Mathematical model for the assessment of fracture risk associated with osteoporosis

Jairson Dinis, Ana I. Pereira, and Elza M. Fonseca

Citation: AIP Conf. Proc. 1479, 814 (2012); doi: 10.1063/1.4756262
View online: http://dx.doi.org/10.1063/1.4756262
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1479&Issue=1
Published by the American Institute of Physics.

Related Articles
Continuous-waveform constant-current isolated physiological stimulator

A new device for performing reference point indentation without a reference probe

A numerical analysis of multicellular environment for modeling tissue electroporation

Some observations on the mechanics and dynamics of tumor heterogeneity
AIP Advances 2, 011001 (2012)

Roles of silica and lignin in horsetail (Equisetum hyemale), with special reference to mechanical properties

Additional information on AIP Conf. Proc.
Journal Homepage: http://proceedings.aip.org/
Journal Information: http://proceedings.aip.org/about/about_the_proceedings
Top downloads: http://proceedings.aip.org/dbt/most_downloaded.jsp?KEY=APCPCS
Information for Authors: http://proceedings.aip.org/authors/information_for_authors

Explore AIP’s new open-access journal
- Article-level metrics now available
- Join the conversation! Rate & comment on articles

Submit Now
Mathematical Model for the Assessment of Fracture Risk
Associated with Osteoporosis

Jairson Dinis*, Ana I. Pereira*,† and Elza M. Fonseca*,**

*Polytechnic Institute of Bragança, Portugal
†ALGORITMI, University of Minho, Portugal
**CENUME, FEUP, Portugal

Abstract. Osteoporosis is a skeletal disease characterized by low bone mass. It is considered a worldwide public health
problem that affects a large number of people, in particular for women with more than 50 years old. The occurrence pattern
of osteoporosis in a population may be related to several factors, including socio-economic factors such as income, educational
attainment, and factors related to lifestyle such as diet and physical activity. These and other aspects have increasingly been
identified as determining the occurrence of various diseases, including osteoporosis. This work proposes a mathematical
model that provides the level of osteoporosis in the patient. Preliminary numerical results are presented.

Keywords: Nonlinear optimization. Risk of fracture. Osteoporosis.
PACS: 02.60.Pn

INTRODUCTION

Osteoporosis is a skeletal disease characterized by low bone mass, predominant in older adults, particularly in
postmenopausal women [1, 2]. This pathology is characterized by the degeneration of bone microstructure, leading to
increasing bone brittleness and susceptibility to fracture, [3, 4]. Osteoporotic fractures are a major cause of morbidity
and mortality in the population [3, 4, 5, 6]. Particularly, hip fracture, has a deep impact on quality of life, increasing
difficulties regarding the activities of daily life [7, 8]. In this work we analyze the influence of some specific risk factors
related with osteoporosis to identify the level of osteoporosis in the patient.

STUDY METHODOLOGY

For the proposed mathematical model we considered 97 women questionnaires aged over than 60 years old. This is
only valid for densitometry examinations marks obtained in the DEXA equipment and software EnCORE 2004 GE
Medical Systems.

We indicate that the patient belong to the Zone I if the patient has a normal bone, Zone II and III, if the patient has
osteopenia and osteoporosis, respectively. We used the reference curve, for the femur anatomical region, to define the
Zone I, II and III (see Fig. 1, where the dashed lines are the boundaries of each zone). This reference curve is indicated
through the DEXA equipment.

Table 1 presents the average of input variables in each Zone.

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Zona I</th>
<th>Zona II</th>
<th>Zona III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age - years (id)</td>
<td>66.3</td>
<td>70.1</td>
<td>73.0</td>
</tr>
<tr>
<td>Age of menopause - years (idm)</td>
<td>49.5</td>
<td>46.8</td>
<td>46.3</td>
</tr>
<tr>
<td>Body mass index - kg/m² (imc)</td>
<td>29.9</td>
<td>28.0</td>
<td>23.4</td>
</tr>
<tr>
<td>Coffee consumption - n (cof)</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Bone mineral density at the femoral neck -g/cm² (BMD_f)</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>Bone mineral density of the ward -g/cm² (BMD_w)</td>
<td>0.8</td>
<td>0.6</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Analyzing the Table 1 we can conclude that in older patients the risk of osteoporosis increases and observe that for
the early menopause, the risk for developing osteoporosis increases also. We also observe that the body mass index is

© 2012 American Institute of Physics 978-0-7354-1091-6/$30.00

Downloaded 17 Oct 2012 to 188.80.15.236. Redistribution subject to AIP license or copyright; see http://proceedings.aip.org/about/rights_permissions
smaller for the women that are in the Zone III (women with osteoporosis). As we expected, the bone mineral density (at the femoral neck and at the wards) is lower in the women group that has osteoporosis.

