</td>
<td>3.948</td>
<td>0.954</td>
<td>1.4E+07</td>
<td>195</td>
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<td>Mix 24</td>
<td>5.833E+13</td>
<td>3.552</td>
<td>0.949</td>
<td>4.6E+06</td>
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<tr>
<td></td>
<td>Mix 25</td>
<td>3.785E+13</td>
<td>3.341</td>
<td>0.990</td>
<td>5.2E+06</td>
<td>162</td>
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<td></td>
<td>Mix 26</td>
<td>1.078E+14</td>
<td>3.584</td>
<td>0.986</td>
<td>7.3E+06</td>
<td>174</td>
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<td></td>
<td>Mix 27</td>
<td>6.720E+17</td>
<td>4.714</td>
<td>0.968</td>
<td>2.5E+08</td>
<td>323</td>
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<td></td>
<td>Mix 29</td>
<td>2.228E+15</td>
<td>4.146</td>
<td>0.984</td>
<td>1.1E+07</td>
<td>180</td>
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<tr>
<td></td>
<td>Mix 31</td>
<td>7.772E+13</td>
<td>3.652</td>
<td>0.992</td>
<td>3.9E+06</td>
<td>145</td>
</tr>
<tr>
<td><strong>Base with high stiffness modulus</strong></td>
<td>Mix 05</td>
<td>1.113E+23</td>
<td>6.452</td>
<td>0.995</td>
<td>1.4E+10</td>
<td>439</td>
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<tr>
<td></td>
<td>Mix 21</td>
<td>8.474E+14</td>
<td>3.873</td>
<td>0.997</td>
<td>1.5E+07</td>
<td>202</td>
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<td></td>
<td>Mix 22</td>
<td>3.402E+15</td>
<td>3.975</td>
<td>0.995</td>
<td>3.8E+07</td>
<td>250</td>
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<td></td>
<td>Mix 28</td>
<td>4.511E+20</td>
<td>5.711</td>
<td>0.998</td>
<td>1.7E+09</td>
<td>368</td>
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<tr>
<td></td>
<td>Mix 30</td>
<td>1.437E+21</td>
<td>5.866</td>
<td>0.993</td>
<td>2.7E+09</td>
<td>384</td>
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<tr>
<td></td>
<td>Mix 32</td>
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<td>4.452</td>
<td>0.991</td>
<td>9.4E+07</td>
<td>277</td>
</tr>
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</table>
4 FATIGUE LIFE OF TESTED ASPHALT MIXTURES

The fatigue life laws of the mixtures tested were plotted in charts which related the fatigue life to the tensile strain in order to observe the quality of the fit obtained as well as the trend presented by the fatigue laws for each group of mixtures.

The first plot of the fatigue laws was made for wearing course mixtures (Figure 1), where it can be observed that the slope of the fatigue laws are the same for all mixtures, in spite of the differences presented in terms of fatigue life. The results show that there exists more than one order of scale between these 3 mixtures. This fact is mainly due to the type of aggregate gradation and the binder used in the mixture.

![Figure 1. Fatigue life of wearing course mixtures](image)

The plot of the fatigue laws for binder course mixtures (Figure 2) also depicts identical slopes for the fatigue law and a large dispersion between the fatigue laws. However, 3 different groups can be observed. The first one, with 7 mixtures, has the same slope and identical interception parameter ($k_1$). The other groups, with only one mixture each, correspond to mixtures with different design parameters.

![Figure 2. Fatigue life of binder course mixtures](image)

The plot of the base course mixtures (Figure 3) presents the largest dispersion of the fatigue
laws, although the mixes present identical slopes. The different interceptions are due to the aggregate gradation and binder used in the mixture design. The different aggregate types also contribute to this dispersion.