In the neighborhood of the reference curves, the proposed mathematical model is not valid. So, the mathematical model is not valid in the following region:

\[ BMD_L = [BMD - \delta, BMD + \delta], \]  

(1)

where the \( \delta \) is the region tolerance and \( BMD \) is the value in the reference curve for the current patient age. Figure 1 represents the Zone I, II and III and the region \( BMD_L \).

\[ f_1(x) = x_1(bmf + bmdw)^2 + x_2 \frac{imc}{30} + x_3 \frac{60}{id} + x_4 \frac{1}{cof} + x_5 \frac{idm}{50} \]  

(2)

\[ f_2(x) = x_1(bmf + bmdw)^2 + x_2 \frac{imc}{30} + x_3 \frac{60}{id} + x_4 \frac{1}{cof} + x_5 \frac{idm}{50} + x_6. \]  

(3)

To identify the optimal solution \( x \), it was solved the following constrained nonlinear optimization problem:

\[ \min_{x} g(x) = \sum_{i=1}^{n} (f_i^j(x) - ts_i)^2 \]

(4)

\[ s.a \]

\[ x_1 \geq x_2 \]

\[ x_2 \geq x_3 \]

\[ x_3 \geq x_4 \]

\[ x_5 \geq x_6. \]

where \( ts_i \) represents the T-score of the patient \( i \), \( f_i^j(x) \), for \( j = 1, 2 \), represents the T-score approximation of the patient \( i \).

The constraints are used to assign different weights to each input variable, risk factors. In this propose, we considered more weight assigned to the BMD of the femoral neck (\( bmf \)) and bone mineral density of the wards (\( bmdw \)), then...
to the body mass index (imc), followed by patient age (id), age at menopause (idm) and finally the coffee consumer (cof).

To identify the optimal parameters, we used three optimization methods from Matlab [9]: Genetic Algorithms (GA), Pattern Search method (PS) and Sequential Quadratic Programming method (SQP).

### NUMERICAL RESULT AND DISCUSSION

To choose the best mathematical model, we use twenty questionnaires. Table 2 shows the objective function values for the three optimization methods.

**TABLE 2.** Objective function values.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>$f_1$</th>
<th>$f_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQP</td>
<td>8.10</td>
<td>1.23</td>
</tr>
<tr>
<td>PS</td>
<td>8.09</td>
<td>1.48</td>
</tr>
<tr>
<td>GA</td>
<td>8.93</td>
<td>3.68</td>
</tr>
</tbody>
</table>

According to the results presented in Table 2 the best function is $f_2$.

The Table 3 presents the results obtained with twenty questionnaires considering the function $f_2$ and different values of $\delta$. The table presents the number of tests that do not belong to the region $BMD_L$ (NTN), the number of questionnaires with a proper identification (NQP), the number of questionnaires with a wrong identification (NQW) and the correct identification rate (CIR).

**TABLE 3.** Numerical results for the function $f_2$.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>Algorithms</th>
<th>NTN</th>
<th>NQP</th>
<th>NQW</th>
<th>CIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>SQP</td>
<td>11</td>
<td>9</td>
<td>2</td>
<td>81.2%</td>
</tr>
<tr>
<td>0.050</td>
<td>PS</td>
<td>11</td>
<td>9</td>
<td>2</td>
<td>81.2%</td>
</tr>
<tr>
<td>0.050</td>
<td>GA</td>
<td>11</td>
<td>8</td>
<td>3</td>
<td>72.7%</td>
</tr>
<tr>
<td>0.025</td>
<td>SQP</td>
<td>13</td>
<td>11</td>
<td>2</td>
<td>84.6%</td>
</tr>
<tr>
<td>0.025</td>
<td>PS</td>
<td>13</td>
<td>11</td>
<td>2</td>
<td>84.6%</td>
</tr>
<tr>
<td>0.025</td>
<td>GA</td>
<td>13</td>
<td>11</td>
<td>2</td>
<td>84.6%</td>
</tr>
<tr>
<td>0.015</td>
<td>SQP</td>
<td>17</td>
<td>14</td>
<td>3</td>
<td>82.23%</td>
</tr>
<tr>
<td>0.015</td>
<td>PS</td>
<td>17</td>
<td>13</td>
<td>4</td>
<td>76.47%</td>
</tr>
<tr>
<td>0.015</td>
<td>GA</td>
<td>17</td>
<td>14</td>
<td>3</td>
<td>82.23%</td>
</tr>
</tbody>
</table>

Table 3 shows that the best result is obtained for $\delta = 0.025$.

To validate the mathematical model twenty new questionnaires of patients aged over 60 years old were used. These data were not used to define the optimal parameters of the mathematical model.

The motivation for using these new questionnaires is to test the reliability of the model. The Table 4 presents the validation results.

**TABLE 4.** Numerical results for function $f_2$ with new questionnaires.

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Algorithms</th>
<th>NTN</th>
<th>NQP</th>
<th>NQW</th>
<th>CIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>SQP</td>
<td>16</td>
<td>16</td>
<td>0</td>
<td>100%</td>
</tr>
<tr>
<td>0.025</td>
<td>PS</td>
<td>16</td>
<td>16</td>
<td>0</td>
<td>100%</td>
</tr>
<tr>
<td>0.025</td>
<td>GA</td>
<td>16</td>
<td>16</td>
<td>0</td>
<td>100%</td>
</tr>
</tbody>
</table>

We can verified in Table 4, that the proposed mathematical model ($f_2$) has 100% of correct identification rate.

### CONCLUSIONS AND FUTURE WORK

Several risk factors for osteoporosis and osteopenia were identified, some of them are considered modifiable, such as, age, coffee consumption and body mass index (imc). The numerical results of the propose mathematical model were satisfactory, since the correct identification rate was 100%. However, the model should be tested with more and new data.
Some ideas for future work are to extend this study to patients under the age of 60 years old and different gender. And propose different mathematical models to approximate the T-score in different anatomical parts of the human body.

ACKNOWLEDGMENTS

The authors would like to thank the financial support from FEDER COMPETE and FCT Project FCOMP-01-0124-FEDER-022674.

REFERENCES