Figure 3. Fatigue life of base course mixtures

The results of the mixtures with a high stiffness modulus (10/20 penetration bitumen), depicted in Figure 4, present a unique situation in which the slope of the fatigue laws are not identical. Two groups of mixtures can be observed, each one with a different slope of the fatigue law. This difference produces completely diverse fatigue lives for low strain levels where the fatigue laws diverge.

Figure 4. Fatigue life of mixtures with high stiffness modulus
5 RELATIONSHIP BETWEEN K1 AND K2

Bearing in mind the form of the relationship between \( k_1 \) and \( k_2 \) proposed by Pell, Figures 5 to 7 present the relationship between \( k_1 \) and \( k_2 \) for the binder course mixtures, the base course mixtures and the high stiffness modulus mixtures, respectively. The relationship between \( k_1 \) and \( k_2 \) for wearing course mixtures was not presented because it only had 3 mixtures with different type of aggregate gradations and binder, including modified binders.

The analysis of these figures allows verifying an excellent relationship between \( k_1 \) and \( k_2 \) for all mixtures. The existence of points which do not follow the general trend indicates that the mixture is different from the others. This case can be verified in Figure 5, where there are two points that do not follow the trend. These mixtures, including those for binder layers, present different design parameters in relation to the other mixtures.

The relationship for base course mixtures, probably due the large number of mixtures considered with different design parameters, presents a small dispersion around the trend line (Figure 6).

The best fit in this work was established by high stiffness modulus mixtures, as there are 6 points (6 mixtures in this group) that follow exactly the trend line defined (Figure 7).

![Figure 5. Relationship between k1 and k2 for binder course mixtures](image1)

\[
y = 0.1311 \ln(x) - 0.68 \\
R^2 = 0.9265
\]

![Figure 6. Relationship between k1 and k2 for base course mixtures](image2)

\[
y = 0.1303 \ln(x) - 0.5773 \\
R^2 = 0.9825
\]
Although the development of the relationship between $k_1$ and $k_2$ should be carried out for each type of mixture or as function of the mixture design because Lytton et al. (1993) stated that the relationship is dependent of the mixture stiffness, the plot of the $k_1$ and $k_2$ coefficients for all mixture was also made and is presented in Figure 8. The analysis of this mixture indicates that a simple global relationship between $k_1$ and $k_2$ exists.

However, discrepancies between some points and the trend line are evident, but this can constitute a first approach to define a relationship between $k_1$ and $k_2$.

Based on the present work, a model is proposed for the relationship between $k_1$ and $k_2$ as:

$$k_2 = 0.1325 \ln(k_1) - 0.6815$$

(5)

In this case, Equation 4 can be written as:

$$N = k_1 \left( \frac{1}{e} \right)^{0.1325 \ln(k_1) - 0.6815}$$

(6)
The determination of the coefficient of Equation 6 only requires one pair of values \((N, \varepsilon)\). This pair of values must correspond to a series of fatigue testing at a unique strain level, a high strain level is preferred, which takes short testing time.

In spite of the excellent correlation obtained between \(k_1\) and \(k_2\) for any type of mixtures and for all the tested mixtures, it is important to verify the fatigue life obtained by the application of those relationships and the influence in the pavement life calculation.

6 CONCLUSIONS

This paper presented the evaluation of the \(k_1\) and \(k_2\) relationship for Portuguese mixtures based on the results obtained from 32 different asphalt mixtures composed by four different types of aggregate gradations.

Relationships between \(k_1\) and \(k_2\) developed for the different group of mixes suggest that they can be used to predict the fatigue life of asphalt mixtures. A relationship for all the tested mixtures in this work was developed showing a good correlation between \(k_1\) and \(k_2\).

The model can be used to produce fatigue testing with only one strain level, once there is only one unknown parameter in the fatigue law \((k_1)\).

In spite of the excellent correlation obtained between \(k_1\) and \(k_2\) for any type of mixtures and for all the tested mixtures, it is important to validate the fatigue life obtained by the application of those relationships and the influence in the pavement life calculation.

7 REFERENCES


